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The Chebydesic pavilion: a path towards flattenable geodesic elastic gridshells

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Abstract

This research develops a design strategy for geodesic gridshells flat assembly, focusing on measuring the stresses in the connections and investigating the influence of geodesics layout on the flat assembly feasibility. The objective is to study the influence of geodesics orientation on a given surface to suggest a combination of Chebychev and Voss nets. The first is known to be the ideal layout to assemble a flat grid with isotropic cross-sections, while the latter has the benefits of drawing a geodesic mesh with quadrangular flat panels. The optimised grid presents the benefits of short on-site assembly, cost-effectiveness and limited material waste. Isogeometric and nonlinear structural analysis algorithms are implemented to model the flattening process of prestressed grids, and measure stresses in the connections at each stage. Finally, theoretical results are illustrated on a large-scale timber pavilion named "Chebydesic", built in September 2021. The design, assembly, and erection stages are presented as well as physical tests and scans to validate computational results.

Keywords: elastic gridshell, geodesics, flattening, isogeometric analysis, nonlinear structural analysis

1. Elastic gridshells made of laths

1.1. Elastic gridshells

Gridshells are freeform structures, generally doubly curved, made of one or more grids behaving like a shell. Elastic gridshells are obtained by the elastic deformation of an initially flat structural grid. Therefore, such structures offer not only the mechanical performance and lightness of shells, the efficiency of planar construction, but also the reversibility of prefabricated structures. In a way, they embody the confluence between architects and engineers, with the structure being both aesthetically pleasing and functionally efficient.

The German architect and structural engineer Frei Otto [1] spent a decade developping gridshells. His first large-scale project, the Multihalle of Mannheim built in 1975 is arranged along a Chebychev net, obtained by the deforming a planar quadrangular net and preserving the edge length. Frei Otto [1] developed the "Compass method" to create Chebyshev nets with two non-degenerated curves on a surface, see Figure 1. Many studies have been done on this topic, such as [2], [3], [4].

Despite Otto's success, the lack of numerical tools remained an obstacle in the development of gridshells. Since the 2000s, the improvement of numerical calculation methods and the advent of new materials allowed for gridshell projects to spread across architecture. Gridshells have been built in various materials,

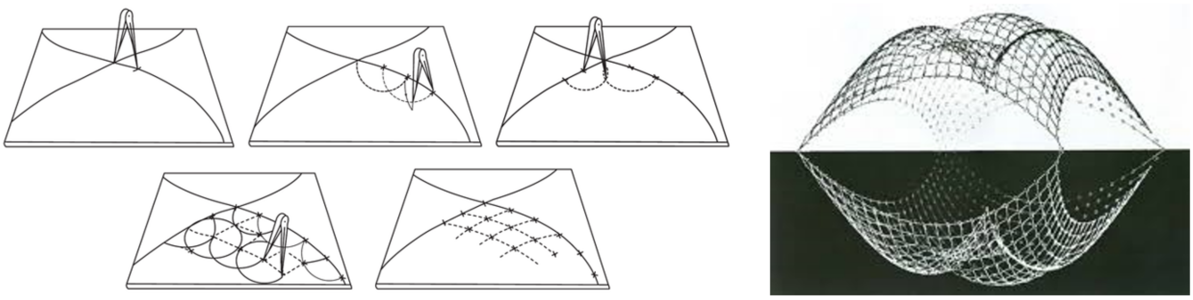


Figure 1: Compass method to generate Chebychev nets

such as timber[5], steel [6], composite [7], [8] and reclaimed skis [9], [10].

1.2. Geodesic and Voss gridshells

Pirazzi et al. [11] first tackled the design of geodesic shells. By orienting the beams tangent to geodesic lines of a target surface, it allows to use anisotropic cross sections. Indeed, by aligning the beams minor axis with the geodesics curvature vector, major axis bending is avoided and high curvatures can be reached with a minimum bending energy. This is particularly useful in timber construction as materials are often manufactured in panels or beams with rectangular cross-section.

This study deals with the design and construction of an elastic geodesic gridshell based on Voss surfaces [12]. A Voss surface presents two sets of geodesic curves whose intersections form planar quadrangular panels, called Voss net.

Therefore, Voss net properties are very important for the design of envelopes. Making curved glass panels can be difficult and expensive, so the discretization of a surface into planar faces is often tackled in design projects. This study aims at extending [13]'s thesis by adding a detailed consideration of the flat assembly process, running exhaustive design analyses and building an elastic gridshell pavilion based on a Voss net in September 2021.

1.3. Flattening geodesic gridshells

Assembling a net on a doubly curved surface at height and with hundreds of connections is time-consuming and expensive. The optimal scenario usually consists in assembling the gridshell on the ground and subsequently lifting it to its final position. The only work at height is the connection of the bracing system. However, designing a geodesic gridshells is more restrictive due to the asymmetric bending behaviour. Indeed, flattening a geodesic grid removes its geometric properties, major axis bending arises, potentially preventing the assembly on the ground. So, its feasibility has to be first assessed. [14] studied the construction of elastic geodesic gridshells, assembling a flat grid made from rectangular cross sections and deploying it to get its final shape. This approach eases the prefabrication, transportation and erection of gridshells.

[10] studied the feasibility of geodesic gridshells using parametric modelling and 2D relaxation. The ground assembly feasibility is achieved by comparing the 2D grid major axis bending to the section properties. However, the study does not account for members prestress. This study aims extending its research by modelling a Voss geodesic elastic gridshell, accounting for members initial prestress and assessing the ground-assembly feasibility.

2. Modelling gridshells flat assembly

2.1. Flattening strategies

Following this first attempt [10], the strategy is improved in this paper by using NLSA and beam members to flatten the grid and determine the initial prestress required to connect the members together. This new approach consists in relaxing the deformed grid to a minimum bending energy configuration without forcing the grid to be fully flat. It is done by releasing the support conditions, controlling the cross-sections orientation and maintaining the connection between crossing beams. Reversely, it consists in visualising the grid being assembled in a position where the beams are connected with the minimum energy, then elevated while the beams ends are pulled to their final location. The allowable height for ground assembly without any scaffolding is set at $1.5m$.

2.2. Numerical methods

The study is done using Isogeometric Analysis (IGA) to run models using curved beams with a limited number of calculation nodes. Using *Kiwi!3D* [15] plugin, IGA is associated with nonlinear structural analysis (NLSA) to account for the large displacements and prestress of gridshell structures. As NLSA is based on load and displacement increments, internal forces can be assessed at any iteration and any location along the beams. At each joint and each load step, the forces in the two crossing beams are projected on the joint local axis system. In timber gridshells, connections are often pinned. Consequently, axial and shear are measured during the deformation.

3. Geodesic gridshells flat assembly assessment

3.1. Geodesic nets

The flat assembly feasibility with anisotropic cross sections is not straight forward. Major axis bending is avoided on the deformed shape, but arises in any other configuration. Indeed, a geodesics layout depends on the surface curvature. If the curvature varies with the grid being assembled, the laths are no longer aligned with the geodesics, stresses increase significantly and may lead to material failure.

The elastic geodesic gridshell studied by [10] in the previous article, called ”Tunnel”, is reworked to compare previous results to a more complete and accurate NLSA model.

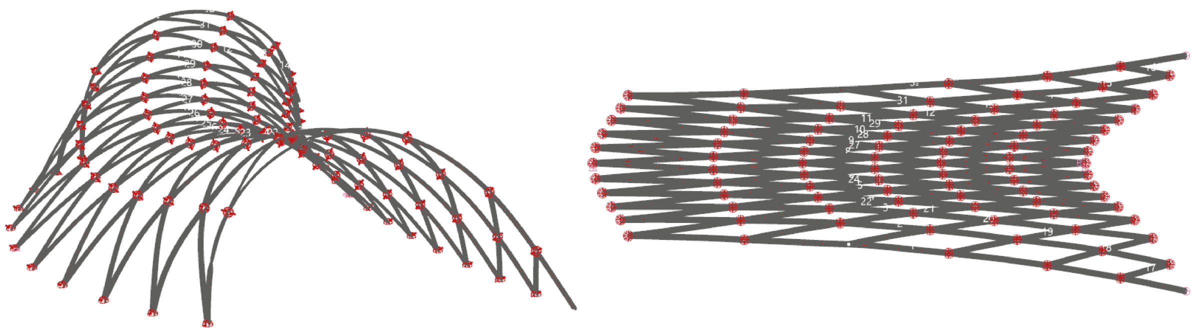


Figure 2: Deformed and flat shape

In this example, the grid total height is reduced from $3.7m$ to $0.7m$, representing a decrease of 84%. This simulation is useful as a scale model was already built and could be used as a comparison with the numerical model. It is presented on Figure 3 and validates the accuracy of the NLSA model.

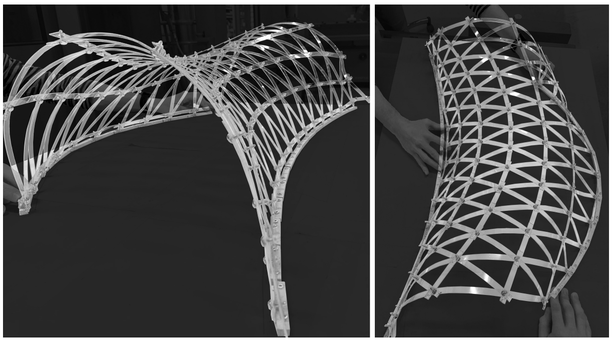


Figure 3: Grid deployed and braced

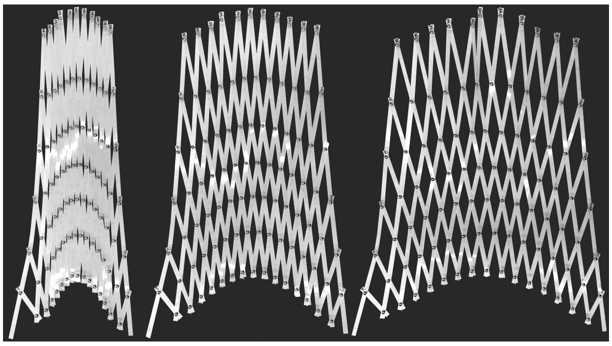


Figure 4: Flat deployment of the grid

Moreover, it showed that this particular typology and this particular arrangement of geodesics are efficient for flat assembly.

3.2. Voss nets

Surface approximation with Voss net

Using [16]'s plugin *Otter*, a target surface S is approximated by a Voss surface S^* using Gauss mapping. First, the target surface S is meshed, and normal vectors at the mesh vertices are projected on the unit sphere. The result is the Gauss image of S. The scatterplot forms a surface S_G which is meshed by a Chebyshev net to define the resulting linear space of Voss nets. An optimisation process is made by deforming and combining eigenmodes to find the closest Voss net S^* approximating S. The target shape and its approximation are shown on Figure 5.

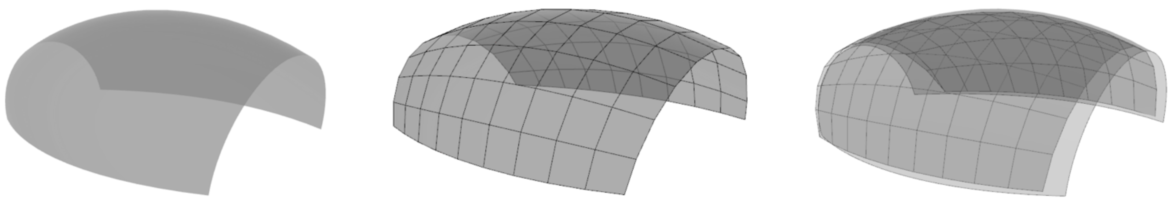


Figure 5: Target shape S, approximation S^* and comparison

In this example, the average distance between S^* and S is 101 mm and it is at most 215 mm representing a maximal discrepancy about 10%.

Relaxation to equilibrium shape

As S^* is created numerically with geometrical criteria, a structural analysis is run to introduce members prestress and reach the equilibrium shape S^{eq}. Results are displayed on Figure 6. Notice that the relaxed members have deformed from an arch-like configuration to a shape resembling that of an elastica.

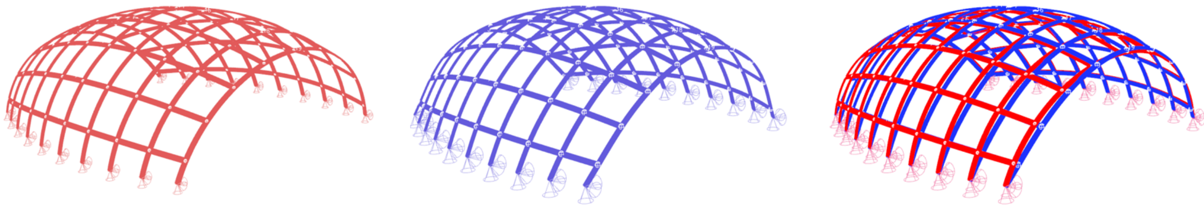


Figure 6: Comparison between initial grid in red, and relaxed grid in blue

Then, the relaxed grid does not strictly meet Voss net definition any longer. The maximum angle deviation is $0.1\circ$ and the face planarity deviation is $2\ mm$ on average and $11\ mm$ at most. Considering materials and connections tolerances, it highlights that the relaxed grid still has the properties of a Voss net.

Flattening:

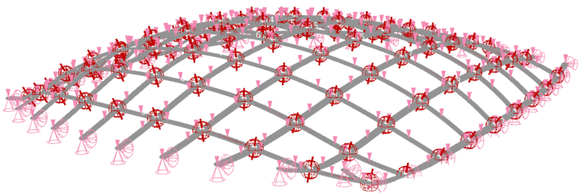


Figure 7: Relaxed ”flat” configuration

The flat grid height reduces from $2.50\ m$ to $1.85\ m$. At this height it is not possible to assemble the gridshell while remaining on the ground. This result is higher than the previous one, the height decrease being only 35% against 84%. One reason would be that geodesics arrangement have an influence on the flattening feasibility. This is investigated by projecting geodesic nets with different orientations on the same surface, then flattening them and measuring the height decrease, see Figure 8. Results highlight wide variations between mesh orientations with an optimum around 30°. Moreover, in the Tunnel gridshell studied in the previous paragraph, the angle between geodesics had a similar value, so, for the rest of the study, 30° is assumed to be the optimal configuration of a geodesic grid.

Thus, it seems that Voss nets are poor candidates for elastic flat-assembled gridshells. Nevertheless, a new concept of geodesic gridshells emerges, consisting in superposing two elastic gridshells. The inner grid is a geodesic non-Voss elastic gridshell whose beams orientation is optimised for flat assembly. And the outer grid is made from a Voss net and connected on top of the deformed elastic (inner) grid. As the Voss net has by definition a different layout than the elastic net, it acts as a bracing system. Moreover, the Voss grid carries the cladding made of planar quadrangular panels. This idea is implemented for the first time in the pavilion described in Section 5.. Using NLSA, the forces in the connections are assessed at each increment, especially to check that the stresses do not significantly increase during the assembly and the erection.

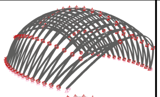
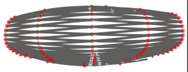
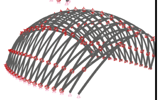
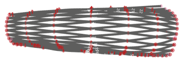
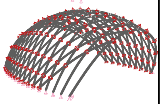
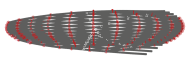
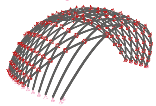
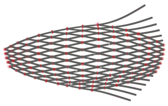
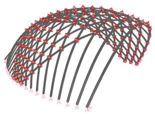
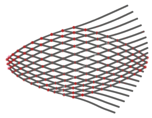
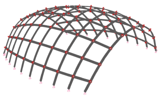
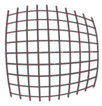
Angle (°)	Initial grid	Relaxed grid	Height (m)	Decrease
14°			1.14	54%
21°			0.78	69%
34°			0.76	70%
47°			1.07	57%
59°			1.14	54%
Voss			1.5	40%

Figure 8: Influence of geodesics orientation on the grid flattening feasibility

4. Pseudo-Chebychev geodesic gridshells

4.1. Performance assessment of pseudo-chebyshev with geodesics

This study highlights the influence of the geodesics arrangement on a given surface. Depending on how open or closed the quad panels are, the relaxation does not provide the same results. Especially, it seems that Voss nets are not compatible with flat assembly. To optimise the geodesic warps and wefts layout, the strategy is to combine it with a Chebychev net. Approximating a Chebychev net with geodesics is a way for a grid made of anisotropic cross sections to approach an ideal configuration for which a grid made of isotropic cross-sections can be assembled fully flat.

4.2. Performance assessment of pseudo-chebyshev with geodesic patchs

The strategy above is arduous to apply using continuous geodesics. The principal curvatures vary a lot across a freeform surface, so it's not easy to guarantee a relevant approximation from one support to another. Moreover, timber laths are cut from standard panels up to 5m long. Both reasons lead to a piecewise geodesic approach using geodesic patchs, inspired by the work of [17], called "zip-patches".

This allows to present a workflow on Figure 9 to design an elastic gridshell combining Chebychev nets and Voss surfaces. This process is applied to a pavilion named "Chebydesic".

5. Illustration with Chebydesic pavilion

5.1. Design principles

Once the concepts of geodesic flattening feasibility are assessed, they are put into practice with a scale 1 pavilion. Several independent algorithms are developed along the research and are then associated. The design process starts with a target shape, which is the input in our chain of algorithms. It is first approximated by a Voss surface, then meshed with a Chebychev net, divided in 3x3m patches, and finally approximated by geodesics.

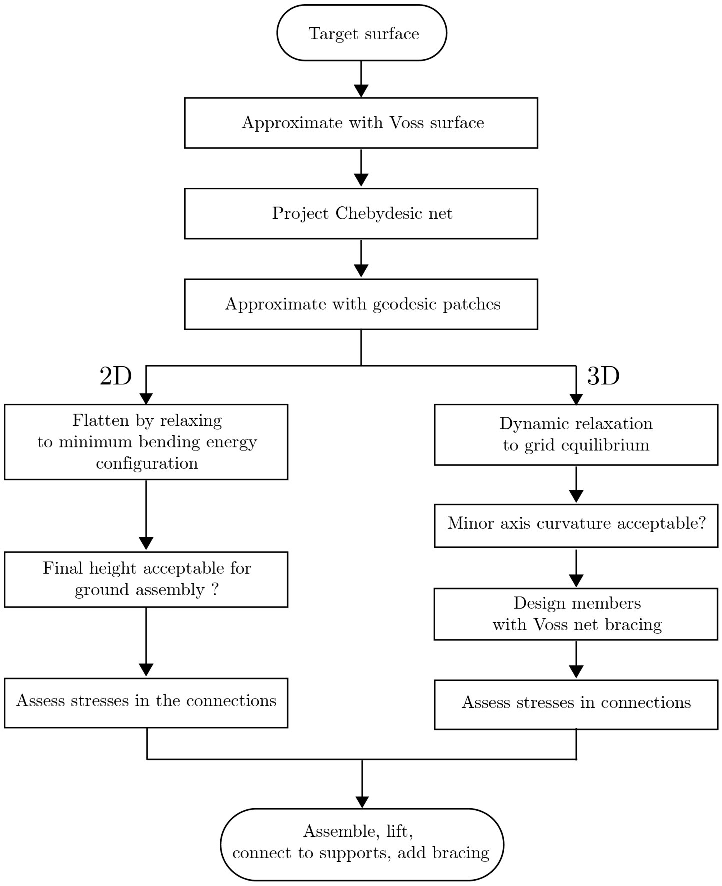


Figure 9: Design process using Voss surfaces

5.2. Geometric design

Inspired by the Berlin Zoo House for Hippopotamus, designed by Gribl architect and SBP engineering, an approximated dome-shaped Voss surface, called "Chebydesic", is generated, as shown on Figure 12. The design of this $2.5m$ high prototype led to the construction of a pavilion in mid-September 2021.

Two grids are designed: one being an elastic geodesic non-Voss grid designed to be assembled flat and deployed to its final position using the results from Section 3.2.; the other one being a Voss grid installed on the elastic grid. Not only is this second grid carrying the cladding made of quadrangular panels but it also acts as a bracing system. The facade is made of two Voss surfaces, inviting to enter the gridshell.

5.2.1. Mesh generation and patching

Following the flowchart above, the egg-like shape is approximated by a Voss surface, see Figure 10.

The process to generate the "zip-patches" is presented on Figure 11 and the resulting grid is described on Figure 12.

5.2.2. Flattening

The flattening of the Chebychev patched grid is assessed first with a mockup, see Figure 13. It shows that the most curved patches, on the right and left edges,to should be removed to enable flat assembly. Knowing the flattening cinematic of the grid helped to implement the NLSA model. The final height is $0.8m$, corresponding to a 84% decrease.

The NLSA model also allowed to calculate the stresses in the connections during the assembly, to com-

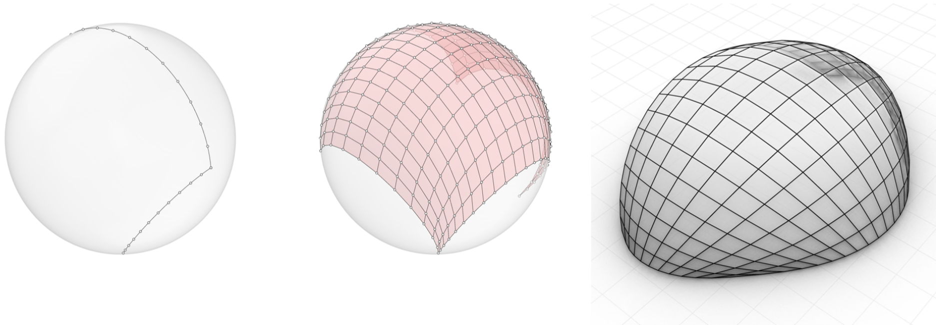


Figure 10: Generation of the pavilion surface: from Gauss mapping to Voss net

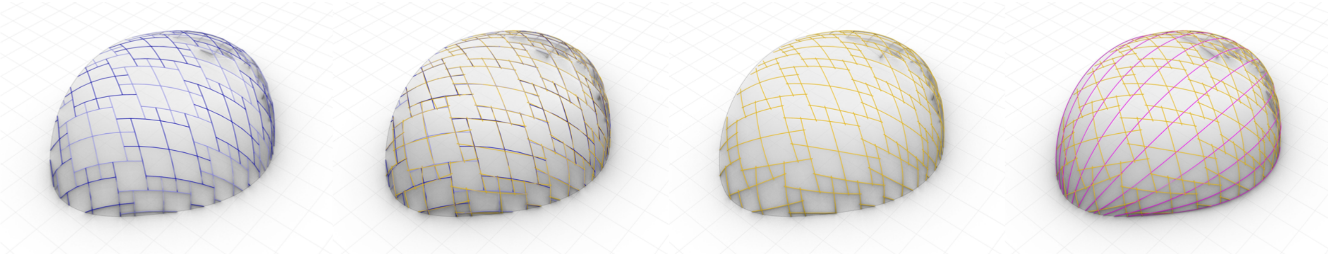


Figure 11: Generation of the pavilion grid: from Chebychev net to geodesic patchs

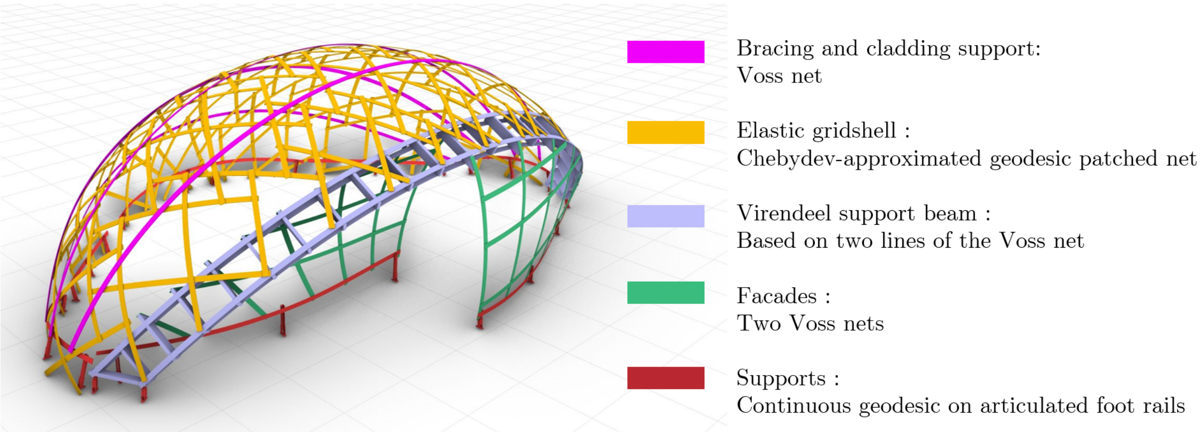


Figure 12: Description of pavilion structural elements

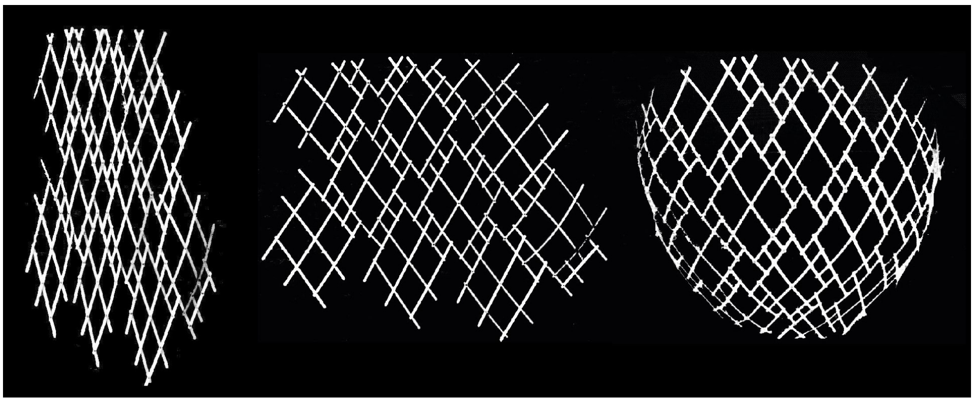


Figure 13: Chebydesic mockup of the whole grid, from flat to deployed configuration

pare them to the stresses in the deformed configuration and to check that they remain acceptable, see Figure 14.

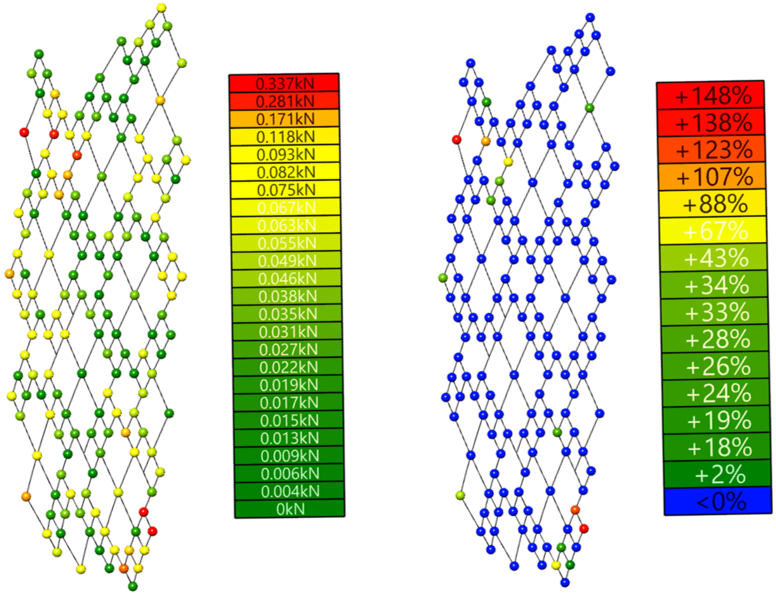


Figure 14: Chebydesic *Kiwi!3D* flat model after removing side patchs displaying the connections axial force values and increase

Pictures of the final pavilion are presented on Figure 15.

Figure 15: Chebydesic pavilion

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