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Topology Optimization of Two-dimensional Tensile Trusses using Different Materials

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Abstract

This paper addresses the discrete topology optimization problem of tensile trusses by proposing a bi-objective genetic algorithm (GA) that integrates continuous prestress optimization for individuals within the GA. The "tensile truss" is defined as the truss carrying prestress, either compression or tension, in its members, even when no external loads are applied. To achieve lightweight and large-span structures, we introduce high-level prestress to the tensile trusses, enabling the use of tensile materials with high tensile stiffness for constructing the tensile members. Prestress optimization aims to find the optimal linear combination of prestress modes to increase the proportion of tensile materials used. The selection of the initial population of GA, a crucial step in the optimization process, significantly impacts the optimization results. We utilize a plastic design method featuring the member length penalty to provide an initial population with diverse statically determinate structures for the bi-objective GA. Our examples demonstrate the effectiveness and efficiency of this method. The entire optimization process results in tensile trusses with diverse topologies that are distributed along a Pareto front. Comparisons between them and conventional truss structures show that tensile trusses have superior structural performances.

Keywords: Topology optimization, Prestress optimization, Tensile truss, Genetic algorithm, Plastic design

1. Introduction

The "tensile truss" is a practical concept defined as the truss carrying prestress, either compression or tension, in its members, even when no external loads are applied. Compared to conventional truss structures, tensile members play an important role in withstanding loads in tensile truss structures. A special type of tensile truss is tensegrity, consisting of isolated compressive members and continuous tensile members. By definition, the tensile truss is much closer to the beam string and suspend-dome with no isolation restriction for compressive members. By introducing high-level prestress to tensile trusses, prestressed compressive members can be constructed by bars, and prestressed tensile members can be constructed by cables without worrying about the sign change of axial forces in them due to varying external loads. This opens up the possibility of utilizing high-performance stiffness material to reduce the sizes of the tensile members, aiming at a lighter structure. For instance, the tensile elastic modulus and tensile strength of carbon fiber-reinforced polymer (CFRP) cables can be more than twice those of steel [1], therefore, the same stiffness and strength of a member can be achieved by twice small sizes. Furthermore, with the development of materials, CFRP has been widely used in reinforcing or retrofitting old buildings and has great potential for creating innovative architectural forms [2].

Topology (layout) optimization of truss (-like) structures has been well studied for a long time. Michell developed one of the earliest truss layout optimization approaches [3]. He found the optimal distribution of materials in a truss applied with a single load case called the Michell structure. Michell's work has set the foundation for research into the optimal layout of trusses with materials of the same or different strength in tension and compression [4]. However, since the Michell structure is impractical due to its infinite number of members, Dorn et al. [5] developed the ground structure (GS) method to overcome it. Ohsaki [6] applied the genetic algorithm (GA) and GS method to the truss topology optimization and introduced a topological bit to indicate the existence of members in GS. For the optimization of tensile truss, existing studies concentrate on finding the special topology satisfying the isolation restriction of tensegrity structures [7]-[9]. Different materials for different types of members have not been considered in these studies.

This study solves the topology optimization problem for tensile trusses by proposing a bi-objective GA that integrates prestress optimization for GA individuals. Specifically, we employ a bi-level optimization strategy that combines a global optimizer, which uses bi-objective GA for discrete (0/1) topology optimization based on the GS method, and a local optimizer, which uses the search algorithm for continuous prestress optimization. The topology optimization minimizes the total volume and strain energy of the structure. The prestress optimization aims to find the optimal coefficients of the linear combination of prestress modes to increase the proportion of tensile materials used. It is important to note that the structure must have prestress modes, implying that it should be statically indeterminate. It differs from conventional truss optimizations, which yield statically determinate structures, e.g., Barta [10], Rozvany et al. [11]. However, we found that the results of this method have forms close to the Michell structure and statically determinate trusses derived by continuous truss optimization, inspired by which we propose a plastic design method featuring the member length penalty to generate individuals for the initial population of GA. Various forms of statically determinate structures are generated by varying penalty factors to preserve the diversity of the initial population. Finally, to restrict generating long compressive members that are easy to buckle in GA, the penalized member length is utilized to calculate the volume of any potential compressive member. Several examples will be presented to demonstrate the effectiveness and efficiency of this method.

2. Tensile trusses

The optimal design of a tensile truss starts from a given ground structure (GS) containing *N* nodes and *M* potential members. The topological variables $x_i \in \{0,1\}(i = 1, 2, \ldots, M)$ are used to indicate the existent states of members: $x_i = 0$ for non-existent state; $x_i = 1$ for existent state. The number of existent members is denoted by m, and the number of nodes connected by existent members is denoted by n.

2.1. Prestress

The axial forces in existent members are denoted by $\boldsymbol{s} \in \Re^m$, where a positive force ($s_i > 0$) stands for tension and a negative force ($s_i < 0$) for compression. The axial forces $\boldsymbol{s}$ can be divided into the forces $\boldsymbol{s}_\mathrm{e} \in \Re^m$ due to external loads and those $\boldsymbol{s}_\mathrm{p} \in \Re^m$ due to prestress; i.e.:

$$\boldsymbol{s} = \boldsymbol{s}_\mathrm{e} + \boldsymbol{s}_\mathrm{p} \tag{1}$$

Using the equilibrium matrix $\boldsymbol{D} \in \Re^{2n\times m}$ and the external loads $\boldsymbol{P} \in \Re^{2n}$, we have the following relations [12]

$$\text{Self-equilibrium: } \boldsymbol{D}\boldsymbol{s}_\mathrm{p} = \boldsymbol{0}$$
$$\text{Equilibrium: } \boldsymbol{D}\boldsymbol{s} = \boldsymbol{D}\boldsymbol{s}_\mathrm{e} = \boldsymbol{P} \tag{2}$$

where the zero vector $\boldsymbol{0} \in \Re^{2n}$ means that the prestress does not contribute to withstanding external loads. For a conventional truss, the prestress vanishes; i.e., $\boldsymbol{s}_\mathrm{p} = 0$. Moreover, for a (tensile) truss that carries prestress in its members; i.e., $\boldsymbol{s}_\mathrm{p} \neq 0$, the number of independent prestress modes is given as

$$m_\mathrm{p} = m - \mathrm{rank}(\boldsymbol{D}) \tag{3}$$

The prestress can be written as a linear combination of the prestress modes, lying in the null-space $\boldsymbol{B} \in \Re^{m\times m_\mathrm{p}}$ of the equilibrium matrix $\boldsymbol{D}$, by using the coefficient vector $\boldsymbol{\beta} \in \Re^{m_\mathrm{p}}$; i.e.:

$$\boldsymbol{s}_\mathrm{p} = \boldsymbol{B}\boldsymbol{\beta} \tag{4}$$

Hence, it is possible to manipulate the force state (tension, compression, or no prestress) by the linear combination of the prestress modes through the coefficient vector $\boldsymbol{\beta}$. In this study, we assume that the force state is determined only by the prestress, for which we introduce two assumptions on the prestress level as discussed in the next section.

2.2. Prestress level

The prestress level, represented by a positive factor a (> 0), does not change the self-equilibrium of the structure because the following equations hold

$$\boldsymbol{D}\left(a\boldsymbol{s}_{\mathrm{p}}\right) = a\boldsymbol{D}\boldsymbol{s}_{\mathrm{p}} = \boldsymbol{0} \tag{5}$$

Because the axial force $\boldsymbol{s}$ is a linear combination of $\boldsymbol{s}_{\mathrm{e}}$ due to external loads and $\boldsymbol{s}_{\mathrm{p}}$ due to prestress according to Eq. (1), a low prestress level could lead to different signs in the axial forces $\boldsymbol{s}$ and the prestress $\boldsymbol{s}_{\mathrm{p}}$. On the other hand, a high prestress level could introduce (geometrical) non-linearity into the problem. This is because the tangent stiffness matrix $\boldsymbol{K} \in \Re^{2n\times 2n}$ is a linear combination of the linear stiffness matrix $\boldsymbol{K}_E \in \Re^{2n\times 2n}$ and the geometrical stiffness matrix $\boldsymbol{K}_G \in \Re^{2n\times 2n}$, which is linearly related to prestress [12]; i.e.:

$$\boldsymbol{K} = \boldsymbol{K}_E + a\boldsymbol{K}_G \tag{6}$$

To avoid the aforementioned problems for simplification of the optimization problem, in this study, we adopt the following two assumptions on the prestress level:

- The prestress level is high enough, such that the signs of the axial forces follow those of the prestress $\boldsymbol{s}_{\mathrm{p}}$, and they will not be changed by considering $\boldsymbol{s}_{\mathrm{e}}$ due to external loads:

$$\mathrm{sign}\left(a\boldsymbol{s}_{\mathrm{p}} + \boldsymbol{s}_{\mathrm{e}}\right) = \mathrm{sign}\left(\boldsymbol{s}_{\mathrm{p}}\right) \tag{7}$$

- The prestress level is (relatively) low enough, on the other hand, such that it has no significant influence on the tangent stiffness of the structure:

$$\boldsymbol{K}_E + a\boldsymbol{K}_G \approx \boldsymbol{K}_E \tag{8}$$

2.3. Different materials

After the introduction of high-level prestress, we can determine the force state only by signs of prestress in members:

$$\text{Tension: } s_{\mathrm{p}i} > 0; \quad \text{Compression: } s_{\mathrm{p}i} < 0; \quad \text{No prestress: } s_{\mathrm{p}i} = 0 \tag{9}$$

Therefore, we can construct prestressed compressive members by bars and prestressed tensile members by cables without worrying about the failure of cables due to varying external loads.

In this paper, we leverage the lightweight and high-stiffness properties of carbon fiber-reinforced polymer (CFRP) cables to construct the prestressed tensile members. We utilize steel bars to construct prestressed compressive members and members with no prestress because when prestress $s_{\mathrm{p}i} = 0$, the external loads will influence the force state to become either compression or tension.

To distinguish these two materials in the optimization process, members made of CFRP cables have smaller cross-sectional areas ($A = 1/c$) and larger elastic modulus ($E = b$) than members made of steel ($A = 1; E = 1$), where c and b are material coefficients larger than 1. In addition, compressive members are sensitive to buckling, and its Euler's buckling load P_{cr} is given as

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{10}$$

where I and L are, respectively, the moment of inertia of area and the member length; by contrast, tensile members do not have the buckling problem. Hence, we propose a penalty method to consider the buckling effect: when calculating the volume of bar members, we use squared member length L^2 instead of member length L. This method will restrict the generation of too long bars and will increase the proportion of cables in the results if we previously defined that the grid spacing of GS is no less than 1.

The cross-sectional properties of members using different materials are presented in Table 1, where L_i^* is penalized member length, and $\bar{l}_i$ is the original member length which remains constant after the GS is defined.

Table 1 Cross-sectional properties assignment

Existent state	Prestress state	Material	Cross-sectional area (A_i)	Elastic modulus (E_i)	Member length (L_i)	Penalized member length (L_i^*)
$x_i = 1$	$s_{\text{pi}} > 0$	CFRP	$1/c$	b	$\bar{l}_i$	$\bar{l}_i$
$x_i = 1$	$s_{\text{pi}} < 0$	steel	1	1	$\bar{l}_i$	$\bar{l}_i^{\,2}$
$x_i = 1$	$s_{\text{pi}} = 0$	steel	1	1	$\bar{l}_i$	$\bar{l}_i^{\,2}$

3. Optimal design of tensile trusses

3.1. Optimization method

In the tensile truss optimization, we have two objective functions. Objective 1 (Obj. 1) aims at a stiff structure, while objective 2 (Obj. 2) aims at a light structure. These two objective functions are conflicting with each other, so any acceptable solution would be a trade-off between them. We use a bi-objective (GA) to derive Pareto optimal results. For each individual of GA with a determined topology, we undertake a search algorithm to optimize the linear combination of prestress modes. The discrete topological variables and continuous linear combination coefficients are treated separately in the upper-level topology optimization, and lower-level prestress optimization problems, as illustrated in Fig. 1.

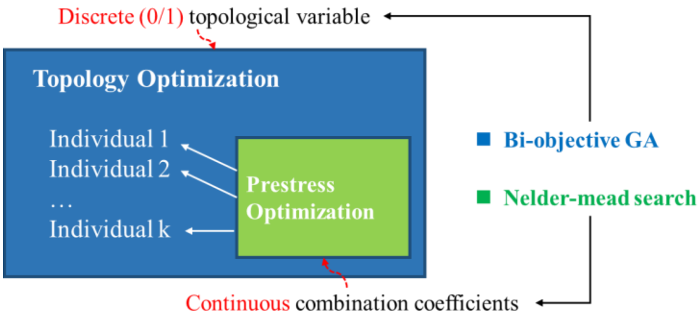


Figure 1. Proposed algorithm for the bi-objective optimization of tensile trusses

3.1.1 Upper-level problem: topology optimization

The goal of upper-level problem is to find the optimal topology of structure based on GS method, the design variable are the existent states $x_i (i = 1, 2, \dots, M)$ of potential member, which can be represented by the integers 0/1; i.e., $x_i \in \{0, 1\}$.

- $x_i = 1$ for all members of the GS;
- $x_i \in \{0, 1\}$ for coding $\{\text{non} - \text{existent}, \text{existent}\}$ states of members in the optimization process.

The optimization problem can be described as follows

$$\text{Opt1-upper-level:}$$
$$\begin{aligned} &\text{Find } \boldsymbol{x} \in \{0, 1\}^M \\ &\text{Minimize } f_1 = \frac{1}{2} \boldsymbol{P}^{\mathrm{T}} \boldsymbol{K}^{-1} \boldsymbol{P} / \bar{W} \\ &\text{Minimize } f_2 = V^* / \bar{V} \end{aligned} \tag{11}$$

where $\bar{W}$ and $\bar{V}$ are respectively the strain energy and the volume of the reference (ground) structure. And V^* represents the optimal structural volume of the prestress optimization which will be introduced in the next section. From the definition of prestress level, the tangent stiffness matrix can be calculated as follows

$$\boldsymbol{K} \approx \boldsymbol{K}_E = \overline{\boldsymbol{D}}\overline{\boldsymbol{K}}\overline{\boldsymbol{D}}^{\mathrm{T}} \tag{12}$$

where $\overline{\boldsymbol{D}} \in \Re^{2N\times M}$ is the equilibrium matrix of the GS, and $\overline{\boldsymbol{K}} \in \Re^{M\times M}$ is the diagonal matrix of axial stiffness [12]; i.e.:

$$\overline{\boldsymbol{K}} = \mathrm{diag}(\frac{E_1A_1}{L_1}, \frac{E_2A_2}{L_2}, \dots, \frac{E_MA_M}{L_M}) \tag{13}$$

Here E_i, A_i and L_i of existent members are cross-sectional properties determined in Table 1. To avoid instability of the structure or singularity of the stiffness matrix when calculating the strain energy, we assign $E_iA_i = 0.000001$ instead of $E_iA_i = 0$ for non-existent members. This value is small enough so that its influence on strain energy can be neglected.

Since cross-sectional properties are functions of signs of prestress, normally, both Obj. 1 and Obj. 2 can be obtained only after the prestress optimization. However, in this study, we adjust the material coefficients, typically, $c = b = 2$, to make all existent members have the same axial stiffness of $\frac{1}{L_i}$ in any force state for simplicity. Then Obj.1 becomes solely a function of topology.

We adopt bi-objective GA to solve Problem (11). GA simulates the process of genetic evolution of organisms, and it has a good ability to make the global search for discrete optimization problems. However, it should be noted that the initial population of GA has a significant impact on the final results.

3.1.2 Lower-level problem: prestress optimization

The goal of the lower-level problem is to find the optimal linear combination of prestress modes. The design variables in this lower-level procedure are the continuous combination coefficients $\boldsymbol{\beta}$. As mentioned in the previous section, by adjusting the material coefficients, Obj. 1 only relates to the topological variables. Therefore, we only need to optimize the structural volume V in the prestress optimization. The optimization problem can be described as follows

$$\begin{aligned}
&\text{Opt1-lower-level:}\\
&\text{Find } \boldsymbol{\beta} \in \Re^{m_{\mathrm{p}}}\\
&\text{Minimize } V = \textstyle\sum_{i=1}^{m} L_i^{(*)}(\mathrm{sign}(s_{\mathrm{p}i}))A_i(\mathrm{sign}(s_{\mathrm{p}i}))\\
&\text{Subject to } \boldsymbol{s}_{\mathrm{p}} = \boldsymbol{B}\boldsymbol{\beta}
\end{aligned} \tag{14}$$

where member length (or penalized member length) $L_i^{(*)}$ and cross-sectional area A_i are functions of signs of the prestress based on Table 1. However, these functions are not differentiable for $\boldsymbol{\beta}$, resulting in difficulty in solving the problem. We employ the Nelder-Mead algorithm [13], a search algorithm that does not require gradients of functions and has high robustness and efficiency, to solve Problem (14).

3.2 Initial population selection

The initial population of GA is important for successful convergence and the presentation of the Pareto front. With all initial individuals randomly generated, the GA yields structures with too complicated topologies and large volumes. To derive lighter structures with simpler forms, a proper selection method for the initial population is needed.

3.2.1 Weighted sum method

One method is to replace some individuals with structures derived by a single-objective GA biased towards Obj. 2 using the weighted sum method. In particular, the objective function of single-objective GA is formulated as follows

$$f = w_1f_1 + w_2f_2 \tag{15}$$

where the weight coefficients w_1 and w_2 satisfy $w_2/w_1 \gg 1$, which means we prefer structures with smaller volumes.

3.2.2 Plastic design method

In this study, we propose the plastic design method [14] featuring member length penalty to efficiently derive initial individuals for bi-objective GA, the optimization problem of plastic deign is formulated as follows

$$\begin{aligned}
&\text{Opt2:}\\
&\text{Find } \boldsymbol{T}_p,\ \ \boldsymbol{C}_p \in \Re^M\\
&\text{Minimize } f_p = \boldsymbol{e}^{\mathrm{T}} \begin{pmatrix} \boldsymbol{T}_p \\ \boldsymbol{C}_p \end{pmatrix}\\
&\text{Subject to } (\boldsymbol{H} \quad -\boldsymbol{H}) \begin{pmatrix} \boldsymbol{T}_p \\ \boldsymbol{C}_p \end{pmatrix} = \boldsymbol{P}\\
&\text{Subject to } \boldsymbol{T}_p \geq \boldsymbol{0},\ \ \boldsymbol{C}_p \geq \boldsymbol{0}
\end{aligned} \tag{16}$$

where $\boldsymbol{e}$ is a vector of length $2M$, whose entries are all equal to 1. $\boldsymbol{T}_p$ and $\boldsymbol{C}_p$ represent the vectors of tensile and compressive member volumes, considering member length penalty, respectively. And $\boldsymbol{H}$ is the corresponding equilibrium matrix. The definitions of them are presented as follows

$$\begin{aligned}
&\boldsymbol{H} = \boldsymbol{D}\boldsymbol{L}^{-1-\alpha}\boldsymbol{\sigma}\\
&\boldsymbol{T}_p{}^{\mathrm{T}} = \left(\frac{s_1^+ {l_1}^{1+\alpha}}{\sigma_1}, \frac{s_2^+ {l_1}^{1+\alpha}}{\sigma_2} \ldots, \frac{s_M^+ {l_1}^{1+\alpha}}{\sigma_M}\right);\\
&\boldsymbol{C}_p{}^{\mathrm{T}} = \left(\frac{s_1^- {l_1}^{1+\alpha}}{\sigma_1}, \frac{s_2^- {l_2}^{1+\alpha}}{\sigma_2} \ldots, \frac{s_M^- {l_M}^{1+\alpha}}{\sigma_M}\right).
\end{aligned} \tag{17}$$

where $\boldsymbol{L}$ and $\boldsymbol{\sigma}$ are diagonal matrices of member lengths and allowable stress, respectively. s^+ and s^- represent magnitudes of tensile and compressive member forces, respectively. And α is the member length penalty factor, which can be any real member. By varying α, we can generate various structures with differing proportions of long or short members, so the diversity of inidividuals can be guaranteed.

Take the cantilever beam structure for example. We conduct Opt2 for the problem as illustrated in Fig. 2 (a) and (b). The GS has in total $N = 45$ nodes and $M = 632$ members, and the grid spacing is 1. As the post-processing, we delete members with absolute axial forces smaller than 10^{-4}, and when presenting the results, we combine two collinear members that connect a single free node (node that is not located at the support and load positions). We use the "dual-simplex" algorithm provided by the optimization toolbox in MATLAB. The results using different penalty factors are shown in Fig. 2, where red lines represent compressive forces, blue lines represent tensile forces, and the widths of lines represent the magnitudes of forces. We can see that all results are statically determinate structures, which is in line with existing research on optimal truss optimization. As the penalty factor increases, the proportion of long members decreases, and larger absolute factors can result in a larger deviation of member volume from the optimal structure. Although these resulting structures are conventional trusses, their topologies are good candidates for the initial population of GA in tensile truss optimization.

(a) Boundary conditions

(b) GS

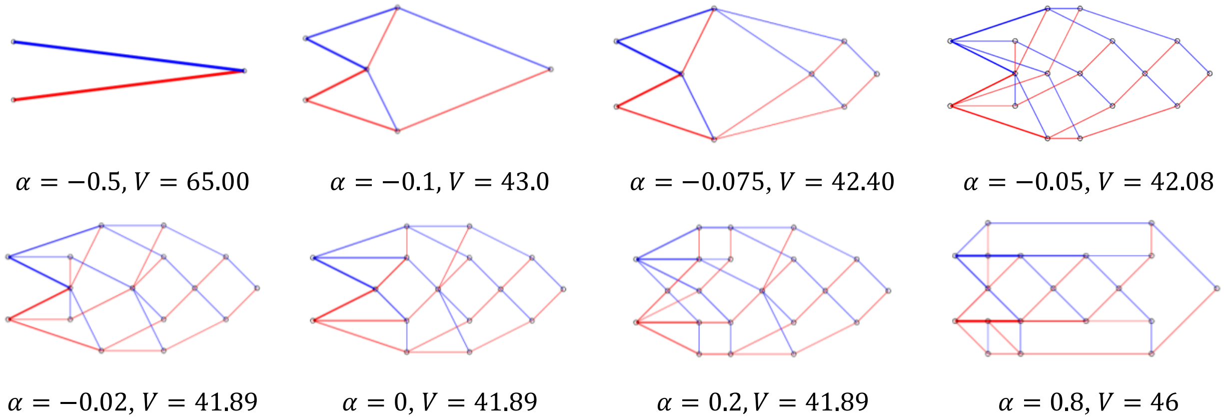


Figure 2. Plastic design for two-dimensional cantilever beam structure using different member length penalty factors.

4. Numerical examples

In the following examples, for the upper-level topology optimization, we adopt the multi-objective GA (NSGA-Ⅱ) provided by the optimization toolbox in MATLAB, where the population size in each generation is set as $k = 600$, and other GA parameters are set as the default values [15]. The single axial symmetric condition is considered. For the lower-level prestress optimization, we adopt the unconstrained and derivative-free simplex search method (Nelder-Mead simplex method) provided by the optimization toolbox in MATLAB, and the initial point is set as the zero point or zero vector [15].

Example 1. Cantilever beam structure

In this example, the initial population of multi-objective GA consists of 400 individuals derived by the weighted sum method and 200 individuals randomly generated. To preserve the diversity of the population, when selecting the initial population, we run single-objective GA several times with different weight ratios $w_2/w_1 \gg 1$, then choose one optimal individual in each run, and randomly choose individuals from all generations in all runs to form the group of 400 individuals. Some solutions are shown in Fig. 3, where dashed black lines represent members with no prestress. Solution data are presented in Table 2.

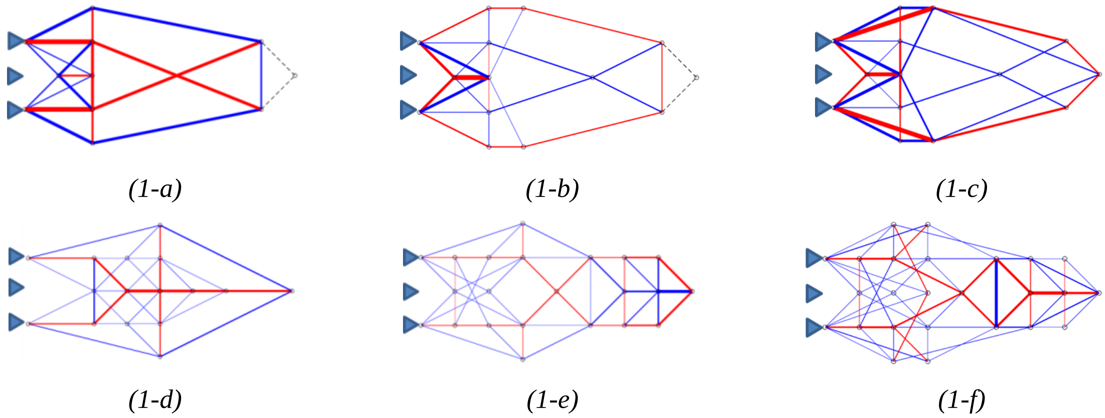


Figure 3. Solutions for Example 1; *(1-d~f)* are those considering buckling effect.

Table 2. Solutions of tensile truss optimization for Example 1

Structure	Volume (V)	Work (W) $\times 10^{-5}$	No. prestress mode (m_{p})	No. member (m)
(1-a)	35.998	23.264	2	22

(1-b)	39.660	21.150	2	28
(1-c)	47.134	15.769	3	31
(1-d)	39.467	24.515	6	37
(1-e)	48.663	22.417	5	49
(1-f)	69.779	13.550	10	65

Let's compare the tensile trusses derived by multi-objective GA and the conventional trusses derived by plastic design method, e. g., tensile truss *(1-b)* and statically determinate truss ($\alpha = -0.075$). From the force distribution of *(1-b)* due to the external load, as shown in Fig. 4 (a), we can find that despite the

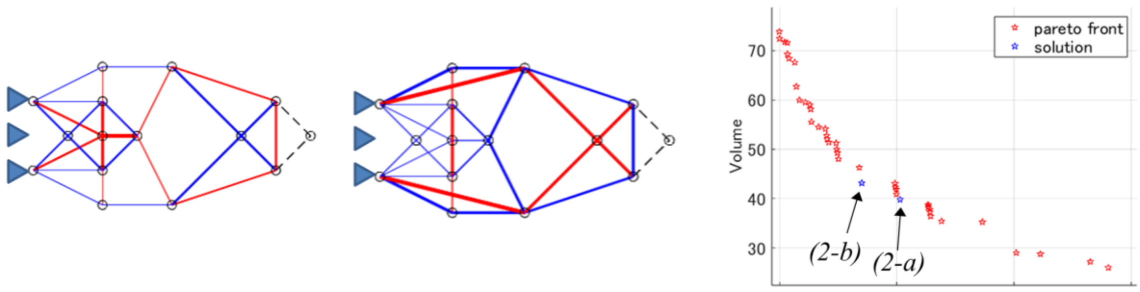
(a) Force distribution ($\boldsymbol{s}_e$) of *(1-b)* due to external load (b) Result ($\alpha = -0.075$) of plastic design

Figure 4. Tensile truss and statically determinate truss

circled four slender members, it is a statically determinate structure, and has the topology the same as Michell structure [3], these four members make small contribution to withstanding external loads since their force magnitudes are very small, but are necessary for the existence of prestress. From the result of the plastic design as shown in Fig. 4 (b), we can see that it has a similar topology as the Michell structure, besides, it has a form close to the tensile truss *(1-b)*. Inspired by this, we try to use the results of the plastic design method to form the initial population of tensile truss optimization, expecting that evolutionary operations (crossover, mutation, and selection) can supplement some members to these statically determinate trusses to derive optimal tensile trusses.

Example 2. Cantilever beam with initial population derived by plastic design method

In this example, the initial population of multi-objective GA consists of 120 individuals derived by plastic design method, 10 individuals as the GS with full connectivity, and 470 individuals randomly generated. In particular, we select six results of the plastic design method ($\alpha = -0.1, -0.075, -0.05, -0.02, 0, 0.23$) and replicate each structure twenty times to form the group of 120 individuals. No buckling effect is considered. The results and their locations on the Pareto front are shown in Fig. 5. Solution data are presented in Table 3.



(2-a) *(2-b)*

Figure 5. Solutions for Example 2 and their locations on the Pareto front

Table 3. Solutions of tensile truss optimization for Example 2

Structure	Volume (V)	Work (W) × 10^{-5}	No. prestress mode (m_{p})	No. member (m)
(2-a)	39.819	20.284	2	30
(2-b)	43.173	17.014	3	32

Comparisons to benchmark

We compare the results in Example 1 and Example 2 with the (global optimal) benchmark of optimal truss topology optimization derived by branch and bound method, see Achtziger [16], as illustrated in Fig. 6. We can see that all Pareto fronts lie below the benchmark. Even on the highest Pareto front (Example 1 considering the buckling effect), the solution *(1-d)* performs better than the benchmark in both objective functions. Therefore, we can conclude that our method can create tensile trusses that have better structural performances than conventional trusses.

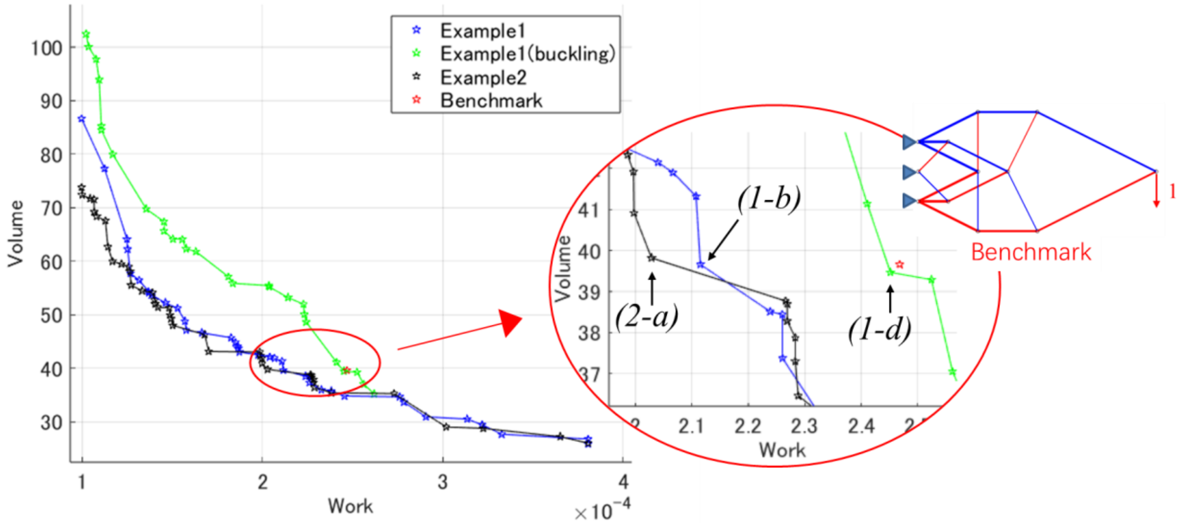


Figure 6. Comparisons to benchmark

To provide a more intuitive comparison, we select one solution with a volume close to the benchmark on each front and compare it to the benchmark, as shown in Table 4. Compared to the solution *(1-b)* of the method with the initial population derived by the weighted sum method, we find that with close volume, the solution *(2-a)* of the method with the initial population derived by the plastic design method has a smaller strain energy. Moreover, there is a significant advantage to using the plastic design method: On average, one run of the weighted sum method requires 87.10 minutes, while the plastic design method only requires 0.160 seconds. The solution (1-d), considering the buckling effect, has higher strain energy and a more complex topology, but it has fewer long compressive members that are easy to buckle.

Table 4 Comparisons of solutions by different methods

Structure	Volume (V)	Work (W) $\times 10^{-5}$	No. prestress mode (m_{p})	No. member (m)
(1-b)	39.66	21.15	2	28
(1-d)	39.47	24.52	6	37
(2-a)	39.82	20.28	2	30
Benchmark	39.66	24.68	0	18

5. Conclusions

By using different materials and the optimization method proposed in this study, we obtain tensile trusses with lightweight and high stiffness. Our examples demonstrate the effectiveness and efficiency of this method. The entire optimization process results in tensile trusses with diverse topologies, which are distributed along a Pareto front. Designers can achieve a more flexible design by balancing between multiple Pareto optimal solutions. Comparisons to conventional trusses show that tensile truss has higher structural performances. Finally, introducing the buckling penalty reduces the number of long compressive members that are easy to buckle.

Acknowledgments

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