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Two units for the design of modular geodesic gridshells

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Abstract

This paper focuses on investigating two quadrilateral units characterised by a grid made of straight rods which differ in how the rods are arranged within the edge quadrilateral. These units start from a flat configuration and deflect to form a geodesic gridshell whose mid-surface is characterised by constant Gaussian curvature. Deflection is controlled by a one unique degree of freedom.

The objective herein is to propose a procedure based on differential geometry that allows designers to define the arrangement of the flat grid so that the deflected configuration lays on a target surface which is either spherical or hyperbolic, while maintaining constant constraint conditions. This is achieved by a suitable application of the law of cosines for non-Euclidean spaces.

The design procedure is validated by Finite Element Method as well as tabletop models.

Keywords: Geodesic gridshells, Modular units, Form-Mobility relationship, FreeGrid benchmark.

1. Introduction

Gridshells are lattice shells built by one-dimensional thin elements made of steel, aluminium or wood that represent multipurpose structures capable of achieving target surfaces [1, 2] and cover very large spans. These structures represent a creative and generative process that intricately combines structural contributions with explorations in form [3].

Based on the construction method employed, gridshells can be categorised as either unstrained or strained. Unstrained gridshells refer to shells that, in their initial state, are free of stress (apart from that induced by their own weight) and are constructed from an assembly of relatively short straight or pre-bent members. The evolution of the shape of unstrained gridshells often reflects aesthetic, geometrical, physical, and constructional considerations. Structurally, challenge lies in the determination of a three-dimensional surface which eliminates any bending, relying solely on membrane actions [4].

In contrast, strained gridshells are built from an initially flat grid typically consisting of wooden laths chosen for their excellent bending characteristic. These laths are bent on-site to create the desired curvature of the gridshell. Utilising frictionless pinned connections enables the rotation of members, facilitating the propagation of distortion by the grid, resulting in intricate curved shapes [5]. Their primary advantage in using strained gridshells lies in the streamlined transport and assembly. Nevertheless, the challenging design and form-finding aspects of these structures continue to pose significant challenges [6, 7].

In this paper, two quadrilateral units characterised by an internal grid made of straight rods are investigated. The arrangement of internal rods within these two units is delineated by employing equations derived from the law of cosines for non-Euclidean spaces, establishing relationships between the side-lengths and angles of the grid. Consequently, this approach enables an exploration of how the two units, starting from flat configurations, transition towards curved surfaces (spherical or hyperbolic) contingent upon the arrangement of internal straight rods within the quadrilateral framework.

Computational models based on the Finite Element Method (FEM) and physical tabletop models are realised to validate the proposed approach by comparing the target surfaces of the two gridshells.

2. Design of grids that bend to surfaces of uniform curvature

Geodesic strained gridshells are obtained by assembling rods composing the planar grid with a slender rectangular profile, aligned horizontally. Thus, in their curved state, curvature along the tangent plane of the shell's mid-surface can be neglected, and the beams are regarded as geodesic lines on this surface. This property can be employed to describe the grid as a series of adjacent quadrilaterals either lying on the horizontal plane, when the gridshell assumes its flat configuration, or on a curved surface representing the shell mid-surface when the shell is fully deployed.

Accordingly, differential geometry can be used to derive some relationships between the side-lengths and angles of the grid. To this end, it is useful to decompose each quadrilateral mesh of the grid into two adjacent triangles and describe them using the law of cosines, which is applicable to triangles lying on either the horizontal plane or curved surfaces.

The law of cosines states a relationship between the sides and angles of any Euclidean triangle of sides a, b and c, respectively opposite to the inner angles α, β and γ, see, e.g., Figure 1(a):

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos(\alpha) \\ b^2 &= c^2 + a^2 - 2ca\cos(\beta) \\ c^2 &= a^2 + b^2 - 2ab\cos(\gamma) \end{aligned} \tag{1}$$

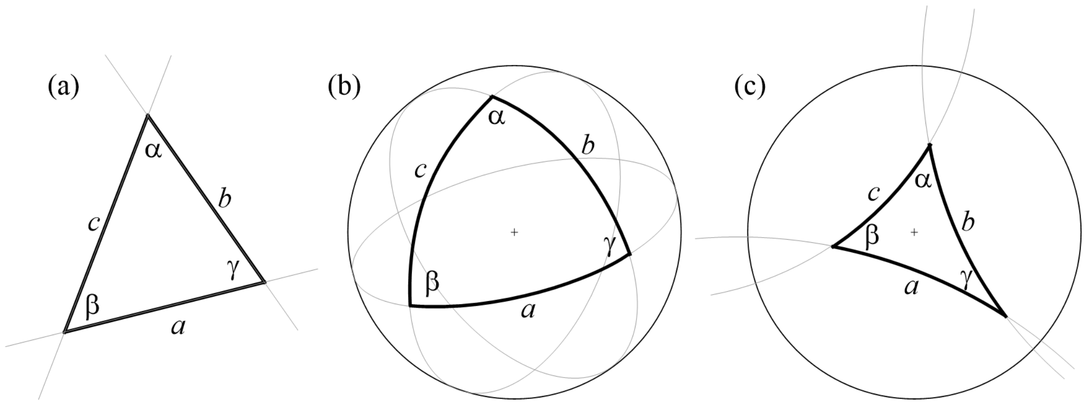

Figure 1: Triangles: (a) on the Euclidean plane, (b) on a spherical surface and (c) on a hyperbolic space graphically rendered by the Poincaré disk.

Generalisations of previous formulas to triangles laying either on a spherical surface of Gaussian curvature $1/R^2$ or on a hyperbolic surface of Gaussian curvature $-1/R^2$ are possible. In particular, on the sphere, see, e.g., Figure 1(b):

$$\begin{aligned} \cos(a/R) &= \cos(b/R)\cos(c/R) + \sin(b/R)\sin(a/R)\cos(\alpha) \\ \cos(b/R) &= \cos(c/R)\cos(a/R) + \sin(c/R)\sin(a/R)\cos(\beta) \\ \cos(c/R) &= \cos(a/R)\cos(b/R) + \sin(a/R)\sin(b/R)\cos(\gamma) \end{aligned} \tag{2}$$

while on the hyperbolic surface, see, e.g., Figure 1(c):

$$
\begin{aligned}
\cosh(a/R) &= \cosh(b/R)\cosh(c/R) + \sinh(b/R)\sinh(a/R)\cos(\alpha) \\
\cosh(b/R) &= \cosh(c/R)\cosh(a/R) + \sinh(c/R)\sinh(a/R)\cos(\beta) \\
\cosh(c/R) &= \cosh(a/R)\cosh(b/R) + \sinh(a/R)\sinh(b/R)\cos(\gamma)
\end{aligned}
\tag{3}
$$

These formulas are used to describe a grid of 4×4 quadrilateral meshes. Compatibility between the edges of each mesh and between the lines that form the grid is imposed by setting additional relationships between the angles formed by pairs of rods. This set of equations describes the geometry of the entire grid in terms of a few parameters representing the inner angles of the outer quadrilateral of the grid and the inclination of inner lines. Such a set of parametric equations is solved by an iterative procedure to obtain the geometry of a flat grid that bends to a target surface of constant Gaussian curvature.

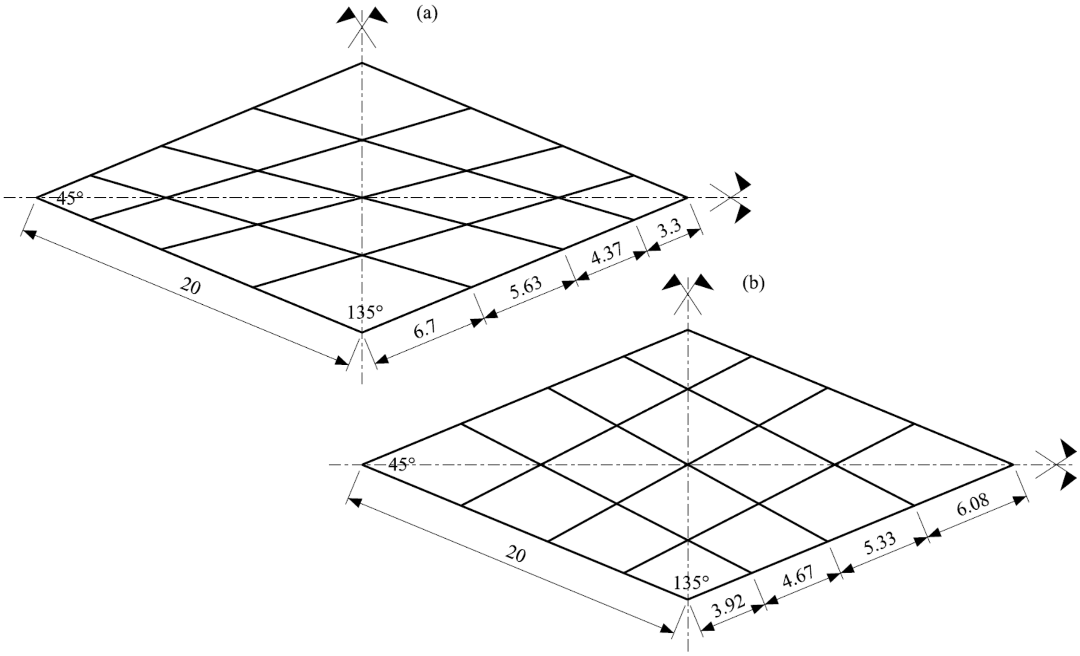

Figure 2: Planar grids that bend to the target surface of uniform Gaussian curvature equal to $1/R^2$ (a) and $-1/R^2$ (b).

As an example, Figure 2 shows two planar grids that bend to target surfaces of uniform Gaussian curvatures either equal to $1/R^2$ or $-1/R^2$, respectively, with $R = 20$. Such grids are designed to form a square on corresponding target surfaces, i.e. their outer edges have equal lengths (set to 20) and equal inner angles (respectively equal to $107.36°$ and $77.67°$ for the two models). This property is verified below by FEM analysis and physical tabletop models.

3. Validation by FEM analysis

Finite Element models of the two grids of Figure 2 are modelled in ABAQUS to validate the design procedure described in the previous section. This is done by analysing the deploying process by means of geometrically non-linear analyses.

The straight rods of both units are modelled in ABAQUS as mono-dimensional elements obeying to shear-flexible beam formulation. These elements are assigned a rectangular cross section of size 0.8×0.1 cm^2 and a material characterised by Young's modulus of 21 MN/cm^2 and Poisson's ratio of 0.3.

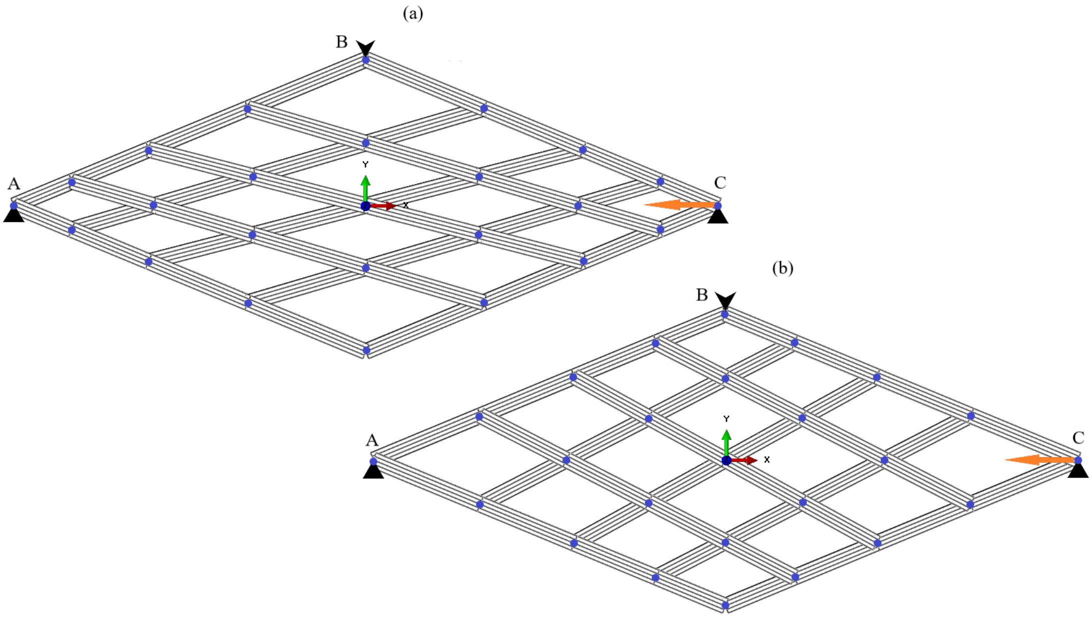

Figure 3: ABAQUS models of the two planar grids with associated boundary conditions.

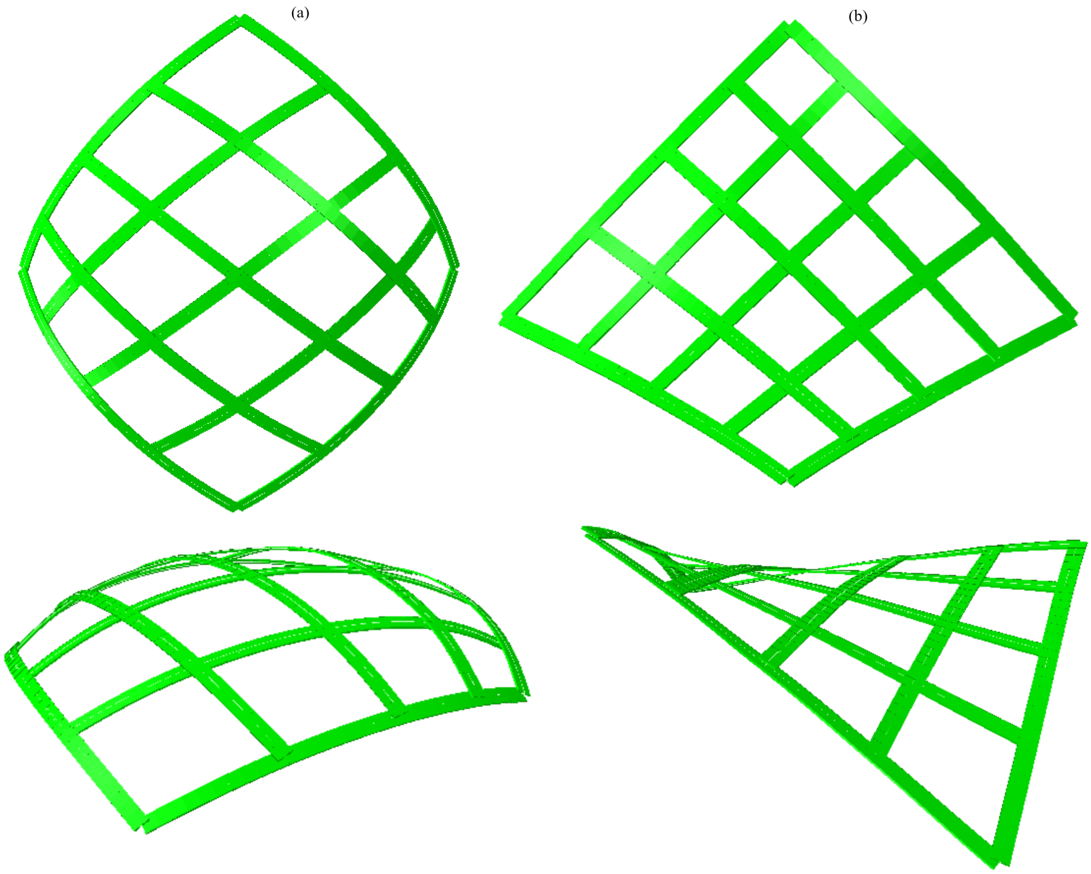

Figure 4: Deployed configuration of the two FEM models. The two grids adhere to surfaces of uniform Gaussian curvature equal to $1/R^2$ (a) and $-1/R^2$ (b).

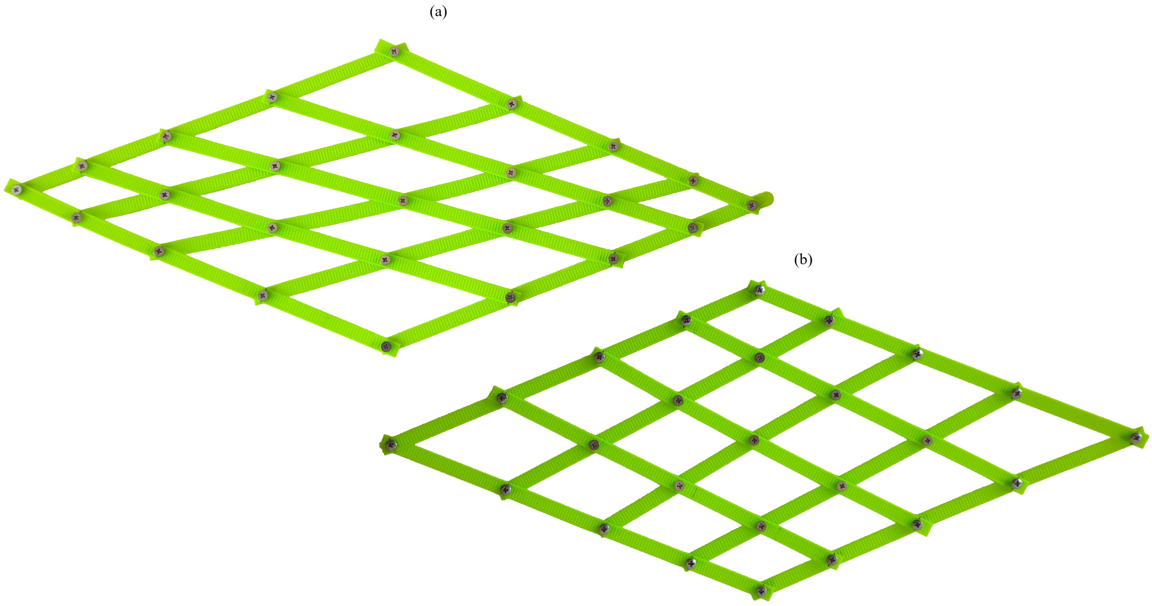

Figure 5: Tabletop models of the two planar grids.

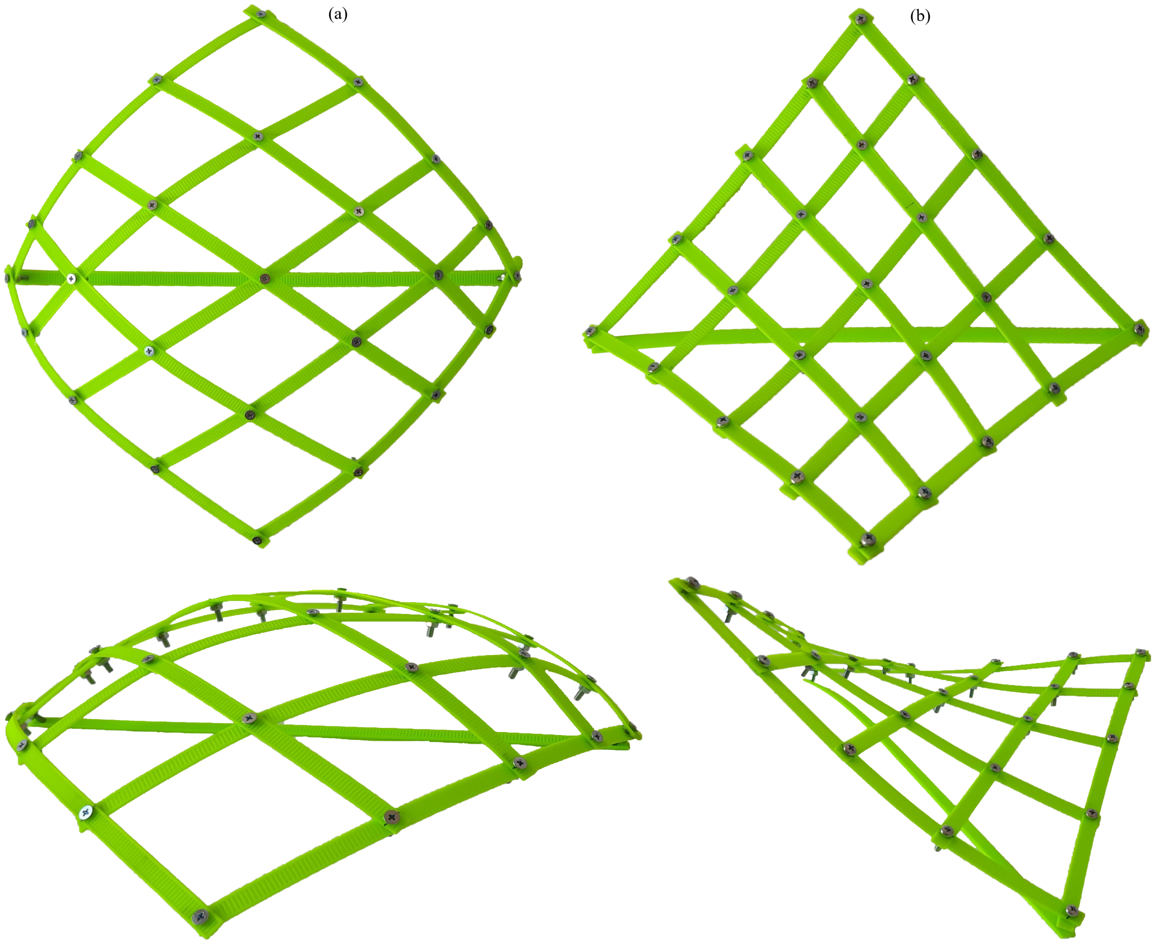

Figure 6: Deformed shapes of the two tabletop models having uniform Gaussian curvature equal to $1/R^2$ (a) and $-1/R^2$ (b).

Elements pertaining to the two orders of rods are arranged on two horizontal layers distant 0.1 cm to avoid interpenetration. The connection between rods of the two layers, see, e.g., the blue dots in Figure 3, are modelled as connector hinges that constrain any relative motion except the relative rotation about the pin connection. The global rigid motion of the entire grid is avoided by adding 3D hinged supports at nodes A and C and a vertical roller support at node B.

Deployment of each model is induced by the horizontal displacement applied to the constraint at node C, whose value is controlled by Rick's arch-length method.

Figure 4 reports plan and perspective views of the deployed configurations of both gridshells. The results of FEM analyses confirm the proposed design approach since both gridshells adhere to target spherical (a) and hyperbolic (b) surfaces of uniform Gaussian curvature.

4. Validation by tabletop models

The proposed design approach is further validated by physical tabletop models. The two models pictured in Figure 5 are built by assembling a set of Veramyd6 cable ties of rectangular cross section 8×1 millimetres. Cut and pierced elements are connected by M3 bolted joints, see, e.g., Figure 5.

Figure 6 shows the deformed shapes of the two physical models achieved by inducing a relative displacement between end nodes A and C. To keep models in their deformed configurations, these two end nodes are then connected by an additional diagonal cable tie loosely connected to the end nodes bolts. Deformed configurations of both models validate the results of FEM analyses and confirm the feasibility of the proposed design approach.

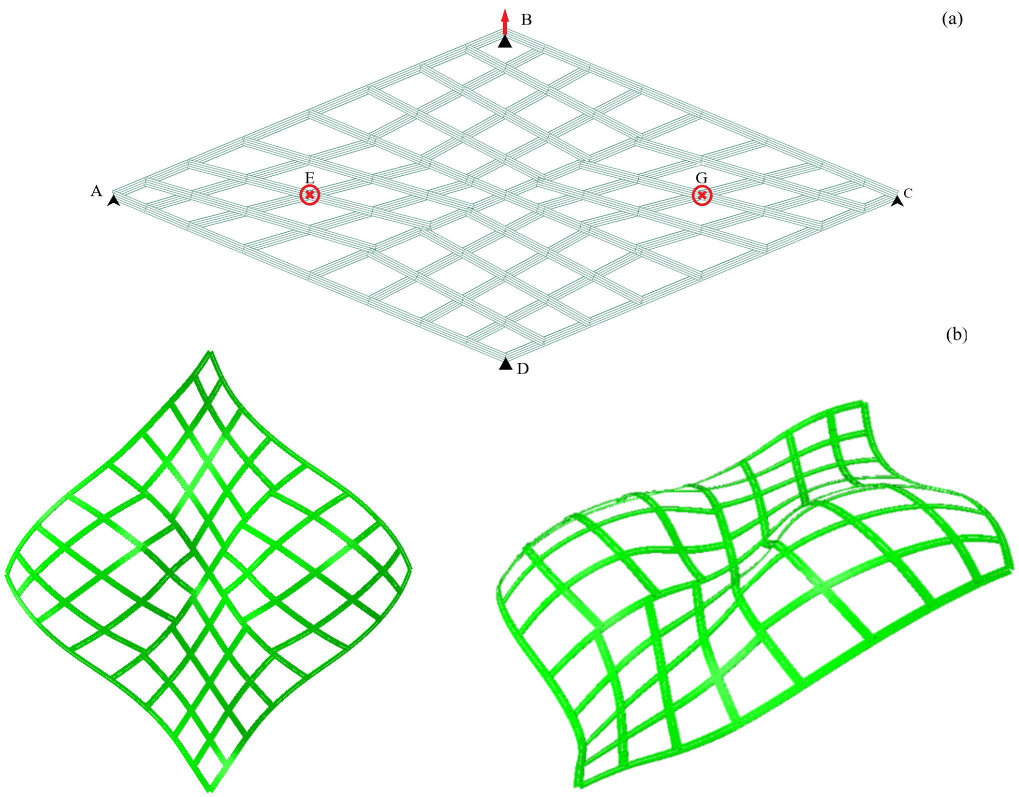

Figure 7: Model consisting of repetitions of the two units (a) and its deformed shapes, observed from both planar and perspective views, obtained by FEM analysis (b).

5. Conclusions

We have formulated a set of equations derived from differential geometry to define the relationships between the side lengths and angles of a gridshell composed of straight rods. This set of equations is employed to define the geometry of planar grids that deploy to target surfaces of uniform Gaussian curvature. The proposed design approach is validated by FEM and tabletop models to verify that deployment of designed quadrilateral grids bends towards surfaces having positive and negative Gaussian curvature, respectively. Extension of the proposed design procedure will regard the design of grids having a target surface of non-uniform Gaussian curvature. For instance, by assembling the two modules proposed in this paper according to a chessboard scheme, see, e.g., Figure 7(a), it is possible to obtain a grid that features a deployed configuration characterised by non-uniform curvature, see, e.g., Figure 7(b). Rigid motions are constrained by adding 3D hinged supports at nodes B and C and vertical rollers at nodes A and C, see, e.g., Figure 7(a). The deployed configuration is reached by applying a vertical displacement at node B and out-of-plane displacements at nodes E and G. Notice that the deployed configuration shown in Figure 7(b) is just one possible solution due to the multistable property of such gridshell.

Future studies will explore the use of strained geodesic gridshells to serve as provisional or permanent support for the restoration of heritage masonry vaults.

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