



Capture, optimization and transformation of physical funicular models in the digital environment: Methodological framework and application in structural design

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Abstract

A light, economical, safe, and expressive structure arises from a harmonious balance between its form, forces, and matter. The acknowledgment that the creation of a structure must be rooted in both aesthetic values and a comprehensive response to the forces and materials at play leads to a re-evaluation of design workflows. Given these considerations, this paper introduces a methodology that involves the use of physical form-finding models and their transposition to the digital realm. The process is carried out through the design of funicular structures with the Polyfuniculator, a physical tool composed of suspended ball chains anchored to a horizontal plate and connected to each other with ball magnets. Using photogrammetry or motion capture, the centre of the balls on which the thrust lines can be drawn are captured in the digital environment. The first method apprehends the structure in static equilibrium, while the latter records its deformation under various impulses, such as wind, ground movements and others. Subsequently, dynamic relaxation can be used to correct any inaccuracies of the survey and to add, remove, or modify the funiculars within the system.

Keywords: Polyfuniculator, physical models, form-finding, photogrammetry, motion-capture, reverse engineering.

1. Introduction

The inherent principle of any form-finding model is simple: the greater the material capacity for deformation, the more its shape will adjust to the applied forces. Understanding this concept unveils various methods to design efficient structures based on the behaviour of materials — whether solids (like articulated chains and fabrics for the creation of funicular structures), liquids (such as surfactant soap film for the conception of prestressed structures), or gaseous (like air supported membranes for pneumatic structures). This paper will focus exclusively on funicular models.

Even though physical form-finding models are three-dimensional prototypes that enable a tangible understanding and manipulation of form, forces, and materials, they are not widely used in design and analysis today. The obsolescence of these processes resulted from the rapid development of personal computers and software that replicate their behaviour in a faster, cheaper, and more accurate manner, enabling the rapid quantification of the elements, the analysis of the structural behaviour and, eventually, the manufacture with computer-aided machinery. For a comprehensive comparison of physical and digital form-finding processes, consulting S. Adriaenssens et al. [1] and M. Rippmann [2] is recommended.

Although there appears to be a shift from physical to digital methods, these approaches are not necessarily exclusive; they can complement each other at various stages of design and structural analysis. Recognising this potential, the article presents an operative methodology that bridges the gap between physical models and the digital set.

2. Polyfuniculator

In 1675, Robert Hooke (1635-1703) encapsulated in one sentence the most efficient shape of an arch: "As hangs the flexible line, so but inverted will stand the rigid arch" (Hooke [3], Block et al. [4]). In other words, Hooke discovered that the shape of a chain affected by certain weights in tension (funicular) is symmetrical to an arch that supports the same set of weights in compression (antifunicular). This discovery outlined a scientific approach that, alongside graphic statics, provided a rigorous understanding of the path and intensity of forces.

Until the end of the 19th century, funicular models were predominantly two-dimensional and, except in rare situations, used exclusively to assess the stability of compressive structures. However, the works and research of architects and engineers such as Antoni Gaudí (1852-1926), Heinz Isler (1926-2009), and Frei Otto (1925-2015) unveiled new methods of structural composition through three-dimensional models of suspended chains, grids, and fabrics (not to mention various models developed for other structural typologies). These processes laid the foundation upon which the techniques of computational form-finding are based. For an in-depth examination of physical models and their historical use, consulting J. Tomlow [5], J. Chilton [6], G. Vrachliotis et al [7] and W. Addis et.al. [8] is suggested.

Funicular models are created through the coordinated manipulation of three parameters: the location of supports and connections, the lengths of the strings, and the magnitude and position of the loads. The adaptability and articulation of the system allow for a composition in which any local change triggers a reaction throughout the structure until it settles into a new equilibrium arrangement.

There are some limitations in constructing a funicular model with conventional methods: the location of the anchor points and connections are fixed or difficult to change, the reciprocal connection between the elements makes it difficult to anticipate the deformations triggered by specific variations, and the exact capture of these geometries can be challenging. For this reason, a physical instrument was developed – the Polyfuniculator (D. Afonso [9] and D. Afonso [10]).

The Polyfuniculator is composed of:

- A horizontally suspended base - The base of the Polyfuniculator can be made of ferromagnetic metal or acrylic. The ferromagnetic material allows magnets to be attracted to its lower surface, which in turn will support the chains. However, the plate is opaque and obstructs the photographic capture of the funiculars from certain perspectives (which can be undesirable for photogrammetric surveying). In contrast, acrylic can have different finishes (transparent, matte, or mirrored). Transparent acrylic is particularly useful for capturing the structure from perspectives that would be inaccessible with an opaque material. Meanwhile, mirrored acrylic allows for the visualization of the inverted form of the suspended chains or fabrics (the antifunicular structure). The chains are attached to the acrylic bases using magnets placed on both faces (Figure 2 and Figure 3).
- Ball chains - There are a myriad of chain configurations and connection mechanisms among their links. Within these, ball chains stand out due to two fundamental characteristics: firstly, the ball chains distribute weight and tension uniformly along their entire length; secondly, when illuminated by a light source near the observer, the spheres reflect light at their central point. This feature is crucial for the photogrammetric survey of the centroid of each sphere (as detailed in subsection 3.1. Photogrammetry). For the same reason, the magnets are also spherical.
- Connections – In small-scale models, the connection between different chain segments can be achieved using magnets. However, it is important to acknowledge that magnets have a finite

resistance (for instance, a 3mm diameter neodymium magnet can support approximately 20g, equivalent to 1.5m of stainless-steel chain with a 2.5mm diameter). Thus, in developing structures with a weight exceeding the magnets' attraction, it is recommended to anchor the chains using articulated connectors (such as jumper rings, double ring clasps and others).

- Grids and fabrics - Beyond the use of chains, it is possible to use suspended grids and fabrics. To design grids of equal segments capable of transforming into continuous surfaces, it is necessary to consider the geometric properties of regular polygons. Triangles composed of similar segments (equilateral) are always equiangular (each angle measures 60 degrees). In contrast, an equilateral quadrilateral is not necessarily equiangular (rhombuses have 4 segments of identical lengths and form pairs of angles that differ from each other). Moreover, equilateral polygons with five or more sides are not necessarily convex (they can be concave and even self-intersecting). Thus, the deformation of grids into continuous surfaces is only feasible if they are composed of polygons with four or more segments.
- Measuring instruments – During the assembly of the model, the length of funiculars can be measured using a ruler. In turn, the weights of the various components can be determined with a precision weighing scale. To position the anchor points, a plan printed on a sheet of paper (Figure 2) or acetate may be used.

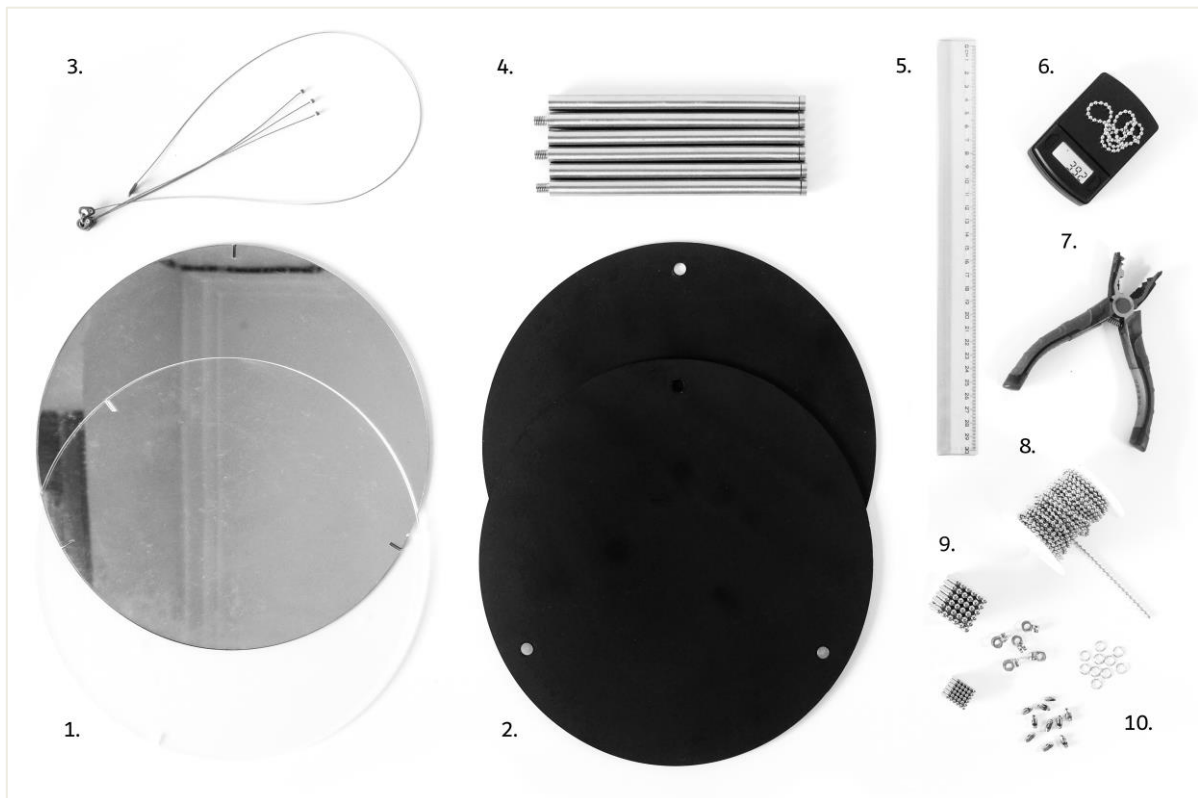


Figure 1: Elements that comprise the Polyfuniculator. 1. Base in mirrored acrylic (top) and transparent acrylic (bottom); 2. Base in ferromagnetic metal painted black (to minimize reflections); 3. Cables for base support; 4. Struts for base support; 5. Ruler; 6. Weighing scale; 7. Plier for ball chains; 8. Ball chains; 9. Ball magnets; 10. Double ring clasps, jump rings and ball chain connectors.

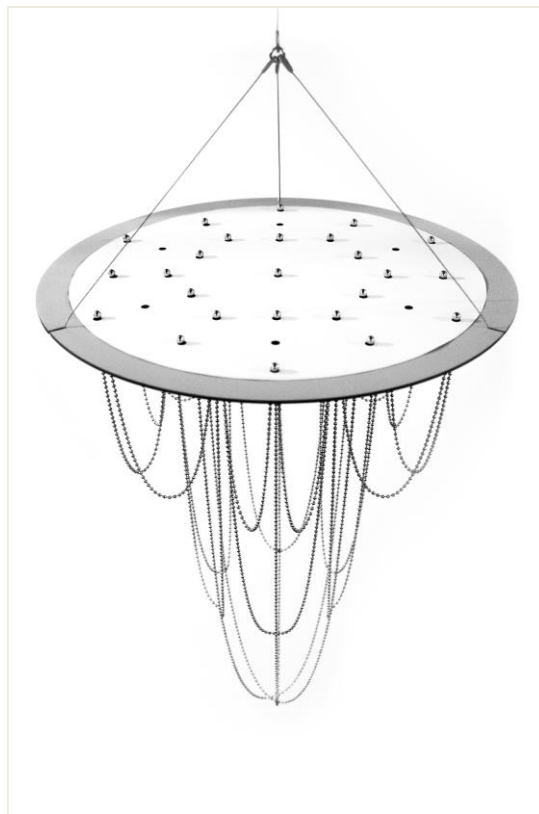


Figure 2: Polyfuniculator with a reference plan.

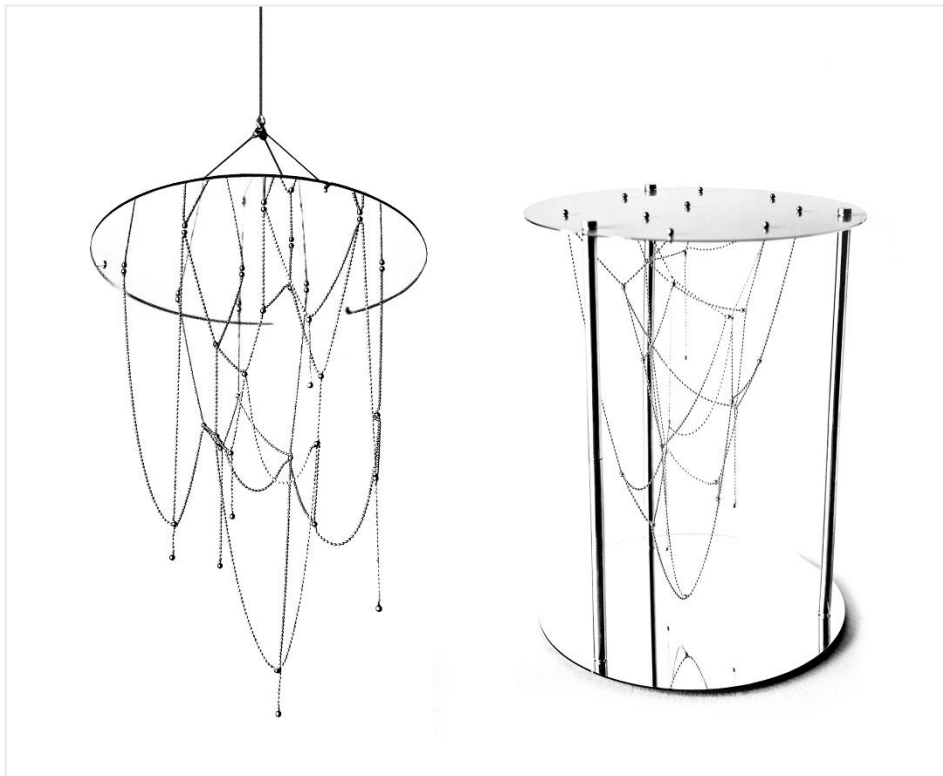


Figure 3: Different support systems of the Polyfuniculator.

3. Bridging the gap between physical and digital models

Over the last few decades, different methods have emerged for capturing physical objects into the digital realm. In this chapter, the use of two passive (non-contact) processes in the recording of funicular models are described: photogrammetry and motion capture.

3.1. Photogrammetry

Photogrammetry is the art, science, and technology of extracting quantifiable information from physical objects through the recording and interpretation of photographic images. From its inception up to present date the technique has been applied across various scales and research fields (ranging from topographical surveys, environmental monitoring, industrial inspection and digital reconstruction of architectural and design objects). In this section, specific techniques to capture funicular models will be addressed.

To avoid the capture of other objects, it is important that the background behind the model is homogeneous and neutral. The slow rotation of the Polyfuniculator (using a rotating table or, if suspended, a gentle manual push) allows photographing the model from multiple angles with a stationary camera, reducing the background area. Additionally, the use of a transparent acrylic base is recommended to prevent any occlusion over the structure.

Capturing the thrust lines of ball chains requires the apprehension of their centroids for subsequent interpolation into curves and surfaces. The photogrammetric survey of specific points is usually carried out using markers placed on the object. However, due to the centralized reflection of metallic spheres, it is unnecessary to apply any markers on the chains. Photographing them using a light source aligned with the camera's axis (such as an integrated flash) serves the purpose.

The removal of saturation and adjustment of exposure, contrast, blacks, and whites are recommended for delineating the reflections of the spheres. Photoshop's Camera Raw plays a significant role in this process as it allows to edit and copy the settings of a single photo across multiple shots simultaneously (Figure 4). This automation speeds up the workflow, ensuring visual consistency across all images.

Throughout this research, various surveys were conducted using PhotoModeler software. PhotoModeler offers a wide range of measurement tools and is particularly efficient at processing specific points with markers. The SmartMatch mode triggers different operations that allow the spatial orientation of multiple perspectives. To acquire the centroids' position, it is necessary to perform the Automatic Target Marking option over the reflections. This action initiates an interface that allows selecting the set of photos to be analysed, the type of marker (in this case, white dots), and their minimum and maximum diameter in pixels. Concluding this step, the Automatic Referencing function is activated to generate the points in the three-dimensional space (Figure 5).

Once the points are mapped, they can be exported to Rhinoceros software. In Rhino is possible to use different interpolation techniques to create the thrust lines, which, in turn, serve as the basis for surface modelling. It is important to emphasize that the form-finding process does not necessarily end with the physical model and that, through Dynamic Relaxation, it is possible to rectify any inaccuracies in the photogrammetric survey and add, remove, or change the funiculars within the system (Figure 6 and Figure 7).

When applied to suspended fabrics, photogrammetry allows for the three-dimensional reconstruction of funicular surfaces. The process is straightforward and does not require markers if the fabric is stretched. However, the presence of folds often makes it challenging to capture the entire surface and fully reconstruct its shape. To mitigate potential occlusions, employing transparent fabrics with markers or using dense grids of ball chains can prove to be beneficial. With these materials, it is possible to capture specific points of the model, enabling the subsequent interpolation onto a continuous surface. Just like models composed of suspended chains, it is possible to correct or change the structure with Dynamic Relaxation.

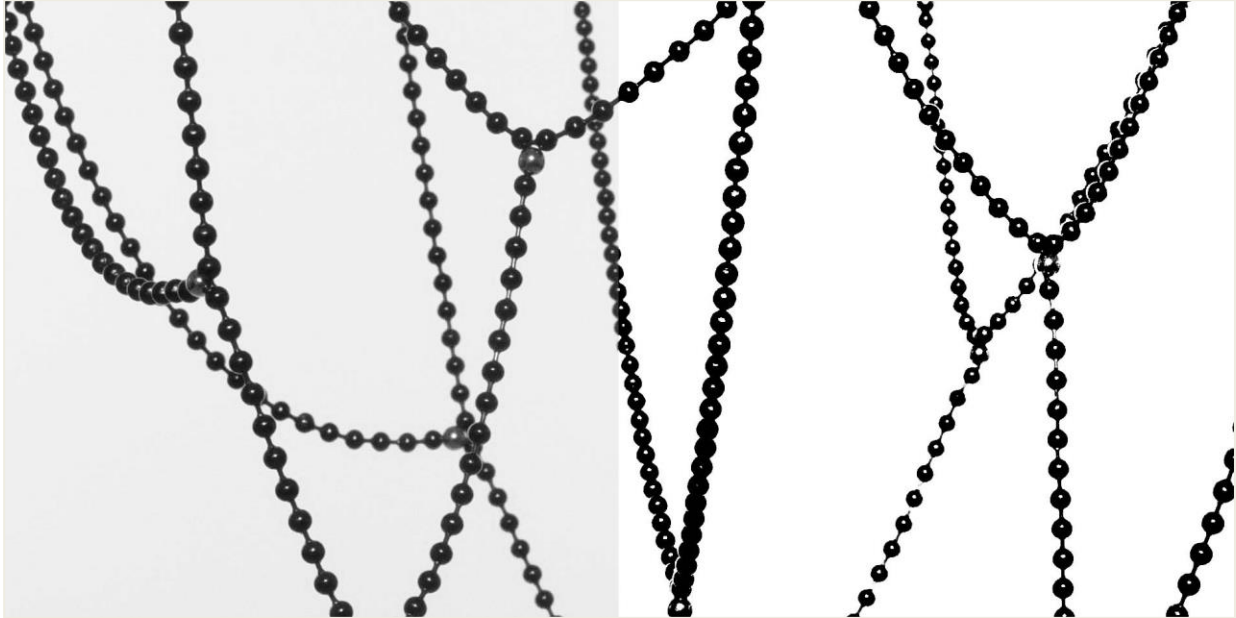


Figure 4: Detail of the original photograph (on the left) and edited (on the right). For greater contrast between the reflection and the shaded surfaces of the spheres, the use of black chains is recommended.

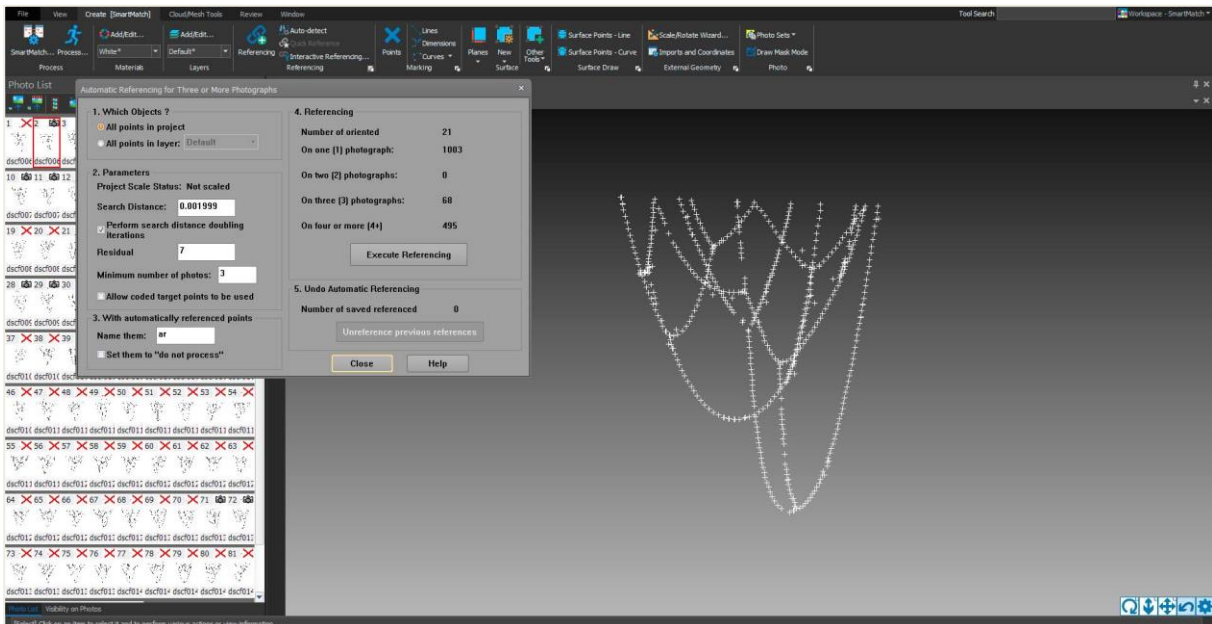


Figure 5: Photogrammetric survey with PhotoModeler.

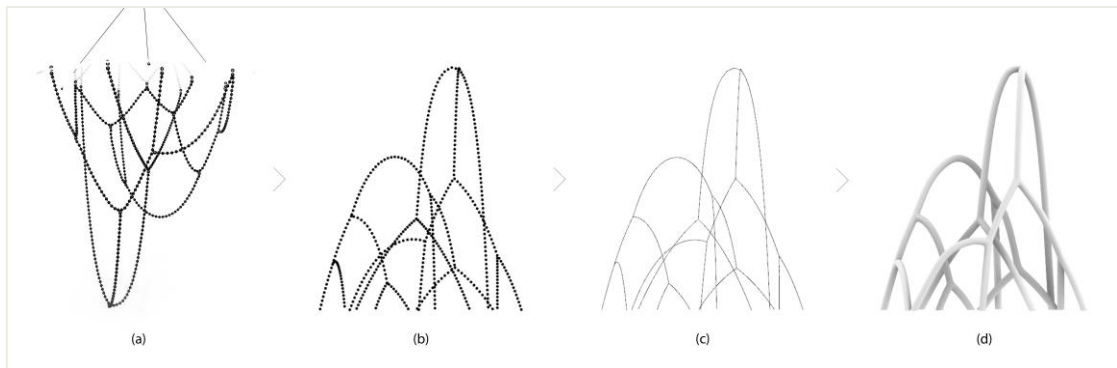


Figure 6: Methodological framework: a) Form-finding with the Polyfuniculator; b) Photogrammetric survey with PhotoModeler; c) Interpolation and correction of the funiculars using Dynamic Relaxation; d) Transformation of curves into surfaces.

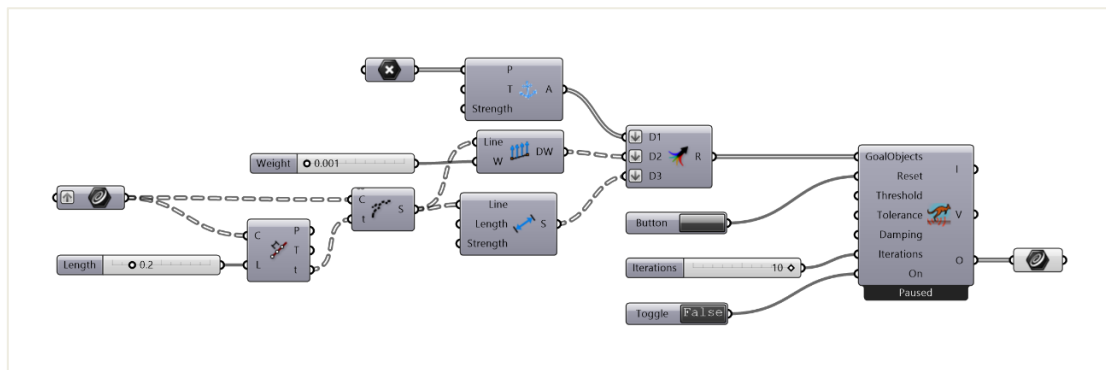


Figure 7: Dynamic Relaxation code to correct potential inaccuracies through the Kangaroo plugin.

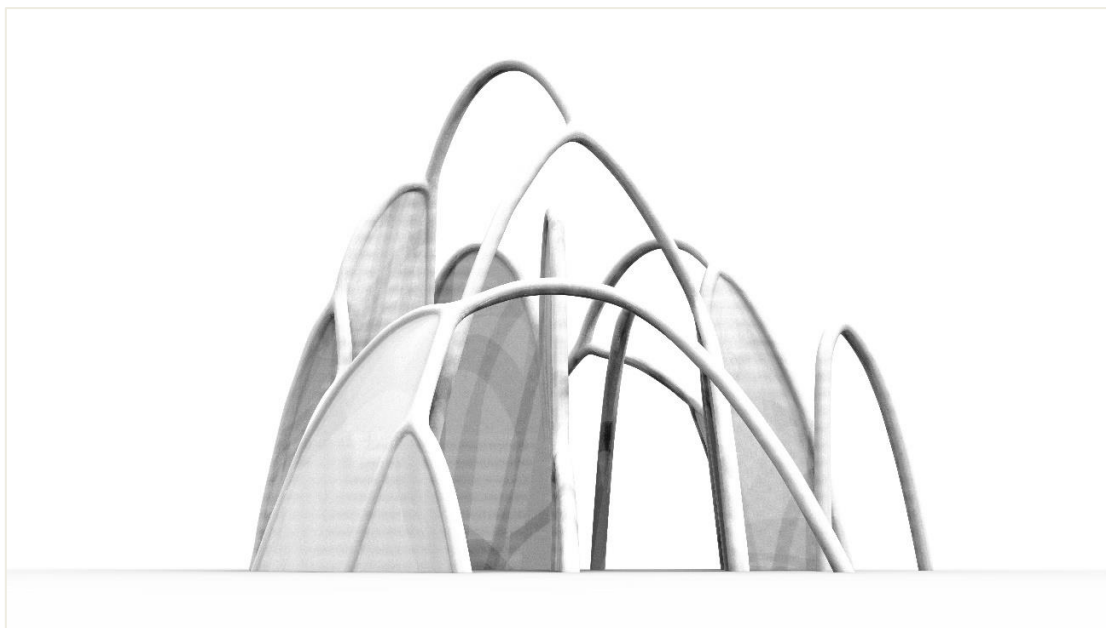


Figure 8: Closed and open surfaces are generated from the thrust lines. The thickness of the structure will depend on the material properties and size to ensure overall stability against buckling and various deformations caused to wind, earth movements and other events.

3.2. Motion Capture

Photogrammetry is an accessible and efficient process to capture static chains. However, for the creation of kinetic structures, assessment of potential deformations, and real-time apprehension of the form-finding process, it is necessary to record the coordinates of the spheres' central points over time. To accomplish this, the application of motion capture is indispensable.

Motion capture can be applied to funicular models in two ways: on one hand, it enables the analysis of the chains' trajectory when subjected to different external forces; on the other hand, it facilitates the generation of static or kinetic structures.

It is important to note that compressed structures are composed of passive materials, which have low elasticity for adapting to new equilibrium conditions. In contrast, structures made from materials that resist tensile stresses (such as steel cables or textiles) are active; they possess higher elasticity and adapt easily to new configurations. For this reason, constructing a kinetic tensile structure (funicular) is easier than a compressive one (antifunicular).

There are several movements that can deform funicular models: radial movements generated by the rotation of the model along an axis (this creates a centripetal force that prevents the dissipation of matter and a "pseudo centrifugal force" that causes its departure (resulting from the inertial force of the chains in the air)); translational movements produced by the linear displacement of one or more points of the model; and vibrational movements made by changes in the direction, sense, and speed of different forces applied on the system.

The capture is not performed on the entire model but on markers strategically placed on it (see Figure 10). To convert spheres into markers, reflective tape is applied over their surface. The tape reflects the infrared emitted by the LEDs positioned around the lenses, which are suited to capture this wavelength. Therefore, it is recommended that the capture volume is free from other bright reflections and light sources.

The captures were made using Qualisys system. For accurate and comprehensive capture of the markers, it is advisable to use at least 6 cameras around the model at different heights. The greater the number of cameras, the greater the redundancy and precision of the capture. Before starting the recording, it is necessary to calibrate the system to identify the zoom of the lenses and their position in space. After this initial step, it is possible to visualize the markers in real-time and record their movements over a specific period (see Figure 10). In this way, the position, velocity and acceleration of the points are monitored for future analysis and design (see Figure 9).

Motion capture files are not compatible with the geometric modelling programs commonly used in architecture. Due to this limitation, the presenting author created a plugin for Grasshopper called Horse, which allows importing the points' data in one or multiple frames and analysing their trajectory over time. Once imported, numerous geometric operations can be performed on the points or on the curve of their trajectory.

The Polyfuniculator enables quick and intuitive manipulation of funicular models using ball chains and magnetic connections. Furthermore, the real-time interaction between the user, the Polyfuniculator, and its digital representation makes the system ideal for exhibitions and the development of participatory projects.

Despite these advantages, there are several drawbacks associated with motion capture. These systems are often expensive, involving the costs of high-precision cameras, reflective markers, software licenses and maintenance. Additionally, they typically require a large space to set up the cameras and ensure an unobstructed view of the markers.

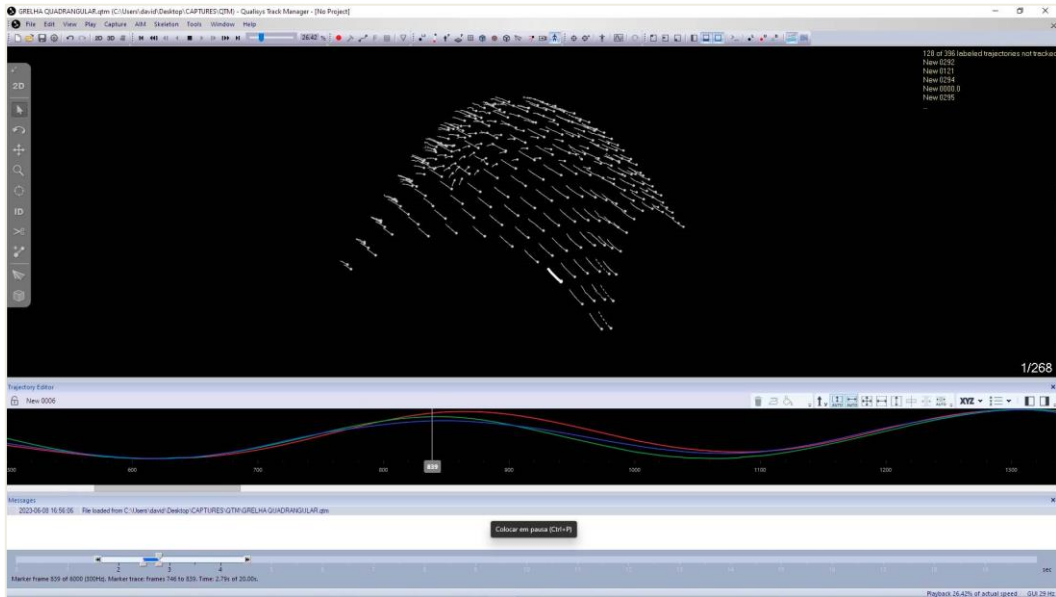


Figure 9: Trajectory of the points of a grid of chains subjected to a vibration. The chart below shows the displacement of the x (red), y (green), and z (blue) coordinates of a point on the grid.



Figure 10: Motion capture is performed using reflective markers placed on the chains (left). This recording system enables the real-time observation of the funicular model on the computer (right).

Conclusion

This article presents an innovative approach to structural design and analysis that seamlessly integrate physical and digital technologies. At the heart of this approach is the Polyfuniculator, a new form-finding tool that enables intuitive and quick manipulation of funicular models using ball chains and magnetic connections.

Subsequently, two methods were explored to capture the models in both static (photogrammetry) and kinetic states (motion capture). Through photogrammetry, it was demonstrated that it is possible to efficiently capture the centroids of the spheres' chains without any markers. Later on, dynamic relaxation can be used to correct any discrepancies in the thrust lines and add, remove, or change the funicular system. The other methodology, motion capture, allows for the monitoring of the position, velocity, and acceleration of markers to study their behaviour or design kinetic structures.

In conclusion, these technologies enable the design and analysis of structures supporting tensile or compressive stresses through the integration of physical form-finding models into the digital environment.

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