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Stiffness-Scaled Models for Asymptotic Gridhshells: Designing and Prototyping.

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Abstract

Asymptotic gridshells, renowned for their structural efficiency and architectural innovation, pose unique challenges in terms of accurate representation and computational modeling due to their non-linear and geometrically complex nature. Asymptotic curves are one of the of three distinct types of curves next to geodesic curves and principal curvature lines, that exhibit significant potential for representation as developable elements [1]. A method for constructing smoothly curved structures involves utilizing the elastic deformation of building components. This approach aims to transform straight or flat elements into the desired curvilinear geometry [2]. The material selection is challenging, as it needs to not only satisfy the structural requirements, but also to account for the permissible bending and torsion of all strip profiles [3]. The proposed methodology involves the formulation of a stiffness scaling technique that dynamically adjusts the properties of the gridshell model based on its structural behavior. Leveraging advanced computational tools and finite element analysis, our method proposes shell analysis as a realistic representation of any asymptotic gridshell of equivalent volume. Through validation exercises and built protypes throughout a 2-day workshop, the efficiency of the proposed method is demonstrated, showcasing its ability to capture the intricate structural responses of potential asymptotic gridshells by structural analysis of the initial surface. Furthermore, the study explores the implications of stiffness scaling on the design and analysis of asymptotic gridshells, emphasizing its potential impact on structural performance, load distribution, and overall stability. The findings from this research contribute to advancing the understanding of asymptotic gridshells and provide a valuable tool for architects and engineers engaged in the design and optimization of such structures. In conclusion, the developed stiffness-scaled models offer a novel approach to enhance the accuracy and efficiency of computational simulations for asymptotic gridshells, paving the way for improved design methodologies and structural innovations in the field of architectural engineering.

Keywords: Asymptotic gridshell, site-sprung construction, structural optimization, multi-objective optimization, performance-driven design, timber, elastic gridshell, computational design

1. Introduction

Gridshells are recognized for their efficiency, lightweight nature, and sustainability [4], while active-bending systems offer a promising avenue to innovate traditional unstrained gridshell design [5]. Presently, research on elastic gridshells predominantly centers on structural systems constructed using site-sprung methods. These methods not only facilitate cost-effective construction of double curved structures with flat elements but also enable scaffold-free assembly. It is surprising that asymptotic curves have scarcely been employed in load-bearing structures [3], given their ability to integrate the benefits of straight unrolling and site-sprung assembly. Elastic timber gridshells built on-site can span considerable distances at minimal material and construction costs. However, their resulting geometries are often limited, and their topologies remain fixed [4]. Moreover, aside from the geometric requirements, the structural analysis complexity of active-bending gridshells frequently hinders their broader application. Could the geometric and structural feasibility of active bending gridshells be assessed solely through shape analysis? This paper proposes a method for selecting subsurface and determining appropriate thickness based on structural shell analysis, offering a grid-independent approach to ensuring structural feasibility for any asymptotic gridshell with equivalent material volume. Our method operates under the assumption that a gridshell with equal material volume can effectively represent shell behavior. We suggest defining asymptotic grids and member sizes based on shell structural analysis, simplifying considerations across various grid variations by defining structural efficiency in terms of overall surface shape and material. We hypothesize that asymptotic 2-directional grids utilizing an equal amount of material with frame connection after assembly mirror continuum shell behavior. This paper outlines the proposed method and presents the construction and testing of two prototypes at a 1:10 scale to validate the hypothesis.

2. Literature review

2.1. Summary of previous research

The mechanical behavior of elastic slats within a grid exhibits a captivating alignment with the geometric properties of curves on surfaces. Remarkably, some of these properties were elucidated as far back as 1897 by the mathematician Sebastian Finsterwalder [5]. Designing asymptotic gridshells involves the generation of asymptotic grids, a process that has been under study for several years. The geometric prerequisites and the method of generating asymptotic curves have been subjects of research. Schling [3] has extensively elaborated on the generation of asymptotic grids on form found and mathematically minimal surfaces. To determine the curvature of a surface, we intersect it with orthogonal planes through the normal vector. The resulting section-curves reveal the principal curvature directions, with the two curves exhibiting the highest and lowest curvature being perpendicular to each other. These principal curvatures, denoted as k_1 and k_2, are utilized to compute the Gaussian curvature ($G = k_1 \times k_2$) and mean curvature ($H = (k_1 + k_2)/2$). When k_1 and k_2 have opposite orientations, meaning their osculating circles lie on opposite sides of the surface, the Gaussian curvature takes on a negative value, characterizing the surface-region as anticlastic or hyperbolic. In cases where k_1 and k_2 possess opposite orientations but equal absolute values, the mean curvature becomes zero. Surfaces with a constant zero mean curvature are referred to as minimal surfaces, constituting a specific type of anticlastic surfaces often observed in nature, such as in the form of soap films [6]. A comprehensive review of the geometric, structural, architectural and aesthetic requirements for design of asymptotic gridhsells is summarized in the Table below [6][7].

Table 1: Design requirements for asymptotic gridshells.

Requirement	**Surface Property**	**Surface Requirement**	**Design Stage**
possible asymptotic grid	anticlastic surface	G < 0 (negative Gaussian curvature) $G = k_1 \times k_2$	base surface selection
Asymptotic grid with perpendicular joints, membrane behavior	(homogenously curved) minimal surface with well positioned singularities	H = 0 (Mean curvature of zero) $H = (k_1 + k_2)/2$ G = 0 where singularities	
structurally efficient	doubly curved surface, well-positioned supports, efficient arched edges		subsurface selection (cutting out part of the base surface), material selection
Requirement	**Grid Property**	**Grid Requirements**	**Design Stage**
constructible	considerable geodesic curvature and geodesic torsion	checks: max bend< allow. bend: $r_1 < r_{1\,max}$ max torsion<allow. torsion: $r_2 < r_{2\,max}$	grid generation, material selection
aesthetic	aesthetic curve network	G = 0 where singularities homogenous curvature?	

Several case studies of asymptotic gridshells base on the anticlastic surfaces: catenoid [3] modified catenoid [6] as well as minimal surfaces Enneper class 3 [3][6][7] and Enneper class 2 [7] has been conducted proving their geometric feasibility to form asymptotic gridshells. The recent research has been enhanced with studies on the kinetic behavior of asymptotic gridshells [9]. Schling [10] has conducted studies on hybrid networks comprising both asymptotic and geodesic paths, expanding grids to double layer and introducing steel as the construction material [3].

2.2. Problem statement

Despite the ability for asymptotic gridshells to cover large spans, with use of a minimal amount of material [11] and a possibility for quick on-site assembly, there are only a few realizations of asymptotic gridhsells as structural systems. One of the obstacles to broader adoption is the geometric complexity of the generation of asymptotic grids and verification of their material and structural soundness. While there are established methods for assessment of geometric feasibility of base surfaces for generation of asymptotic grids, there's a lack methods allowing for selection of structurally promising subsurfaces. We hypnotize that structural efficiency asymptotic grids can be solely assessed by the structural analysis of the base shape. If our hypothesis holds true, grid-independent shell analysis could serve as a primary performance indicator for performance of any asymptotic grid of qual material volume. This approach can reduce the need for sophisticated structural analysis of asymptotic grids variations to single shell, analysis therefore can streamline asymptotic gridshell design and facilitate their practical adoption.

3. Method

3.1. Subsurface Selection

As a base surface the mathematical minimal surface Enneper 3 was selected as suitable for generation of asymptotic grid. The selection of the subsurface was driven by the architectural considerations of minimum height specifying the position of the initial cutting XY plane at height of 2.6m. Subsequently, geodesic curves were computed to connect the flat bottom beams. These generated outlines were then utilized to divide the initial base surface of Enneper 3, enabling the selection of a subsurface featuring flat and elongated bottom support lines, along with edge arches that could be constructed as geodesic beams (Fig. 1).

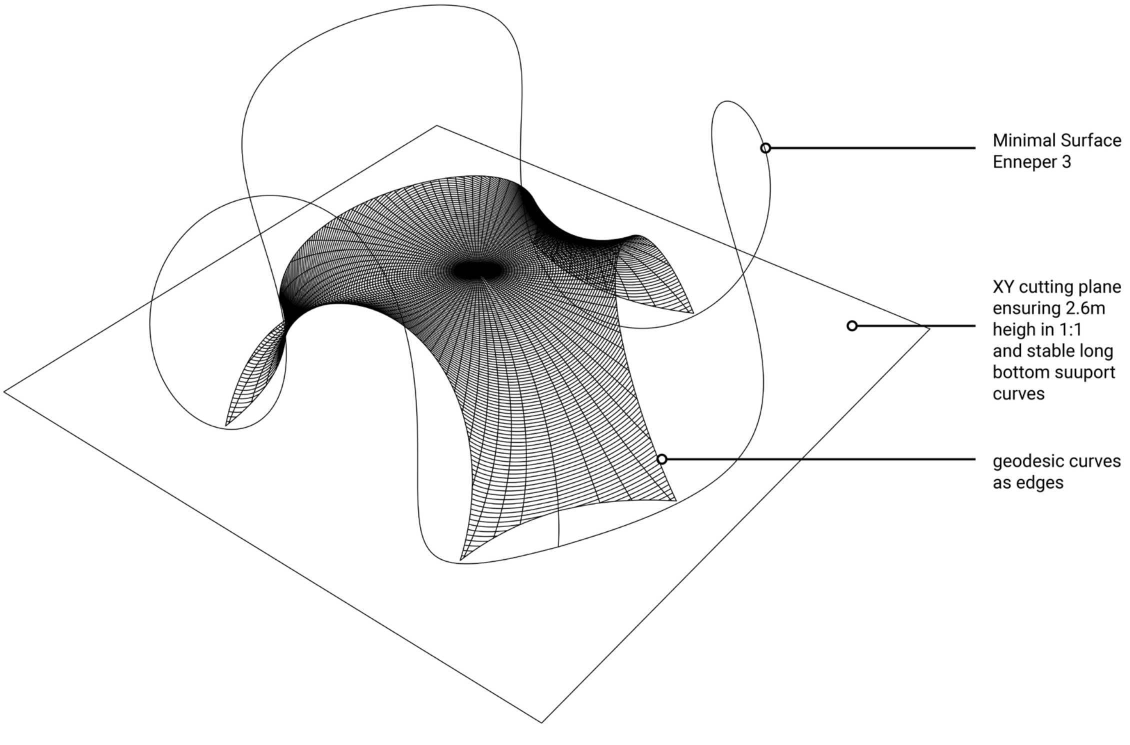


Figure 1: Selecting a subsurface from Minimal Surface Enneper 3.

3.2. Continuum Structural Analysis

To evaluate the structural soundness of the proposed asymptotic gridshells based on the selected subsurface, we employ the equivalent continuum technique [12]. According to this method, a grid-shell can be represented by a continuous shell with the equivalent thickness, which volume satisfies the following relation:

$$\mathrm{V}_{cont} = \mathrm{V}_{grid} \tag{1}$$

$$\mathrm{V}_{grid} = \sum_{n=0}^{n} \mathrm{A}_n \mathrm{l}_n \tag{2}$$

$$\mathrm{V}_{cont} = \mathrm{t}_{cont}\, S \tag{3}$$

Where:
t_{cont} - thickness of the continuum shell
S – surface area of the shell
n – number of laths in gridshell
A – cross section area of the lath
l – length of the lath

The gridshell density is defined as the proportion of the members lengths per m2 of the surface. Let ρ_{grid} be given by:

$$\rho_{grid} = \frac{\sum_{n=0}^{n} \mathrm{l}_n}{S} \tag{4}$$

3.3. Defining structurally sound shell thickness

The shell structural analysis was conducted with Karamba3d. assuming the subsurface is a shell of constant thickness, uniformly supported at all edges (Fig.2), made with birch plywood with the following properties: Elasticity Modulus (**E**) = 8750MPa, Bending Strength, Density (**ϱ**) = 450kg/m3, Compressive Strength (fco) = 38MPa. For this project structurally sound shell is characterized by at L0: 1.4 Dead Load with the Max. displacement <2.83cm, Buckling Factor BF>5 and Utilization U<20% and at LC1: 1.2Dead Load and 1.0 Wind: BF > 5, Util < 25%.

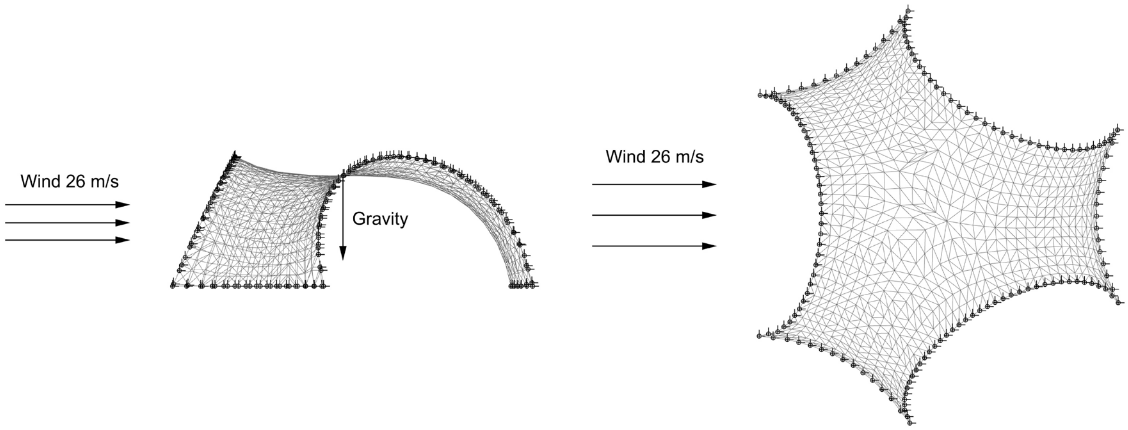


Figure 2: Boundary condition for the shell analysis.

Table 2: Structural shell Analysis – Scale 1:1 –Supports at the outline.

Shell Parameters	Surface Area [m2]	55,706096	55,706096	55,706096	55,706096	**55,706096**	55,706096
	Thickness [mm]	1	2	3	4	**5**	6
	Volume [m3]	0,055706096	0,111412192	0,167118288	0,222824384	**0,27853048**	0,334236576
Shell Analysis - Supports on the outline.							
LC0	Displacement [cm]	0.211122	0.105599	0.080581	0.067418	**0.059006**	0.052993
	BL Fact	0.960528	6.837894	17.707225	33.747099	**54.396208**	79.853513
	Util	-0.4% - 0.4%	-0.2% - 0.3%	-0.2% - 0.3%	-0.2% - 0.3%	**-0.2% - 0.2%**	-0.2% - 0.2%
LC1	Displacement [cm]	5.803574	0.84356	0.295207	0.159617	**0.095024**	0.061008
	BL Fact	-0.005175	0.077361	0.760421	2.732295	**6.542279**	12.202776
	Util	-11.4% - 10.5%	-4.1% - 2.6%	-2.2% - 1.3%	-1.4% - 0.9%	**-1.0% - 0.7%**	-0.8% - 0.5%

The analysis indicated that assigning 5mm thickness to the selected subsurface can serve as a structurally sound shell structure (Table 2.) Structural analysis of the shape in 1:10 scale has indicated that a 0.5mm thick shell in the scaled prototype should achieve the proportional displacements and can serve as a stiffness-scaled model (Table 3).

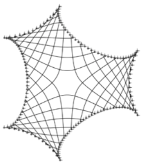
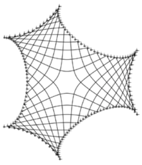
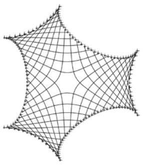
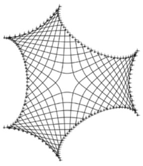
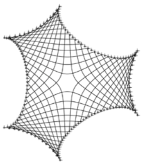
Table 3: Structural shell Analysis – Scale 1:10 –Supports at the outline.

Shell Parameters	Surface Area [m2]	0,557061	0,557061	0,557061	0,557061	**0,557061**	0,557061
	Thickness [mm]	0,1	0,2	0,3	0,4	**0,5**	0,6
	Volume [m3]	0,0000557061	0,0001114122	0,0001671183	0,0002228244	**0,0002785305**	0,0003342366
LC0	Displacement [cm]	0.002111	0.001056	0.000806	0.000674	**0.00059**	0.00053
	BL Fact	11.61368	67.596245	175.737954	335.773556	**541.644465**	795.594206
	Util	0.0% - 0.0%	0.0% - 0.0%	0.0% - 0.0%	0.0% - 0.0%	**0.0% - 0.0%**	0.0% - 0.0%
LC1	Displacement [cm]	0.581437	0.084686	0.029535	0.015975	**0.009513**	0.006109
	BL Fact	-0.0096	0.384445	0.042428	2.7145	**6.513474**	12.154267
	Util	-11.5% -10.5%	-4.1% - 2.7%	-2.2% - 1.3%	-1.4% - 0.9%	**-1.0% - 0.7%**	-0.8% - 0.6%

3.4. Grid Densities & Lath Parameters

In order to find asymptotic grids of equivalent material volume of the proposed shell, a few asymptotic grids variations were generated (Table 4), assuming shell in 1:10 scale of surfacer area 0,557061m2 and 0.5mm thickness, resulting in 0,0002785305m3 of the material volume.

Table 4: Analysis of grid densities and cross section of equivalent volume.

39#		Total Length		Lath Height		Thickness	
		18,3880	m	0,01	m	0,001515	m
				0,012	m	0,001262	m
				0,015	**m**	**0,001010**	**m**
				0,02	m	0,000757	m
				0,025	m	0,000606	m
45#		Total Length		Lath Height		Thickness	
		21,1515	m	0,01	m	0,001317	m
				0,012	m	0,001097	m
				0,015	m	0,000878	m
				0,02	m	0,000658	m
				0,025	m	0,000527	m
51#		Total Length		Lath Height		Thickness	
		24,0075	m	0,01	m	0,001160	m
				0,012	m	0,000967	m
				0,015	m	0,000773	m
				0,02	m	0,000580	m
				0,025	m	0,000464	m
57#		Total Length		Lath Height		Thickness	
		26,8520	m	0,01	m	0,001037	m
				0.012	m	0.000864	m
				0.015	**m**	**0.000692**	**m**
				0,02	m	0,000519	m
				0,025	m	0,000415	m
63#		Total Length		Lath Height		Thickness	
		29,6588	m	0,01	m	0,000939	m
				0,012	m	0,000783	m
				0,015	m	0,000626	m
				0,02	m	0,000470	m
				0,025	m	0,000376	m

Based on the available material stock of Birch plywood: 0.6mm AB/AB Flex, 0,8mm AB/AB Flex, 1.0mm AB/AB 3-ply, 1.2 AB/C 3-ply, 1.5 AB/AB 3-ply, 2.0mm AB/AB 4-ply and 2.5mm AB/AB Flex, the asymptotic grid 39# with 15mm height and 1,01mm thick members and asymptotic grid 57# with 15mm height and 0,069 mm thick members were selected as protypes for testing.

3.5. Material Testing

Before generation of the fabrication files the bending strength of selected materials for the prototypes must be tested. The minimal bending radius for the selected grids was calculated both for the flat and assembled stages. 50cm long stripes of the 1mm and 0.6mm plywood were tested to meet the minimum flexibility criteria specified in Figure 3. Without breaking or exceeding the elastic range 0.69mm thick plywood was bent to the radius of 48mm while 1.00mm thick plywood was permanently deformed at 88mm radius. Both materials testes reached the minimum bending thresholds and were classified as suitable for the selected grids.

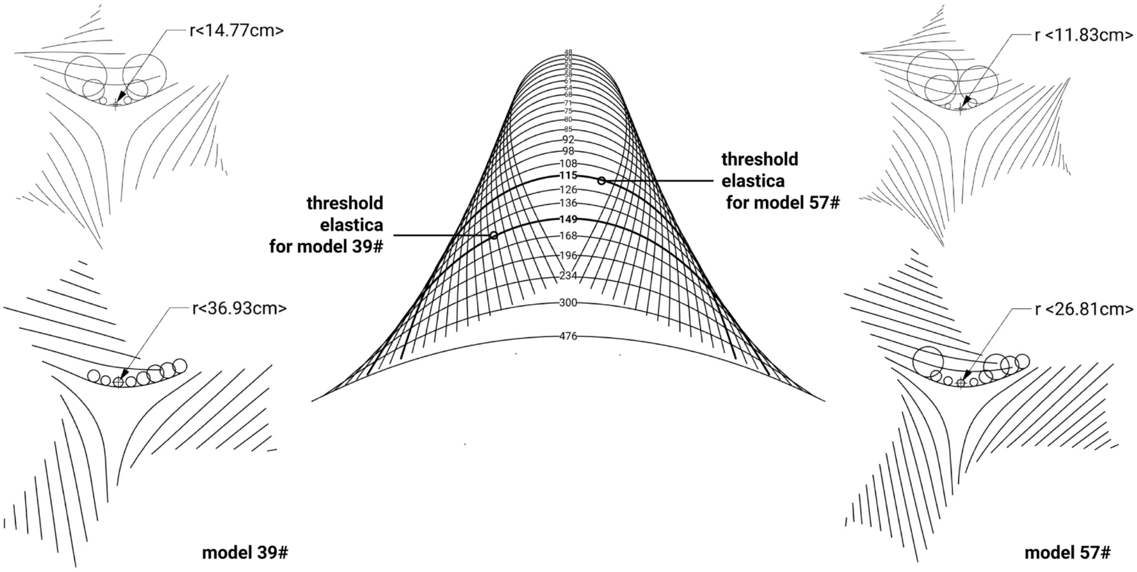


Figure 3: Minimum bending radiuses and material testing.

3.6. Final models

Based on the shell and material testing the following models were selected for construction assuming similar structural behavior and representing as structurally sound 1:1 structures (Fig. 4).

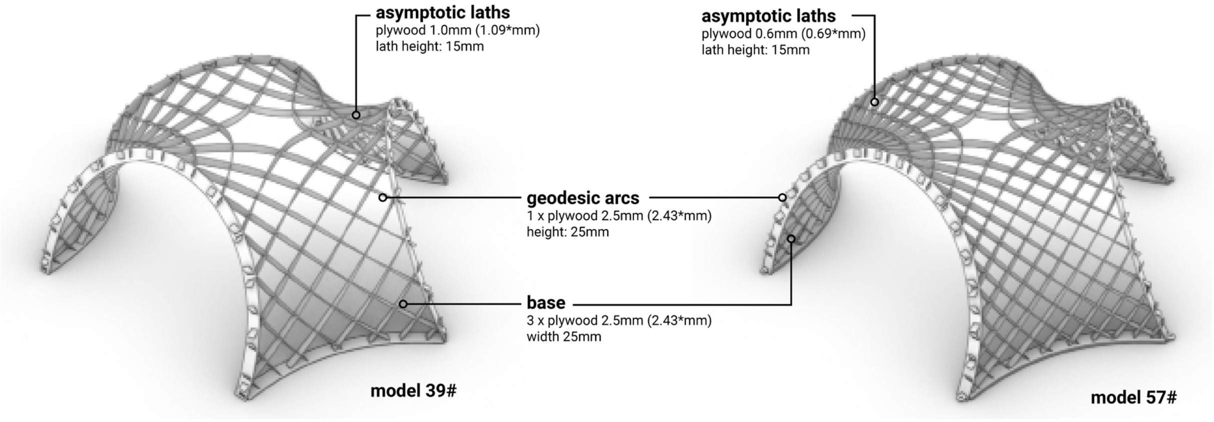


Figure 4: 3D models of the selected prototypes.

3.7 Slot calculations allowing assembly from flat.

To allow the assembly from flat the slots tolerances should allow for the rotation between 90 and 75 degrees (Fig.5), therefore laser cut models should have the 0.84slot for 0.6mm (0.69mm[1]) and 1.33mm for 1.00mm (1.09mm[1]).

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[1] The actual, measured thickness of the selected materials.

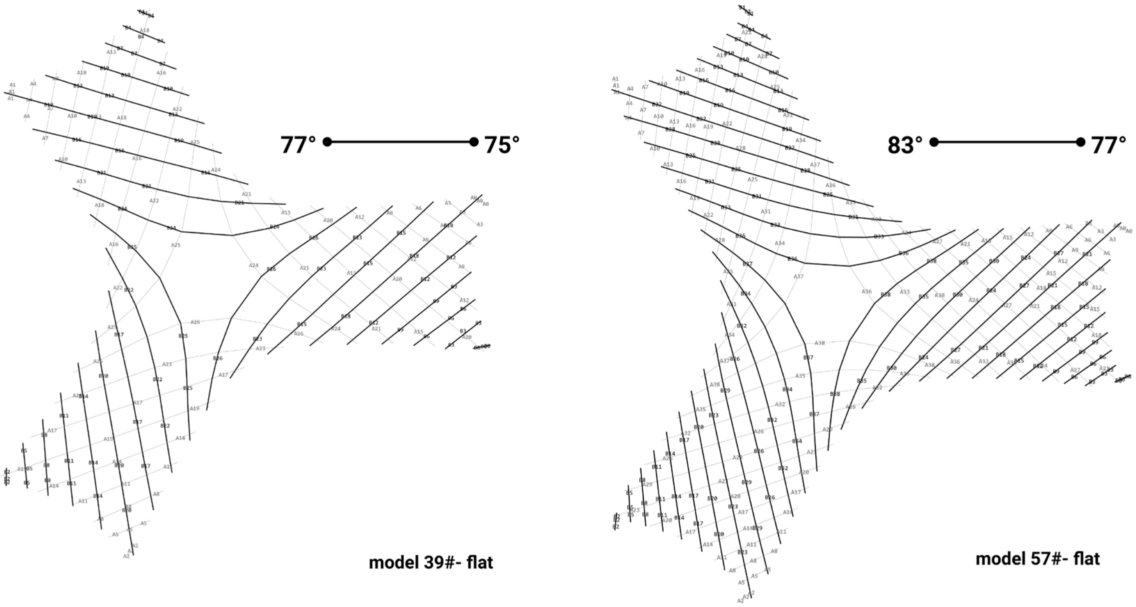


Figure 5: Angles between members in the flat state of the prototypes.

In order to generate the appropriate design files, the kerf of the laser should be accounted for. The prototypes have been laser cut on the Cromak LC5070Z. Test files were cut from both materials, allowing for calculation of the kerf into the design files with the Speed 100 Power 100.

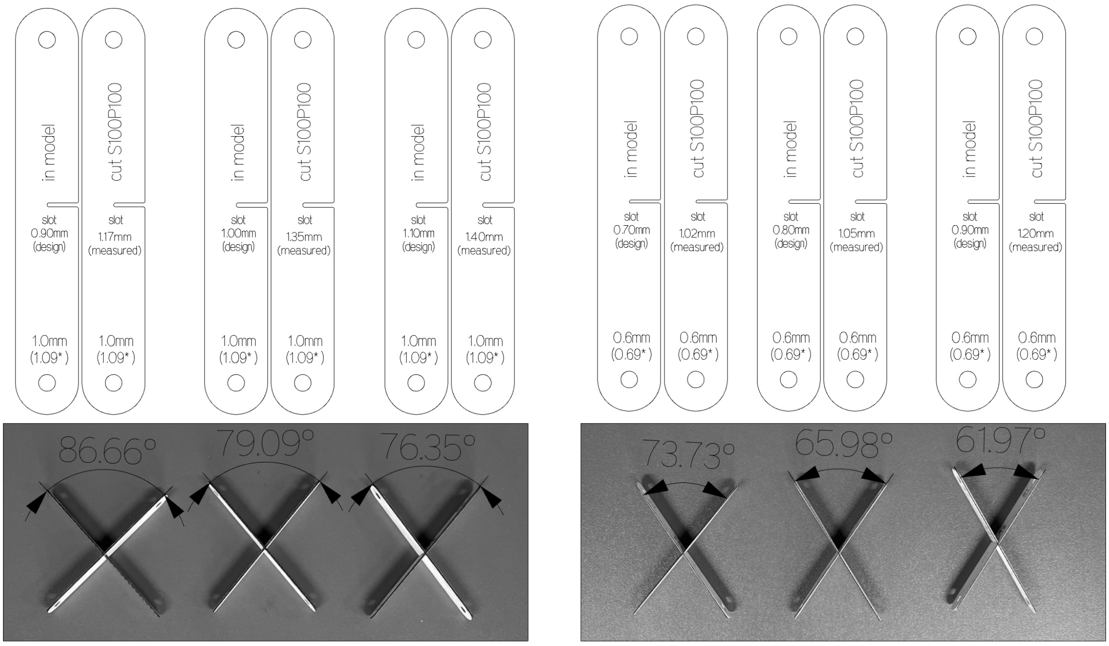


Figure 6: The models: design files and measured cut sizes of the slots.

To allow 75-90˚ rotation the 39# model has design slots of 1.0mm resulting in 1.35mm (79˚) and #57 model has design slots of 0.7 resulting in 1.02 slot. (74˚). The 0.6mm plywood is so flexible that they can rotate more than allowable rotation defined by the slot size. Both protypes were assembled during the OPTIshell workshop (Fig.7).

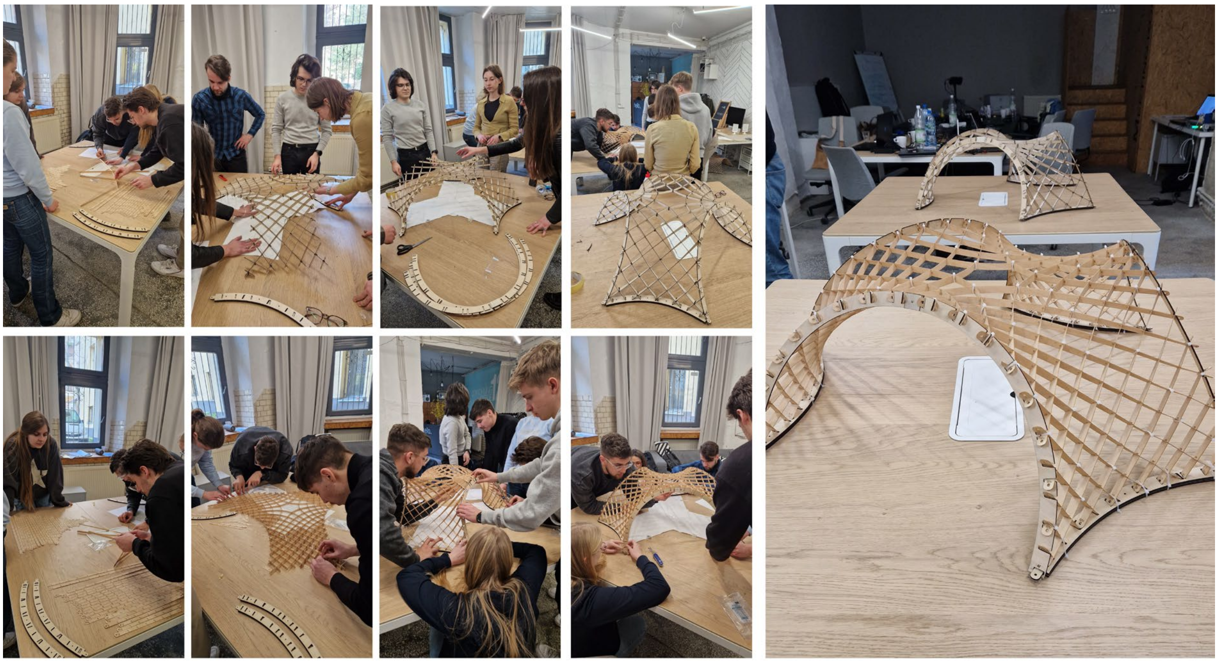
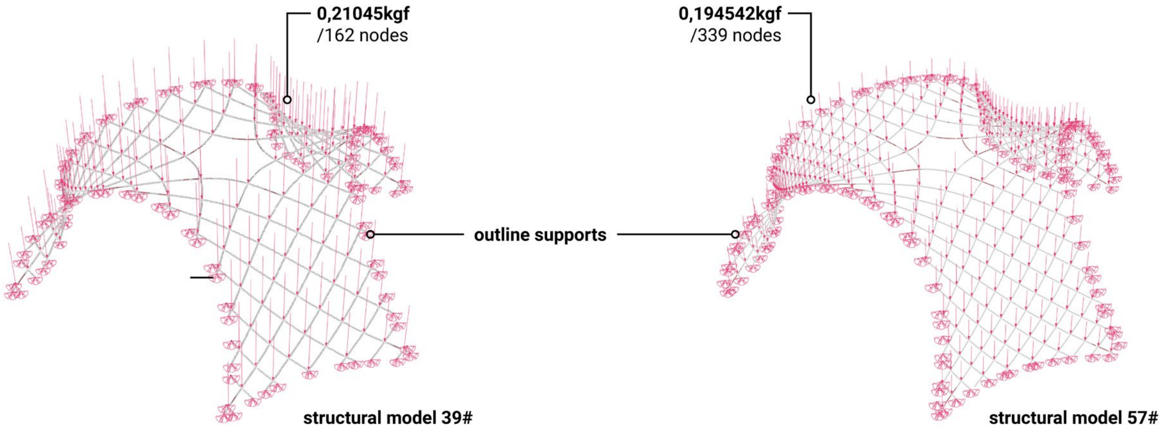


Figure 7: Protypes construction.

3.9. Structural Analysis of the models

In order to verify if the selected asymptotic and built asymptotic gridshell protoypes are structurally sound and exhibit similar structural perfomance both computational structural analysis and physical tests were performed. As the actual thickness of the selected materials varied slightly form the assumed dimensions (Table 4), the volumes of the gridshell protype Model 39# = 0.21045kg and Model57 = 0,194542kg. Simulating dead load we accouted for this proportion in the structural analysis perforemed wtih Kiwi3D (Fig.8). The physical tests were conduted by loading the top central six nodes and measuring the displacement with the rangefinder.



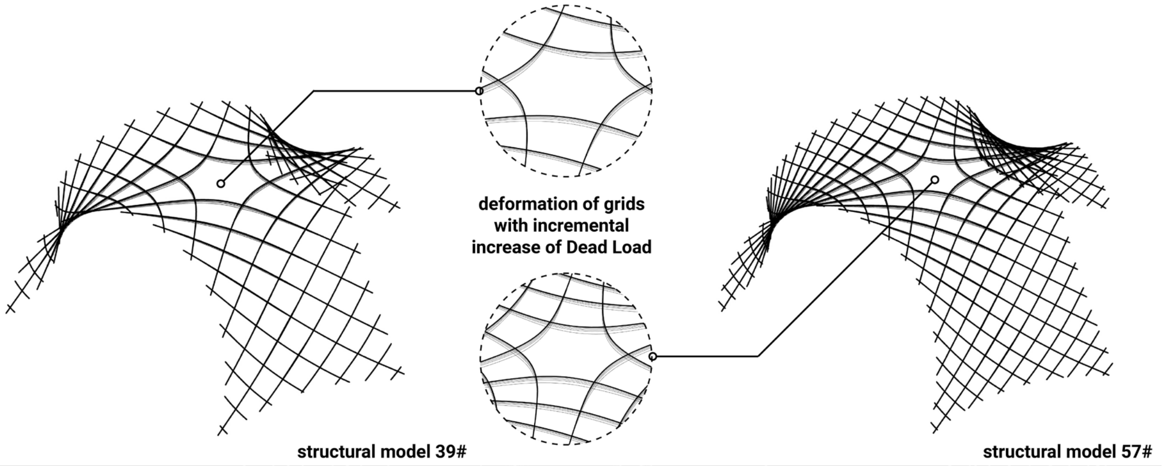


Figure 8: Structural analysis models – Kiwi3d.

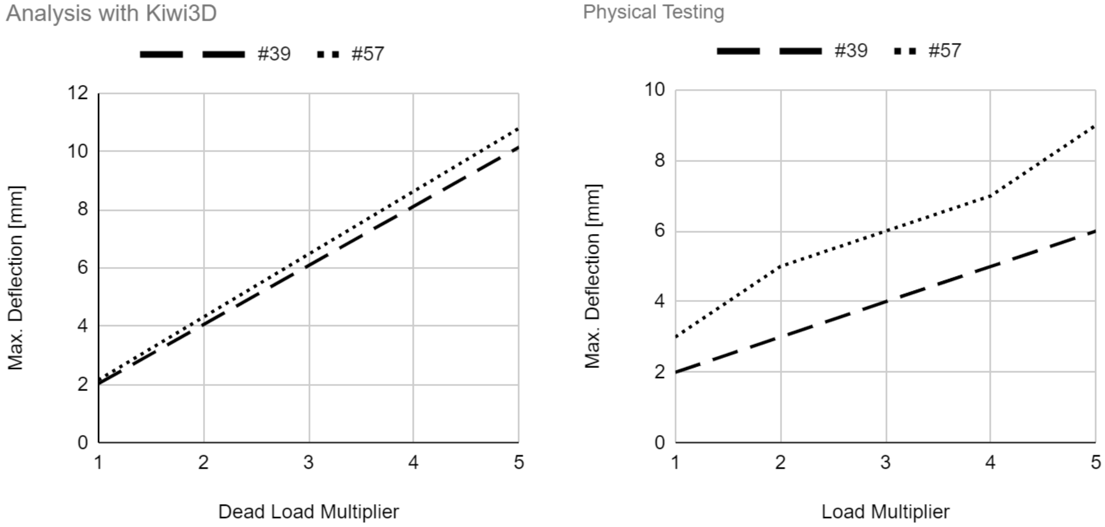


Figure 9: Protypes structural analysis and physical testing results.

4. Conclusion: submission of contributions

The proposed method has demonstrated early success, suggesting that shell analysis based on material volume can reliably determine grid densities and lath sections. Both structural analyses using Kiwi3D and physical testing have indicated that Model 39#, featuring thicker material, exhibits slightly reduced deflections, although overall performance between both models is fairly similar (Fig.9). Analysis assuming frame joints indicated similar structural performance, the physical tests showed slight differences as the 0.6mm members of # 57 model could have bent more than the allowance of the slots, resulting in larger deflections. Assuming the surface shape and topology of a grid has the primary influence on the gridhell behavior and the effects of initial bending and torsion stresses on the bending active gridshells can be omitted, the volume of material for a gridshell, whose grid can mimic membrane behavior, can be evaluated by determining the thickness of the structurally sound shell based on the same surface.

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