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Fornix: the circular platform-frame

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Abstract

This paper explores the potential contribution of stone construction methods to environmental sustainability and modern construction and structural requirements. Drawing inspiration from an unrealized project by architect Pino Pizzigoni (Bergamo, 1944), it examines the spatial equilibrium of interlocking stone modular ribs designed without binders or reinforcement materials. The study addresses the structural performance, customization possibilities, and constructability of these structures by employing an integrated concept development method that combines a parametric design setting with mechanical analysis and physical concept models. The conceptual study indicates that the interlocking ashlars shall be able to maintain the thrust line within the structural section, allowing the structure to absorb external stresses typical in building constructions. Moreover, it suggests the possibility to achieve a three-dimensional equilibrium of mutually self-supporting rigid bodies that are firm even during installation. The findings lay the groundwork for a computational design method to produce resistant, spatially coherent ribs composed of modular elements that self-balance during construction and achieve improved structural capacity under operational conditions.

Keywords: ashlar beams; interlocking blocks, self-balancing structure, platform frame, mechanical analysis, dry masonry, Greek stereotomy, formwork, barrel vault.

1. Introduction

In 1944, Gio Ponti's magazine, "Stile", published an article on page 36 of issue 12 entitled "Una Nuova Struttura Lapidea per la Costruzione delle Chiese" (A New Stone Structure for Church Construction). The author, Giuseppe (Pino) Pizzigoni (Bergamo), also describes in it the project submitted to a competition announced on the occasion of the Eighth Triennale of Milan for the construction of a new church in the experimental QT8 neighborhood, which the Milanese institution had planned as a demonstrative example of post-war reconstruction. The structural concept was that of a barrel-vault like structure, simply supported, where adjacent arches interpenetrated each other along the transversal cross-section. As illustrated in Figure 1, each block had a longitudinal milled profile resulting from the intersection with the corresponding pair of blocks belonging to the adjacent arch.
The project for that church, as well as the patent (Pizzigoni [1]) for the structure that characterized its conceptual matrix, were not followed up.

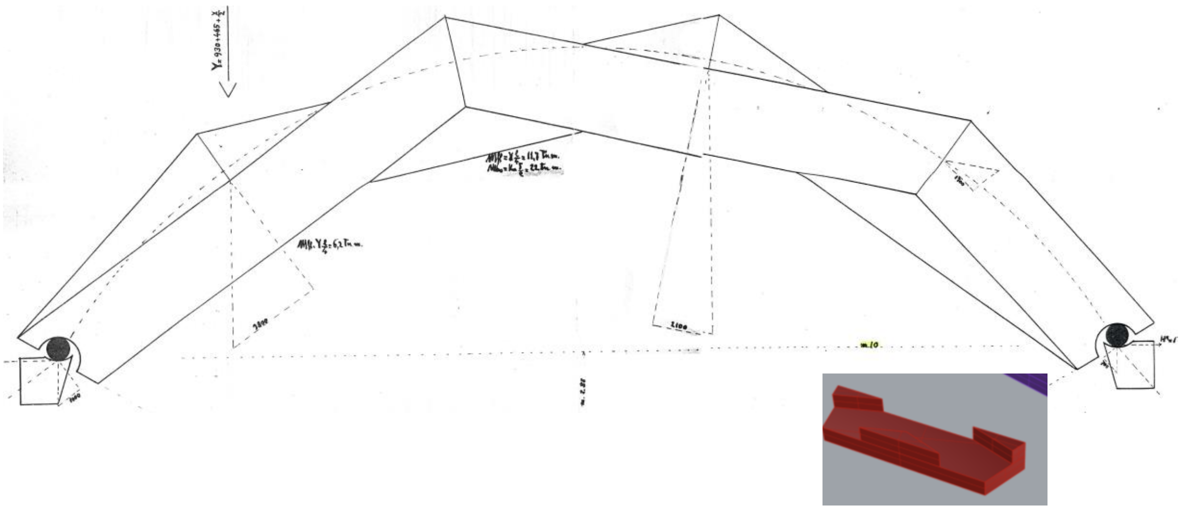

Figure 1 Drawings from the original patent by Giuseppe Pizzigoni [1] and detail of the block highlighting the interlocking features.

The project is interesting because it seems to combine two structural systems that have given rise to constructions of different shapes and purposes, such as arches, domes, pavilion coverings, and so on: those of masonry interlocking blocks and those of reciprocal frame structures.

Reciprocal Frames have been employed for roof construction in ancient Asia, and later on were proposed for flooring by Leonardo, Serlio, and others (see Pizzigoni [2]). Until global stability is ensured, they can be installed without the support of a formwork. They are defined by the fact that the composing elements do not meet at the vertices but superimpose along their length, in such a way that each element is supported and supports the adjacent ones, characterizing a non-hierarchical structural system (see Parigi et al. [3]). Because of the location of the reciprocal supports, transmitted actions are typically bending and shear. Consequently, structures of this type have traditionally been made of timber, bamboo, and other bending-resistant, lightweight materials.

Meanwhile, the proposed usage of stone as a construction material relates the design to masonry structures. These are a subset of compressive-only structures. Contrary to Roman Opus Cementitious, these structures do not rely on binders or reinforcements but achieve stability through the strategic hierarchical organization of each component to directly transmit loads along its axis. They are examples of structures with very high durability and very low embodied carbon. As discussed by Heyman [4], masonry structures display large cross sections and self-weight, which are functional to ensure that the thrust line is contained within the section of the blocks, thus avoiding bending failure and increasing shear frictional resistance (see Beatini et al. [5]). The construction of heavy masonry structures is on the other hand challenging.

The milled design of the stone further characterizes the case study by directly connecting it with the recently rediscovered interlocking contact interfaces. Originating from ancient Egypt (see Beatini et al. [6]), their morphological possibilities were systematically analyzed in the studies on stereotomy by Philbert de l'Orme. Interlocking joints are those that limit the reciprocal movement of blocks due to geometric constraints. As firstly analyzed by Dyskin [7], under the assumption of perfectly fixed supports and rigid materials, the kinematic incompatibilities ensure the stability of the structure. In masonry arch designs, these joints are employed to stabilize along the line of thrust, effectively reducing the risk of shear and bending failures. However, in this instance, the transversal cross-section of the blocks is planar, deviating from common applications. This design relates more closely to the lever principle, articulated by Aristotle and utilized in the construction of heavy masonry structures such as Gothic cathedrals in Northern Europe (see Bruun et al. [24]) and possibly in the tholos of Athena Pronaia in Greece (see Pizzigoni et al. [8], and Aita et al. [9], [10]), and recently in novel dome designs by Pizzigoni et al. [11].

Given the above considerations, the authors have hypothesized that the project under study was meant to ingeniously combine the advantages of reciprocal frames and interlocking masonry joints. If so, this would not only constitute a captivating design but also a contribution to the pressing need for carbon-neutral structures that are structurally robust, can be constructed with locally available materials, appear modular and easy to be dismantled, and employ little or no formwork. Moreover, the manufacturing difficulties that the project would have faced at the time it was proposed would substantially be diminished today, thanks to advances in water jet cutting and 3D printing.

Before delving into a full structural analysis, under the goal to evaluate the potential sustainability gains of the system, a preliminary tectonic, integrated analysis is performed, encompassing structural, design and construction aspects.

In Section 2, a minimal structural unit of the arch is preliminarily analyzed, mechanically, with the aim to discuss its equilibrium conditions. In Section 3, the design is generalized to make it suitable for curves and voussoirs other than the ones in the original project, and the design process is automated. Section 4 discusses aspects related to material strength and constructability, leading to further concept refinements. Conclusions summarize the main findings and briefly outline next steps.

2. Mechanical static analysis

A static analysis has been put in place, to inspect the governing equilibrium equations of the structural module and resulting first assemblies, as underlying basic units for load-carrying capacity and self-support quest during construction.

Toward that purpose, a first configuration was analysed, with three blocks (Figure 2), set as to acquire a symmetric shape in the “vertical” plane. As mentioned in the Introduction, blocks are mutually supported through unilateral constraints, under the assumption of infinite strength. So far, the analysis is confined to a two-dimensional geometry, and associated static equations that are delivered in-plane. The underlying assembly of static equilibrium equations involves the gravity load of each block, the reactions from the external supports, delivered at surface edges that remain un-connected on the sides and, most of all, mutual reaction static actions that are exchanged at the surface interfaces allowing for mutual and global self-support. Once such internal interface actions are devised, in terms of forces and moments at the planar interface, generally allowing for shear components (e.g. in the presence of friction) and positioning along the interface, the set of 3n equilibrium equations of the n-block assembly (n=3, so far), is written. The whole can be numerically implemented, e.g. within a MatLab environment, through a quadratic programming formulation, to allow for systematic numerical treatment. Given the intrinsic nature of the problem, to be highly statically-undetermined, different equilibrium solutions may be sought, in the hypothesis of rigid block behaviour, for instance those that shall minimize selected targets, as that to set to zero local tangential forces and highlighting possible overall statically-determined configurations, for example with pre-imposed locations of force transfer along the contact surfaces (as per a point-joint) (Figure2a).

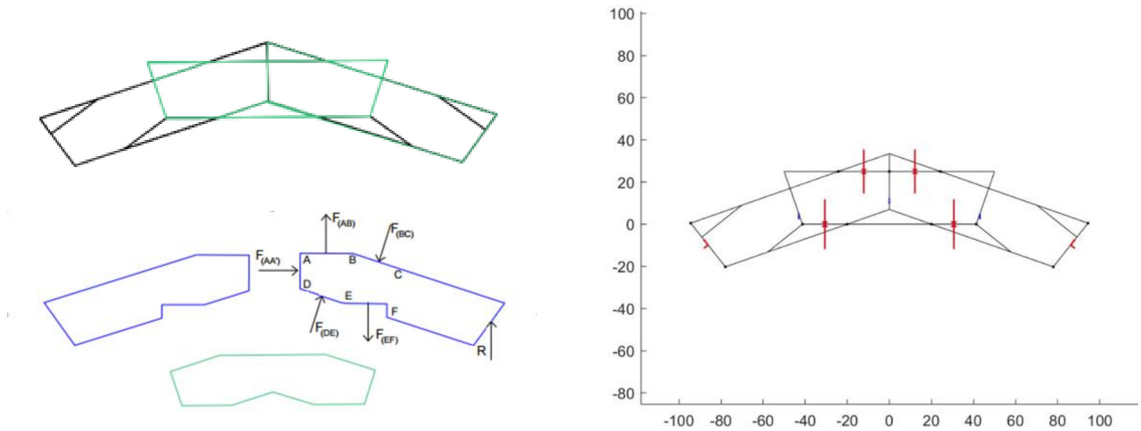

Fig. 2. A minimal structural unit composed of three blocks and actions assumed at the contact interfaces of a block (left). Computed statically determined configuration with external and internal actions (right).

This is done for the problem at hand, according to first trials that are being made. Figure 2b shows an "isostatic" equilibrium configuration (actually "hypostatic", as simply supported at the two extremes, at free horizontal translation) that can be achieved by the system, among the possible equilibrium solutions. Thereby, an equilibrium solution with no tangential forces at the interfaces is foreseen and depicted. In the classical spirit of Limit Analysis, which may conveniently be employed to handle different structural forms, such as 3D truss-frames (see Ferrari et al. [12], [13]) and masonry arches (see Cocchetti and Rizzi [14], [15], [16]), this is rather comforting, in terms of no collapse states, as linked to missing friction, due to a theorem of Drucker [17].
This shall corroborate promising potentialities, for the considered modular structural element and assembled scheme, in generating self-supporting configurations with aspired architectural and mechanical functioning and properties.

3. Generative design

The blocks depicted in Figure 1 could display any desired structural depth. However, their interlocking feature derives from the corrugation created by the superimposition of two parameterized arc curves, expressed by the angle between the blocks, as shown in Figure 3a. By construction, this angle (α) is half the angle subtended by each block (θ). The smaller the angle, the smaller the interlocking feature. If the number of blocks for the given curve is increased, the height and the slope angle of the interlocking feature tend to zero, eventually leading to standard arches without interlocking effects.
In view of custom design, optimization, and fine tuning of desired structural properties, different configurations may be sought to achieve self-supported structures that may display overall static determinacy, no-thrust configurations for global curvilinear geometries (e.g., for arches), reversible up/down use with respect to the gravity forces, and so on. Therefore, a new design procedure has been envisioned, as shown in Figure 3b. Here, the upper (and lower) profiles of blocks lying on adjacent ribs weave around the arc to be approximated. Consequently, the angle between the blocks in the figure is now independent of the angle subtended by each block, and more blocks can be arranged within the original spacing.

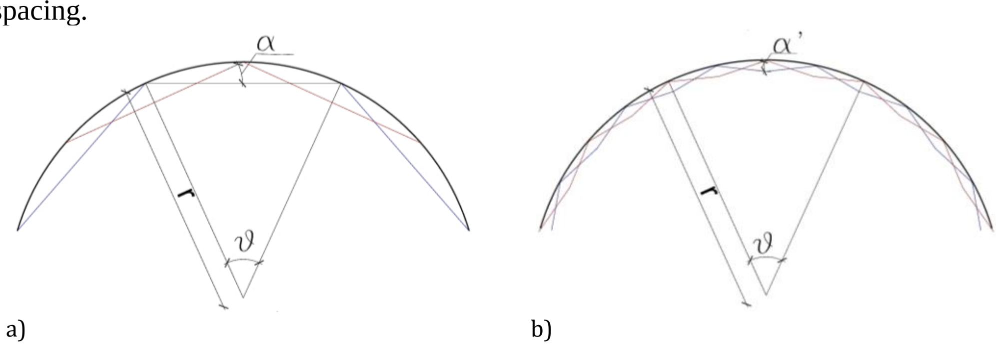

Figure 3 (a) Interlocking profiles created by the superimposition of polylines approximating a reference arc curve and characterized by sloping angle α. (b) Interlocking profiles created by polylines weaving at custom distances from the reference arc curve, characterized by sloping angle $\alpha' = \alpha$.

The construction of the blocks can be easily automated (Figure 4). Given a curve s, divided into intervals of lenght Δs, two weaving curves are created whose points are at distances h_1 and h_2 from the original curve, $h_1 + h_2 = d$ being the depth of the block. Slope angle α is now given by

$$\tan(\alpha) = \frac{(h_1 + h_2)\Delta s}{\Delta s^2 - h_1 h_2}$$

and as such can be set independently from θ.
By offsetting the curves, both the lower and upper profiles of the blocks are generated. The profiles of the protruding features, which are hatched in the figure, are automatically generated. These are then extruded along the binormal to the curve, as defined by the Frenet–Serret frame, and are all parallel to

each other by construction. In alignment with generative design procedures for structural ribs (see e.g. [20]), limit values of the structural depth (d) shall eventually be computed to avoid self-intersection of the curves, functional of the curvature values and structural span Δs, and to preserve the self-stabilizing capabilities of the blocks during the construction phase.

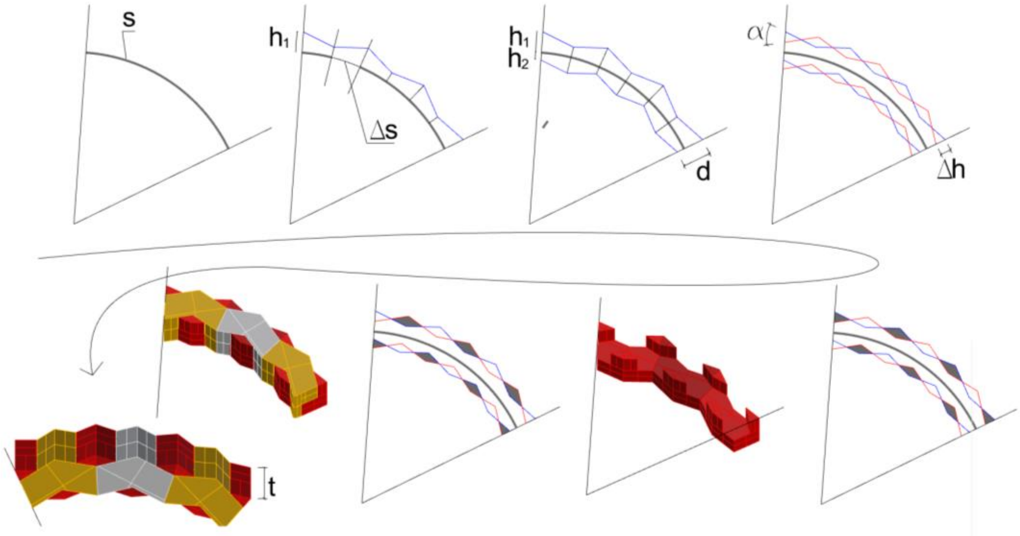

Figure 4 An automated procedure to create interlocking ribs from a given curve. The reciprocal slope of the blocks over each other, their transversal thickess t and their depth d are all independent from the radius of the arc curve, s, and the number of blocks spanning it.

It can be noted that the transversal thickness t of the blocks can independently be set. This opens the opportunity to create spatial structural ribs and bifurcations (Figure 5). In this scenario, as in the previous case, blocks are transversally extruded by simply translating each vertex along the binormal to the reference curve at the closest point on the curve.
In extending the $2D$ case, limit values of the transversal thickness t shall be sought that, for the given torsion of the curve, maintain the aforementioned design objectives.

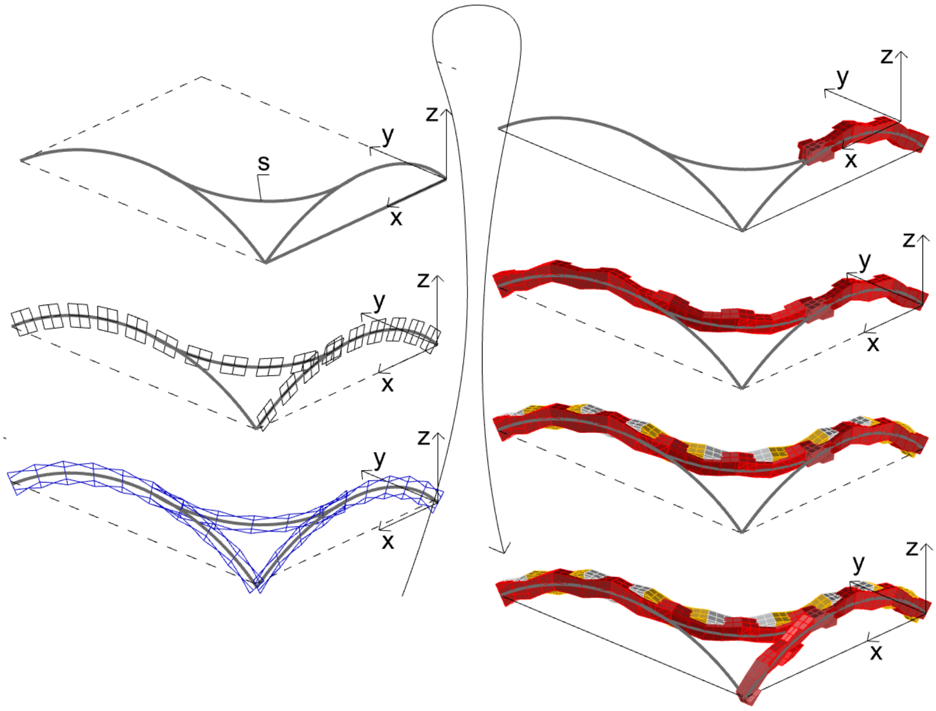

Figure 5 The design process of interlocking blocks allows to spatially arrange the blocks and to produce bifurcating blocks, where two ribs merge.

4. Constructability

Brunelleschi's cupola in Florence (Beatini et al. [18]) and after him Sangallo (Paris et al. [19]) and many other architects of the XVI and XVII centuries in France, reinterpreted architectural classicism and were able to build enormous domes without scaffolding during construction, by orientating simple blocks in such a way that they were locking each other even during construction. Over the years, designers have sought new ways of working on the geometry of structural elements to expedite construction, whether through kinetic structures (see e.g., Beatini et al. [21]), or 3D printed active bending structures (as in Fathollahi et al. [22]) and interlocking designs (e.g. Pizzigoni [23]). Modular self-supporting structural elements and assembled macrostructures may be advantageous considering maintenance requirements. To validate the concept, a demonstration scale model has been printed in PLA, as shown in Figure 6. Through the installation process (visible in the video attached to the oral presentation of this conference), each of the 10 elements composing the circular profile can find self-standing equilibrium conditions during the installation phase, without a sustaining formwork. Each block, during assembly, is not perfectly fixed to the adjacent ones, which allows for construction tolerances, but finds its equilibrium in a similar fashion to a lever, a principle already employed in Greek architecture.
The model provides direct feedback on the feasibility of the construction concept and its ability to withstand basic external actions, such as those due to gravity.

Figure 6 Scale model of the interlocking blocks during assembly, 3D printed in PLA
https://youtu.be/a3v0kSaaMJE

Moving toward real construction, 3D printing allows a much higher control on geometry, now even at the architectural engineering scale, and the possibility to construct precise and complex forms at a fraction of the cost. This can be advantageous if one considers that stress concentration during or after assembly may arise, damaging the structure. Therefore, a new module is now under investigation, that distributes the contacts along a smooth surface. Three of such modules are depicted in Figure 7. The design method extends the wave concept elaborated in Section 3 to 3*D* spirals evolving along the reference curve. In Figure 8, the design process is outlined. The reference curve, here a circle, is divided into intervals of a specific length. The two end protuberances and middle, larger protuberance characteristic of each block are here generated by two consecutive spiral curves *a* and *b* that rotate around the reference curve in opposite directions, enclosed by circles of radius *r*. The depth and thickness of the block are therefore function of *r*, and the profile of the block, exemplified by curves *a* and *b* in the figure, is given by

$$\begin{cases} x = r \cdot \sin(t) \\ y = r \cdot \cos(t), \quad \text{for } 0 \leq s < \frac{\Delta s}{2}, \\ z = z_s \end{cases}$$

$$\begin{cases} x = -r \cdot \sin(t) \\ y = -r \cdot \cos(t), \quad \text{for } \frac{\Delta s}{2} \leq s < \Delta s \\ z = z_s \end{cases}$$

with $0 \leq t \leq \pi$.

At each cross section, two blocks compose the spiral rib. The full design is depicted in Figure 9. To make the picture clearer, consecutive, and interlacing blocks are depicted in red, maroon, grey and yellow colors. However, as before, all blocks are equal, being the curvature of the reference curve constant, and one block can be employed throughout. Considering an implementation with clay or concrete based matrices, material proprieties as modified by the 3*D* deposition method shall be optimized for the structural actions.

Figure 7 Three interlocking blocks with a spiral shape. 3D printed in PLA.

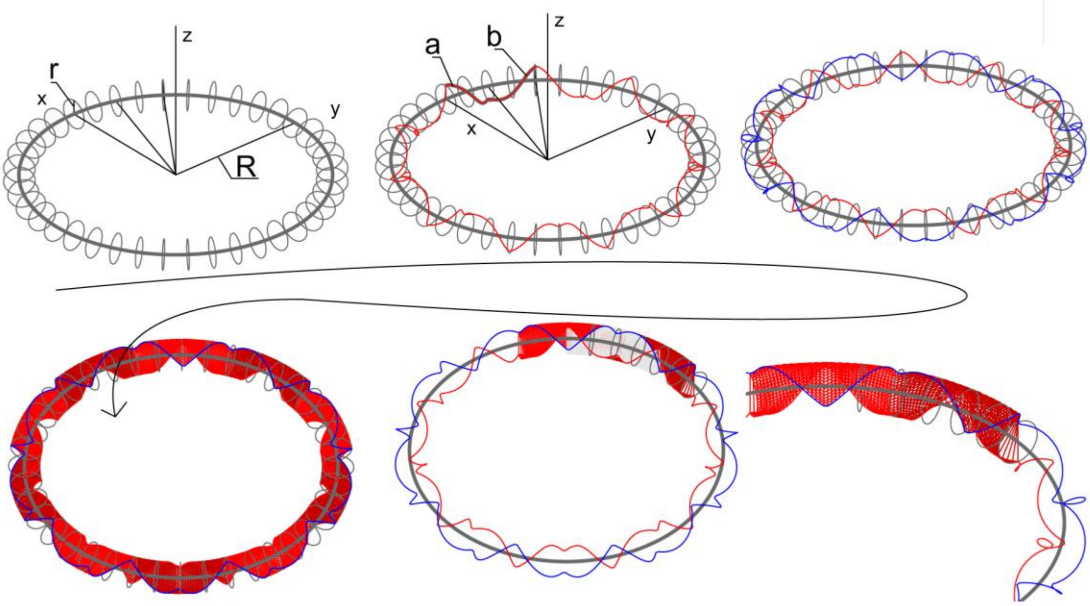

Figure 8 Automated design of interlocking, 3D spiraling blocks generated along a circular reference curve. Blocks are all equal and distinguished by color for visual clarity.

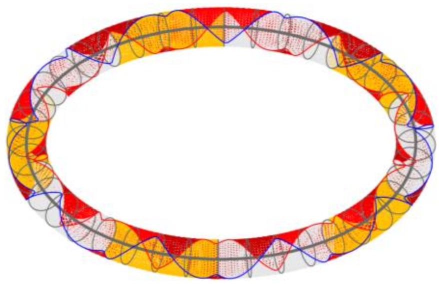

Figure 9 The full design of Figure 8, composed of 20 equal blocks, combined with the underlying reference circular curve and spiral profile, which generates the cross section of the blocks. Blocks are all equal and distinguished by color for visual clarity.

5. Conclusions

This paper builds upon the idea of a stone arch proposed by architect Giuseppe Pizzigoni after World War II as an example of possible post-war reconstruction. The proposed concept study illustrates the intriguing possibility of creating structures that rely neither on binders nor reinforcement for their structural capacity. An integrated concept development was performed through parametric design, physical scale models, and preliminary structural analyses, so far delivered in plane.

Structural studies indicate that the interlocking design may demonstrate enhanced robustness under operational conditions. Three-dimensional strength and stability analyses would further be needed to confirm the finding, especially considering the stress distribution within the interlocking feature.

The introduction of geometrical relationships governing a generative design approach sets the stage for future studies, which will define explicit limit values considering the curvature and torsion of the target curve. Finally, qualitative investigations into constructability indicate that each element during assembly can find its equilibrium state without formwork. This ability does not derive from perfectly matching interlocking profiles on (which would be difficult to achieve in real constructions) but on the self-equilibrium of weights.

The preliminary results underscore the foundational principles of the project, bridging the theoretical groundwork laid by the self-balancing capabilities allowed by internal supports in reciprocal frame structures, the inherent stability provided by the gravitational weight in masonry, and the innovative combination of these principles through interlocking masonry constructions. This synthesis not only reaffirms the project's conceptual underpinnings but also sets a clear directive for future research. A holistic study, encompassing the interaction of all these aspects, is imperative to fully realize the potential of the integrated, tectonic design approach, ensuring results conform to the envisioned potentials.

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