
Natural slope: experimental methods and computational models searching for shape generated by controlled falling sand flows

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Abstract

Controlled falling sand flows are governed by complex mechanisms so that their behavioral model, showing properties of both solids and liquids, represents a barely explored research field, within the contest of form finding. This paper recounts the explorative process on controlled falling sand flows conducted by the corresponding author at the University of Naples Federico II during her Master's Degree thesis research activities. In particular, it presents an experimental study on the sedimentation processes of sand piles obtained by pouring sand from a point of emission with fixed flow rate. After that, the equilibrium condition achieved by the sand flows on a collision plane, defined by Frei Otto as the "natural angle of repose", is observed. The research is focused on the analysis of behavioral reaction of the sand to addition and subtraction. It investigated three typologies of experimental systems, in order to define geometric solutions, consistent with the conical surfaces derived by Euerlian principles, and two computational methods, capable of processing the same physical simulations and of generating approximate solutions in terms of three-dimensional surfaces, although using non-mechanical algorithms. Finally, the presented work highlights the potential of the self-organization of the sand that make ridged and developable surfaces and generate repeatable and controllable solutions, despite the complexity of the shape.

Keywords: form finding, sand flows, natural slope, "discovering by doing", geometric methods, computational design, self-organized material, developable surface.

1. Introduction

"Any granular material falling from fixed point forms a cone on the surface below and a funnel within the granulate mass with the same angle of inclination, the natural angle of repose, 35 degrees". (Frei Otto, 1972)

In some of his experimental models, Frei Otto observed sand and deduced behavioral characteristics that help to understand the complexity of granular material. In the formation of sand piles, showing both solid and liquid characteristics, a physical dualism, due to its inconsistent and adaptive nature, is evident. Its behavior is deeply related to mechanism of self-organization [1], still poorly known and controllable, but, according to which, as demonstrated experimentally, a sand flow generated from a fixed point of emission would form, by "natural slope", conical surfaces with constant slope. The contribution of the presented research is to have complete mastery of the aggregation processes of the material and, as far as possible, to be able to predict the physical-mechanical behavior of the controlled falling sand flows in presence of known boundary conditions. In particular, the goal is to foster the architectural design of sand profiles without the necessity of using complex mechanical analyses, by studying analytical functions, including polylines and Nurbs, for the approximation of natural slopes.

2. Controlled falling sand flows

In the context of form finding, in which the observation of natural processes leads to the definition of new structural languages, the analysis of controlled falling sand flows represents an innovative field. The first experiments on sand were conducted by Frei Otto [2], who defined the "natural angle of repose of 35 degrees" as the equilibrium condition achieved by a sand pile after falling from a fixed point. Since then, the most recent experimental research has been elaborated by Gramazio & Kohler at ETH of Zurich [3], [4], in which a computational algorithm ensures perfect control of the final solution and of the material aggregation processes during the simulation of "procedural landscapes".

To understand the main characteristics of granular matter, an exploratory process on the behavioral patterns of sand flows from a fixed point of emission began. Observing the natural formation of a sand pile on a plane, the fluid emission deposits in a solid form, in which the final conical surface has a constant slope about 35° and a "ridge" as highest point. In this case, an elasto-plastic flow is determined with a dissipation energy that is a function of the variable "z", the distance of the emission point from the collision plane, and that must consider factors such as imperfect axially of the flow, frictional forces, different size of the grains and mutual collision. The incidence of boundary conditions is considered by Frei Otto's sandbox, in which the subtraction holes of the collision plane determine a critical equilibrium state of the sand with two main stresses, a radial stress " σ_r " that distribute the grains from the center outward and a tangential stress " σ_θ " that affects compactness and circularity (Figure 1). This is the reason why the angle of repose is 35° degrees and results lesser than 30° degrees, the angle of soil friction.

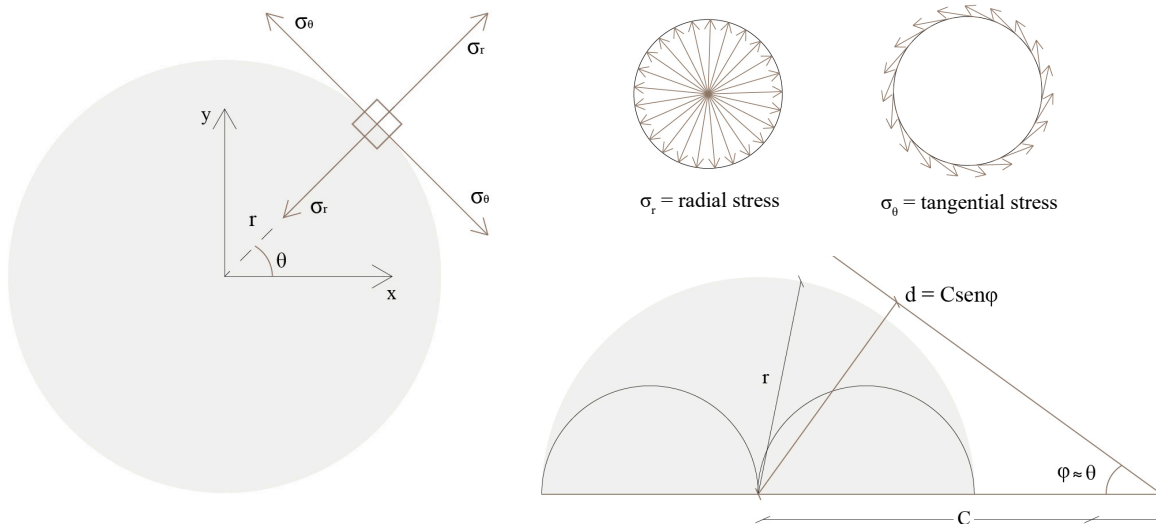


Figure 1: Typical stress state of sand in presence of subtraction holes on collision plane

2.1. Experimental models

The perfect adaptability of granular material suggests the complex nature of sand. To observe its natural behavior, according to a "discovering by doing" approach, three different experimental systems, for producing granular volumes, were investigated. The first one was inspired by Frei Otto's sandbox and featured a collision plane, side panels to contain the emitted material and an underlying box to accommodate the excess (Figure 2). These experiments were useful to understand the kinematics of sand flows and to analyze the natural process of aggregation, which can be traced back to the Voronoi's geometry. It was possible to verify how sand organizes itself at holes and how ridge points are distributed at material subtraction geometries. However, the produced shapes and those which could be generated are complex to control and poorly measurable due to many variables affecting the final configuration of the surfaces: uncontrolled emission, undefined amount of sand, unconstrained flow, uncontrolled flow velocity, excessive dispersion of grains.

1. base panel
2. sand container
3. side panels
4. collision plane
5. removable panel

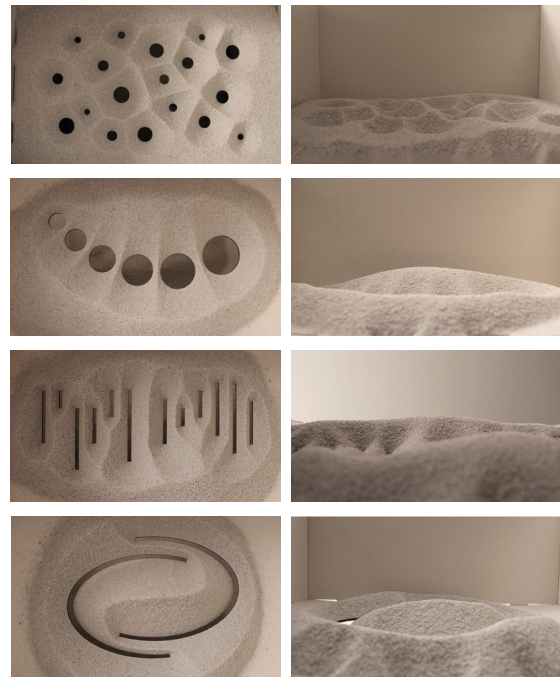
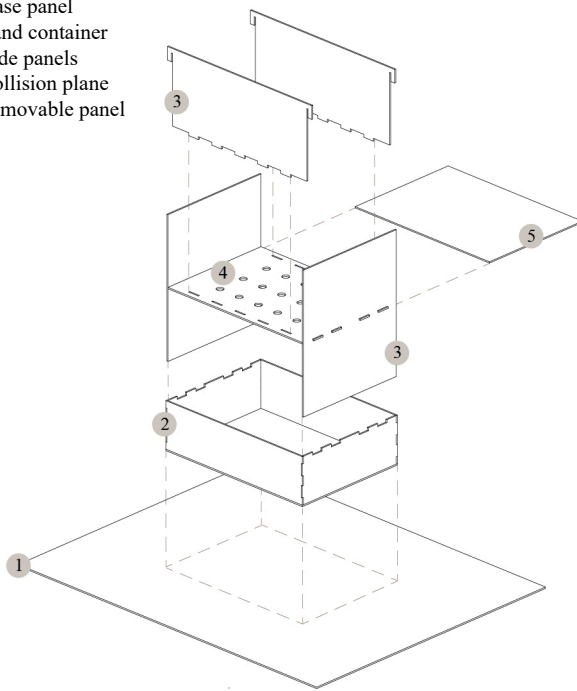


Figure 2: Sandbox components and some experiments

In order to reduce the shown variables of “sandbox”, a second experimental system, called "sandsystem" (Figure 3), was developed. It is planned to control the amount of emitted sand, to measure the amount of material lost after emission and to define the direction of sand flow by introducing a funnel used as emitter and placed 12 cm away from the collision plane. By joining these inputs with geometrically defined shapes on the collision plane, dimensional results could be evaluated, despite discontinuous boundary conditions: lability of emitter supports, approximation of collision center, irregularity of the sand flow due to manual control of the emission, excessive dispersion of the grains.

1. base panel
2. sand container
3. collision plane
4. emitter supports
5. emitter funnel

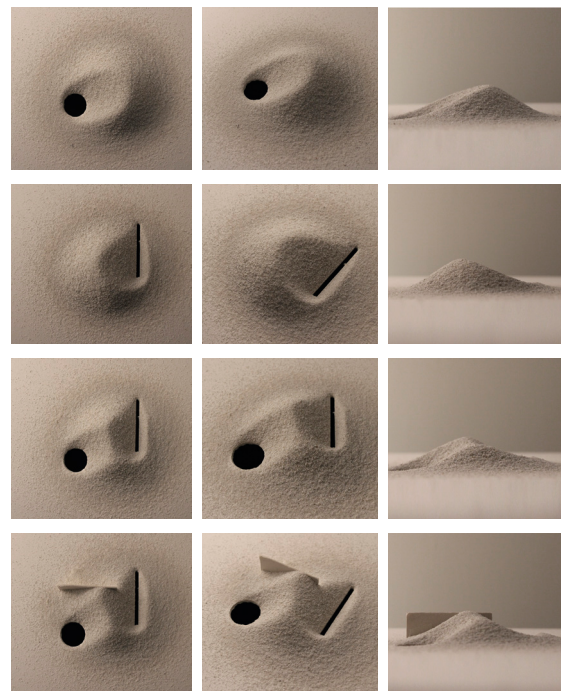
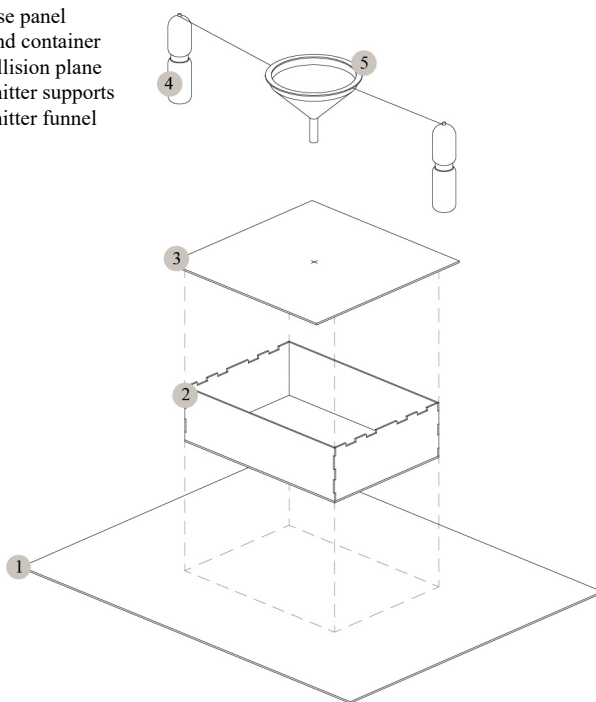


Figure 3: Sandsystem components and some experiments

Searching for an improvement of the second experimental system, "sandmachine" (Figure 4) was designed, using the laser cutter Atomstack X20 pro, as a control system to constrain the point of sand emission in x-y directions. A specific emission element, that could fit into the track normally intended for the laser cutter tool, was designed and some funnel parameters were modified. The nozzle size was halved to 0.5 cm and the distance of the emission point from the collision plane was increased to 18 cm. In addition, thanks to the introduction of a hygrometer, repeated experiments could be carried out, keeping the ambient humidity between 53 and 55%.

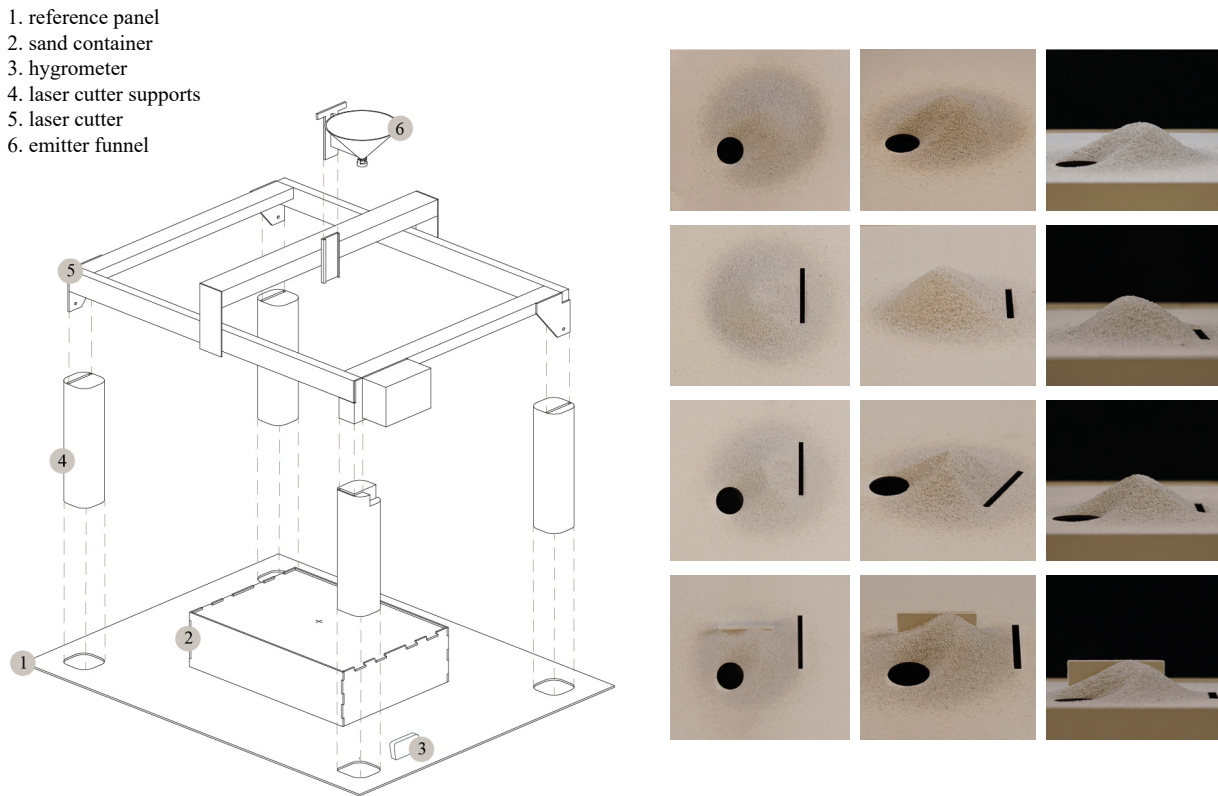


Figure 4: Sandmachine components and some experiments

The device so configured will be used soon to analyze the shape of sand flows emitted from a moving nozzle. As far as this research is concerned, the shown experiments highlighted visual differences between "sandsystem" and "sandmachine", later confirmed by photogrammetric processing of digital images in Metashape (Figure 5). Setting up a photographic scene, imposing shutter times, iso and focal lengths, taking care to indent the box edges, used by the software as reference points, in each shooting area, allowed for a three-dimensional restitution of the experimental models (Figure 6). After the insertion of input data and the point cloud generation, the overlap of meshes, especially in the second system, confirms the analogy of a sand pile with developable conical surface and demonstrates how the presence of a smaller emission hole greatly affects the flow, in terms of trajectory and axiality. Moreover, the dispersion of grains is reduced and a more beveled surface, than the first system tested, is generated.



Figure 5: (a) Sandsystem; (b) Sandmachine; (c) Overlap

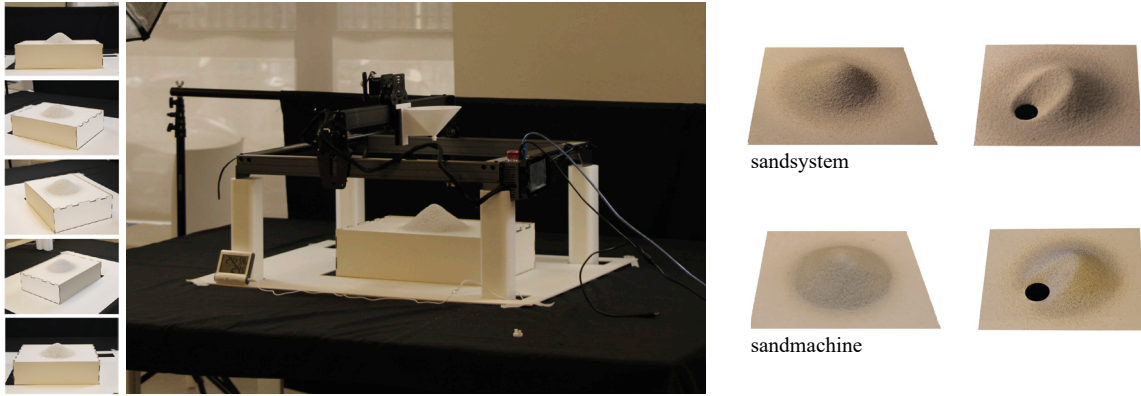


Figure 6: Photographic staging and 3D models

According to Eulerian principles, when the collision plane has circular holes, the subtraction of sand generates a hyperbolic intersection surface between two cones (figure 7). Note the distance between two centers (1), cone parameters (2) and single cone equation (3), it is deduced that the geometric locus of the intersection points between two cones corresponds to the equation of a hyperbola (4).

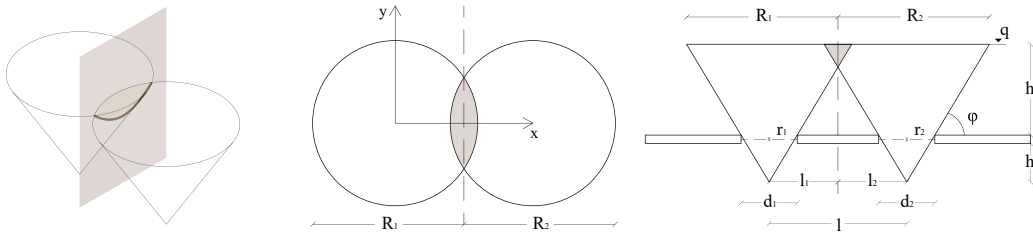


Figure 7: Geometric representation of the theoretical model

$$l_1 = \left(l + d_1 - \frac{d_2}{2} \right) \frac{1}{2} \quad (1)$$

$$R = q + \frac{rtg\varphi}{tg\varphi}; h_1 = q + rtg\varphi; h_2 = rtg\varphi \quad (2)$$

$$x^2 + y^2 + tg^2(\pi - \varphi)z^2 = 0 \quad (3)$$

$$z = \sqrt{\frac{l^2 + y^2}{tg(\pi - \varphi)}} \quad (4)$$

By applying the equations as input parameters in Grasshopper it was possible to confirm the validity of the theoretical model and to obtain a geometric definition of the experimented surface in presence of a circular hole on the collision plane (Figure 8).

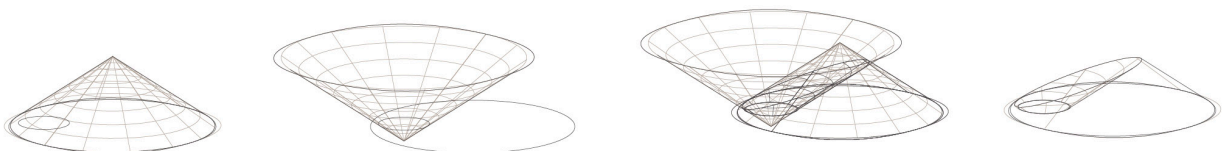


Figure 8: Definition of the theoretical model applied in Grasshopper

2.2. Computational methods

The introduction of the theoretical model into the parametric software was a key step in moving from a purely experimental phase to the computational translation of the tested models. Starting from known values, two algorithms which generate similar three-dimensional solutions, were developed. The first one, "heightfield method", is inspired by the principle of generating a surface by means of a point cloud. This method (Figure 9) involves using the hole size and the diameter of the sand pile, measured in experimentation, as input for generating a plane mesh and identifying intersection points. Each point is translated with a vector "v" having a dimension equal to the distance of the single point from the closest curve multiplied by the tangent of 35° , the angle of repose. By comparing the obtained results with Metashape surfaces (Figure 10), this computational system proved satisfactory and allowed for dimensional matching of all the granular surfaces processed in the experimental phase.

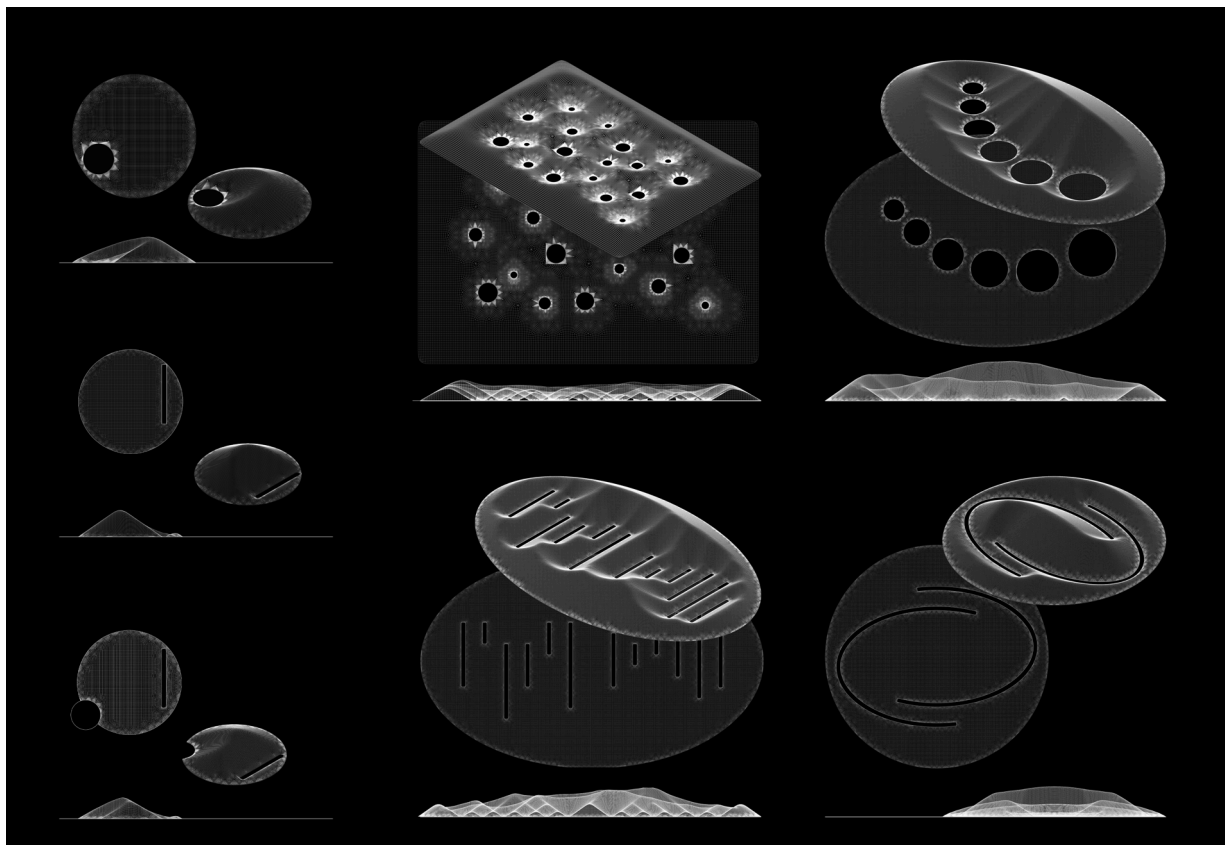
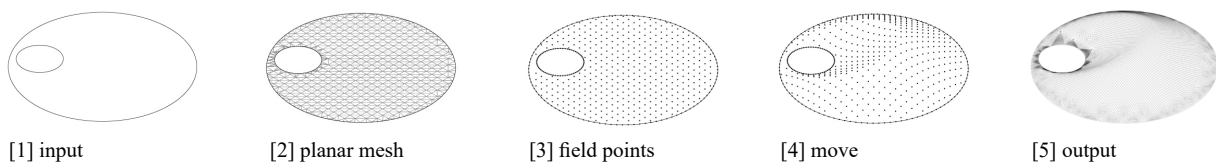


Figure 9: Grasshopper definition of heightfield method and some 3D models

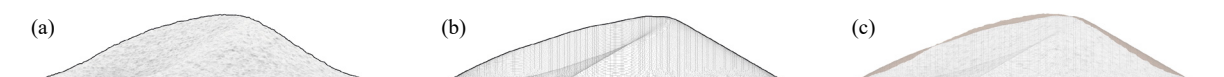


Figure 10: (a) Metashape; (b) Heightfield method; (c) Overlap

The second computational method draws inspiration from the Ph.D. Thesis of A. H. Abouelkheir elaborated at the ENSA Paris-Malaquais [5], considering the analogy between the granular surfaces and Voronoi's system. Then, "Voronoi method" (Figure 11) is tested, introducing the same input parameters of the first method. The known geometries are divided into the same number of points which give the tessellation of the planar surface. The intersection points of the Voronoi cells, shifted by the same vector "v" used in the heightfield method, forms the ridge line of the sand pile. Processing ridged surfaces and transforming polylines into nurbs, the final surface was obtained. As in the previous method, comparison with the Metashape surfaces (Figure 12) is a confirmation of the results obtained by the computational method.

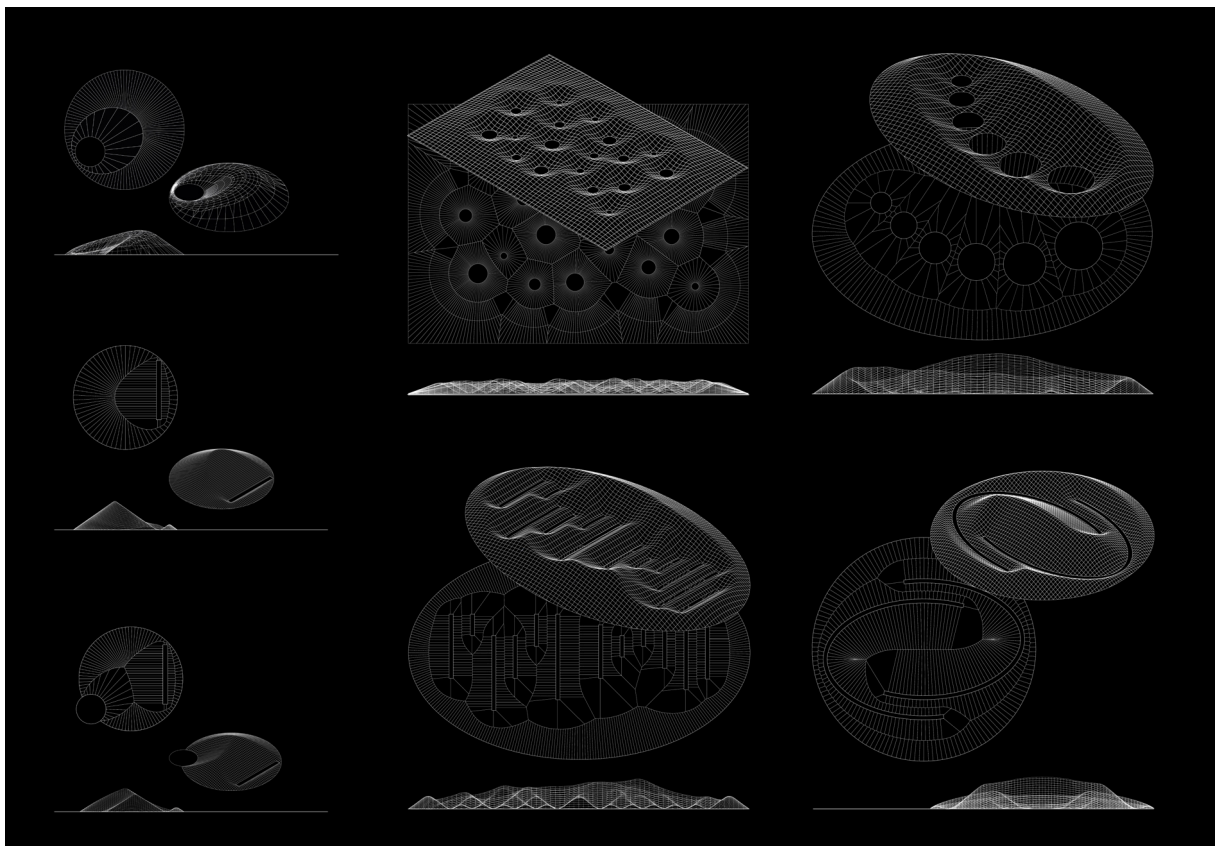
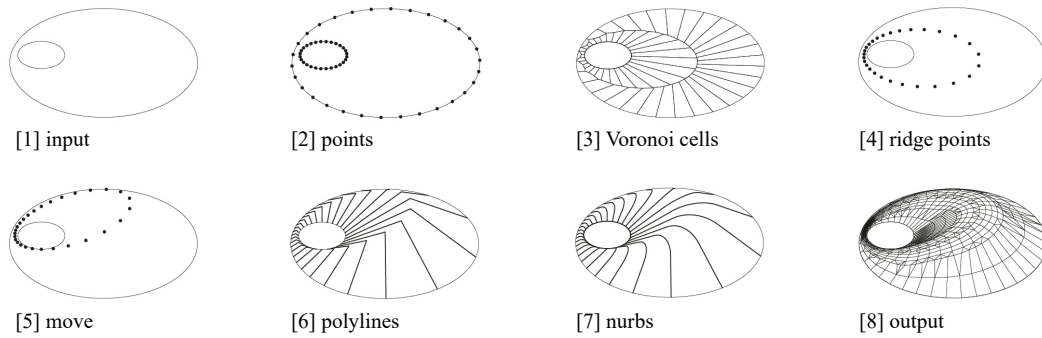


Figure 11: Grasshopper definition of Voronoi method and some 3D models

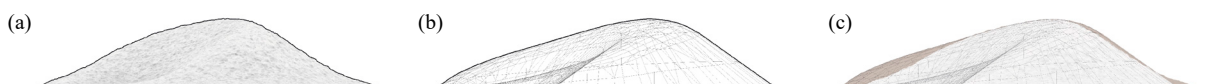


Figure 12: (a) Metashape; (b) Voronoi method; (c) Overlap

The digital reproduction of the experimented surfaces represents the goal and the main contribution of this exploratory process, that confirms the possibility of making repeatable the variety of generable shapes, totally controlling the behavioral reaction of sand flows through physical simulations. In addition, the possibility of controlling sand flows suggests a potential application to real structures, transforming, for example, the designed profiles into building roofs, since the geometric analysis of the granular surfaces confirmed their analogy with developable and, therefore, ribbed conical surfaces. This condition opens new design scenarios in which the shapes generated by controlled falling sand flows can be used as formworks, composed of linear elements, for building shell structures (Figure 13).

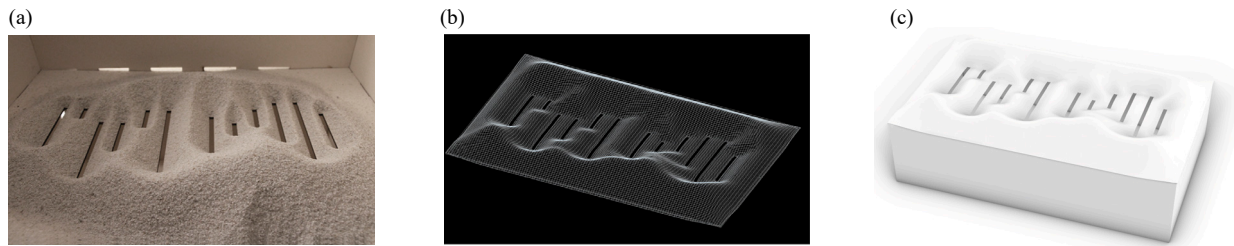


Figure 13: (a) Experimental model; (b) Computational model; (c) Potential application

3. Experimental architecture

The experiments described so far are concerned with the kinematics of dry sand, free from any interaction with other materials. In the previously mentioned Ph.D., in order to solidify the sand, a system, called "hyosand", composed of pentachloride disulfate, was developed. The question then arises as to whether there exists in nature an element that could catalyze the shapes generated by controlled falling sand flows. In conclusion of the research work, further experiments were carried out (Figure 14) by exploring the "fusion" between sand and a common vinyl glue. The reaction between these elements is completely random and uncontrollable. Specifically, by pouring the glue onto the conical surface of the sand pile, only a superficial layer of the pile is crystallized (Figure 15), and the resulting shapes are poorly controllable, since they are affected by variable factors, such as the viscosity of the glue and the grain size of the sand.

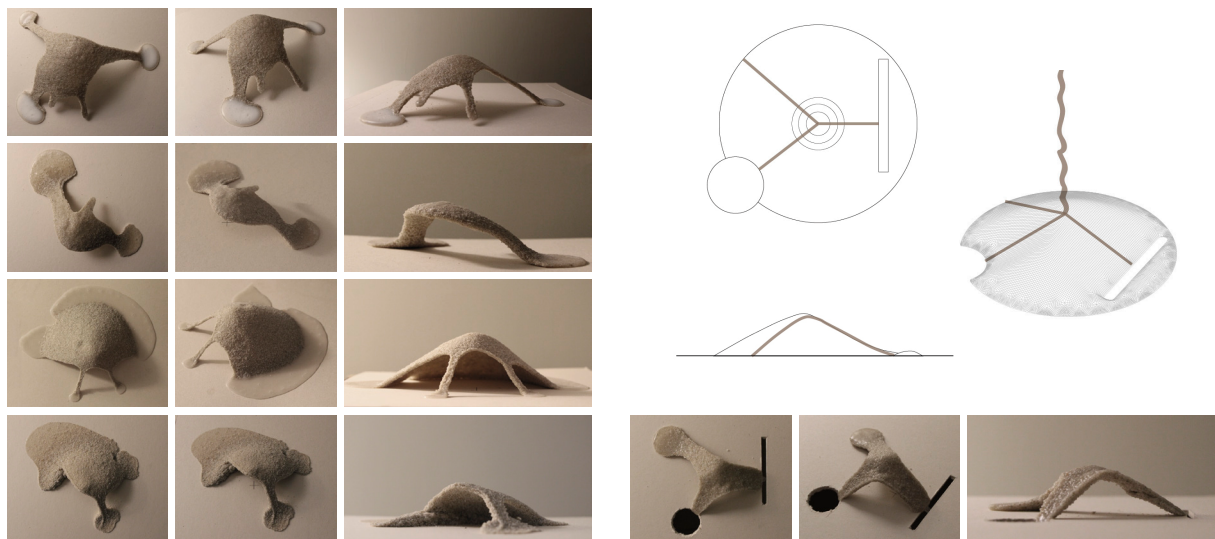


Figure 14: Some experiments pouring glue on sand pile



Figure 15: (a) Sand pile; (b) Glue casting; (c) Final surface

Despite the extreme randomness of this solidification system, a few common elements have been highlighted, including the double curvature and the presence of points where there is more casting of material. By working out a planar discretization of the experimented surfaces in Kangaroo, in which the points of more casting are used as anchor points, length and elasticity of the "arms" are set up and a uniform load is applied, verisimilar three-dimensional models consistent with funicular behavior are obtained (Figure 16). Although the properties of the resulted surfaces are still under investigation, the results show how sand profiles can be successfully used as formworks and how it is possible to obtain structural shells from the fusion of granular surfaces with a catalyst element. These preliminary solutions open interesting prospective about experimental architecture. Such a technique could be very appealing for designing structures in case of difficult environmental conditions and with limited on-site resources.

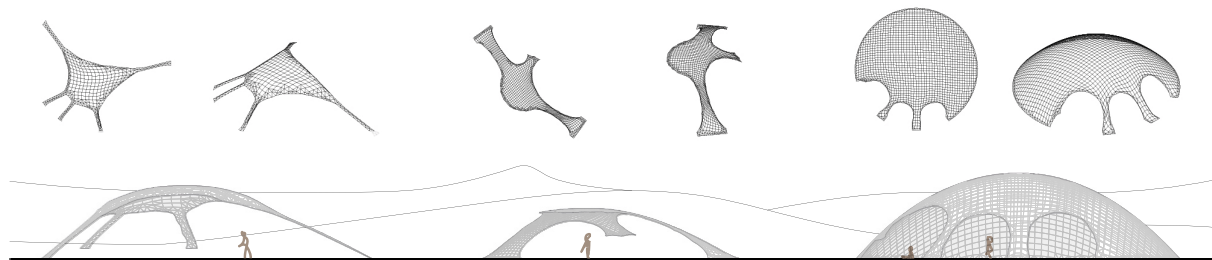


Figure 16: Experimented surfaces in Kangaroo

4. Conclusion: submission of contributions

The presented research investigated the behavior of controlled falling sand flows by means of an experimental campaign and, after a computational evaluation, showed how the obtained sand profiles can be approximated by analytical surfaces, thus avoiding complex mechanical analyses that would make the design process burdensome. Moreover, some preliminary experiments have shown how sand profiles can be used as formworks, although these aspects, as well as the mechanical characteristics of the obtained shapes, are still being studied.

This experience represents a further contribution in the context of form finding by controlled falling sand flows, that although it is still barely explored, it may develop significantly in the future. The technology of architecture is a constantly evolving machine in which the desire to search for new forms to be translated into structural realities merges with the idea of sustainable design. Granular surfaces could be a possible answer to the demand for innovation, and the current challenge is to be able to reproduce and "crystallize" the formal opportunities generated by sand flows, transforming them into architectures that respect their form and nature.

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