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## **Euler Path Structures: design exploration with reconfigurable continuous, flexible material**

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### **Abstract**

The impact of the construction industry, as one of the biggest contributors to the climate crisis, urges designers to explore new approaches and solutions that embrace strategies to circular design and reuse of materials. However, geometrically complex structures such as gridshells, are in many cases assembled from a magnitude of bespoke members, difficult to imagine a second life. Inspired from knitting-techniques, we introduce a novel design approach for spatial, grid-based structures where flexible material follows various reconfigurable patterns in a hypothetically endless, continuous line. We apply specific Eulerian paths that follow routes that pass every edge of a predefined grid pattern only once while they may pass the nodes of the pattern once or multiple times. The applied profile of the material, presently considered as circular and strip type of profiles, significantly influences how the pattern transforms, and consequently determines the emerging three-dimensional form relevant for both architectural/geometrical and structural perspectives. We aim to generate load-bearing structures that potentially surpass traditional lightweight structures in sustainability by enabling the reuse of the same material for creating new configurations.

**Keywords:** Spatial structures, elastic gridshell, Euler Path, graph theory, Gaussian curvature, architecture, reuse, circular design, sustainability.

### **1. Introduction: from Knitting to Reconfigurable Euler Path Structures**

The climate crisis and the high portion of construction industry emerges more interest and attention to circular design and the reuse of materials. Exploring the reuse and the salvage of timber [1–6] demonstrates the significance and potential of reutilizing materials in new structures. Gridshell structures, especially if designed and built with flexible material, are considered lightweight, low emission structures due to lowering the cost and energy for pre-forming the material. Examples include Multihalle Mannheim [7], Weald and Downland [8], inside/out [9], geodesic winding [10], and Portalen pavilion [11] among others. Nevertheless, the challenge of reusing elements persists due to the complexity of structures, remaining largely unresolved.

In the realm of textile arts, knitting and crocheting are distinct techniques, differentiated by their tools but both resulting fabric structures. In knitting, long needles are used to join or knot a series of stitches consisting of one continuous thread. Each loop or knot is connected to another, and when enough loops have been made, the result is a flat or spatial textile artifact. Crocheting is a craft that developed in the 19th century out of a form of chain-stitch embroidery, and instead of a needle it utilizes a single hook to craft knotted yarn patterns [12]. In crochet work the hook is used, without a foundation material, to make a texture of looped and interlinked chains of thread. However, for the purpose of presented study, it is not crucial to distinguish between the different tools, as the critical aspect under consideration is the ability of both techniques to unravel the textile, returning it to its original form: a single continuous yarn. Through specific sequences a structure of intertwined and interlinked thread chains or openable knots

can be created forming a series of patterns and designs. Single knits are crafted using a single yarn and can be produced using machine or hand techniques. As shown in the Textile Logic workshop, a variety of textile patterns can be incorporated using CNC knitting machines to achieve "non-uniform or graded material" in knitted membranes [13]. Finally, an entire knitted piece can be methodically disassembled by pulling on one end, returning the yarn to its original state, allowing it to be reused for another configuration. So, the inherent principle of reusability characterizes the craft of knitting and crocheting. All of these characteristics, (i) the single continuous yarn, (ii) interlinked thread chains, and (iii) reconfiguration are consistent across knitting and crocheting and inspired the proposed novel design approach for flexible structures that eliminates the need for cutting material to discrete pieces. To find a possible path for such structures, we search for a solution by means of graph theory. Graph theory is a field of mathematics and computer science that studies the properties of graphs. A graph, in this context, is a collection of points, called vertices (or nodes), and lines connecting them, called edges [14]. This theory explores the relationships and structures that emerge from these connections. An Euler path, also Eulerian path, in graph theory is a path through a graph that visits every edge exactly once [14]. If such a path exists, the graph is said to be Eulerian. Not every graph has an Euler path. For a connected graph to have an Euler path, it must either have exactly two vertices with an odd degree (number of edges connected to the vertex) or all vertices with even degrees. To solve this, we have used the feasibility of applying the Chinese postman, where the shortest possible total distance is the goal [14]. Eulerian path as a method for assembly of a form has been used in a research by Sun et al. [15]. After determining the paths, they opt to utilize 3D printed elements and connect them together using a single thread, instead of employing a continuous element. In their project, the algorithm for finding the Eulerian path restricts the graph to having nodes with only two or four degrees, despite the fact that an Eulerian circuit can exist in graphs with nodes of any even degree. This constraint necessitates pre-editing the mesh to meet these criteria, potentially resulting in deviation from the input model during simplification. Their algorithm might remove edges or nodes that are essential for preserving the similarity to the original intended shape. The likelihood of these unexpected outcomes occurring increases with the size and complexity of the graph. Their paper suggests a future implementation of the Chinese postman algorithm to handle more complex geometries. Industrial knitting machines are frequently employed for producing intricate forms using yarns [16]. In 3D knitting, elements are produced three-dimensionally as an entire piece, eliminating the need for cutting and sewing the fabric. Narayanan et al. [16] use graph theory and the conversion of a surface into a mesh and represent it as a (directed) graph order to find the knitting path. In their approach, the mesh is segmented into sliced regions using either a user-specified time function or by an automated algorithm using the graph Laplacian- a matrix that helps find spanning trees in a graph [17].

## **2. Aims, Methods and Limitations**

By applying the fundamental characteristics of knitting and crocheting, which are, (i) the single continuous yarn, (ii) interlinked thread chains, and (iii) the disassembly, reuse and reconfiguration of the unchanged material we propose a novel design approach for flexible structures. By the hypothetical use of single thin, continuous, and flexible elements such as rods or strips, which follow the specific logics of Eulerian paths, our research suggests the development of a dynamic system to the design of sustainable, spatial structures that can be assembled into a specific configuration, disassembled and reconfigured into various other configuration without loss of material quantity or quality. In addition to the structural performance that may be less efficient compared to other grid configurations, we are equally interested in possibly enhanced architectural features such as surface curvature, depth of the surface, density and transparency of such structures. Our research employs both digital and physical approaches for extracting the material behavior, the surface curvature of the generated patterns, and the structural performance of the assembled models. In particular, we explore and compare the effect of each Eulerian Path and their different paths, and the effect of the cross-section/profile on the emerging form. Attempting to find an Euler path in a graph representing complex geometry can be challenging. If the graph is non-Eulerian, meaning it has more than two vertices with an odd degree, it is not possible to find such a path. In these cases, some edges may need to be traversed multiple times to cover all edges with a single path. We use the Postman Tour problem, which addresses the challenge of finding the shortest path that covers all edges in a non-Eulerian graph. To solve this problem, we utilize the

Handshaking lemma, which states that in any undirected graph, the sum of the degrees of all vertices is equal to twice the number of edges. Fleury’s algorithm, introduced in 1883, is an elegant but somewhat inefficient algorithm to find an Eulerian path. The algorithm commences by selecting a vertex with an odd degree as the starting point. If no such vertex exists, an arbitrary vertex is chosen instead. Nonetheless, despite the challenges related to time complexity, the decision to use Fleury’s algorithm was based on a multitude of factors. Its elegant nature simplifies the process of integrating changes, modifications, and the introduction of constraints into the algorithm. While there are potential alternatives like Hierholzer’s algorithm with the linear time complexity  $O(E)$  that may offer faster performance, their use could complicate the process of future modifications and extend beyond the current scope of this work. By employing both circular and strip cross-sections made of flexible material, the continuous element follows an Eulerian path and self-organizes into its final shape. In a circular cross-section, the torsion or twist in the profile can be neglected, resulting in a flatter final shape. On the other hand, in a strip cross-section, the rotation of the element along the Eulerian path induces twist in the profile, leading to a greater change in curvature compared to the circular cross-section. We simulate the assembly of the continuous elements using Kangaroo Physics [18], a Grasshopper3D plugin [18]. We also conduct preliminary structural analysis using Karamba3D [19] to check and compare both structural displacement and material utilization. As above-mentioned our studies are presently limited to circular and strip material, as well to a selection of possible Eulerian paths for a given spatial mesh structure. Also, the connections within the shown configurations are all assumed as pinned for now.

### 3. Eulerian Path Structures

When investigating the (digital) assembly method, we found graph theory useful for delineating various paths on planar and spatial grids that can be realized with flexible material, similar to elastic gridshells. We studied the resulting outcomes to gain understanding and predictability, which led us to infer rules or constraints. These rules or constraints guide the path generation, ultimately resulting in the formation of self-organized structures. In this section, we explain two case studies to show the potential of Eulerian path structures: the House of Nikolaus and a 5 x 5 grid. In both of these case studies, we examine two types of cross-sections: circular and strip, to replace the continuous yarn of the knitting process. The architectural features of surface curvature, depth of the surface, density and transparency, in addition to structural stability and maximum displacement are explored and compared for each case study pattern, each path, and each cross-section.



Figure 1: The House of Nikolaus – exploring all 44 possible Eulerian paths.

#### 3.1. The House of Nikolaus

The game "House of Nikolaus", also called House of Santa Claus, is a puzzle consisting of eight edges and five vertices, that is often used to introduce concepts in graph theory [20]. It involves drawing the shape of a house (or a series of interconnected lines) without lifting the pen from the paper and without retracing any line. The challenge lies in figuring out whether it's possible to draw the entire figure under these constraints. This game is an illustration of the Euler path problem, where one has to find a path that visits every edge exactly once. It serves as a practical and engaging way to understand the basic

principles of Eulerian paths in graph theory. Depending on the starting vertex and the selected route, the House of Nikolaus can be generated by 44 different Eulerian paths. We assume a rectangle of 10 cm x 15 cm with the House of Nikolaus inscribed. Therefore, the total length of the path is 82.46 cm for all possible variations of the Eulerian paths. It's worth noting that the paths are depicted with fillet corners to ensure clearance, although the simulation and reality lack these fillet corners. The formal varieties depicted two-dimensionally in Figure 1 emphasize the diverse possibilities of achieving either a more centric or linear arrangement, depending on the chosen paths.

We applied both circular and strip cross-sections of flexible material to all 44 Eulerian path routes. The resulting forms are simulated by means of elastica curves as shown in Figure 1 to mimic natural behavior. Initially, a straight element with a length of 82.46 cm is positioned on the canvas, and the vertices of the pattern are then marked on the element before being adjusted to their corresponding positions on the pattern. Figure 2 shows an example of how one geometrical pattern is generated with strip and circular cross-sections in the digital environment. The result is then a relaxed equilibrium shape of the pinned flexible strip or rod material.

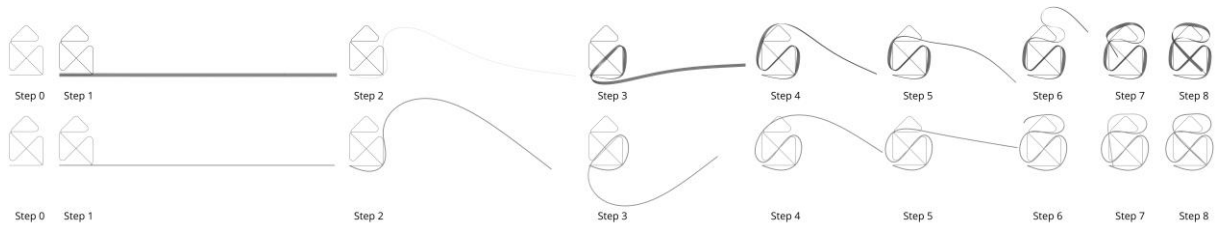


Figure 2: The simulation process for path number 0 of the House of Nikolaus for a strip (top) and circular cross-section (bottom).

We used the same total (amount of) volume of material for both strip and circular cross-sections. To extract the radius of rods for simulation, we consider the same length of the material as needed for the path generation and extract the radius by equalizing the two volumes of a rectangular box ( $V_s$ ), representing the strip, and a cylinder ( $V_r$ ), representing the rod. In below formulas,  $L$  represents the length of the element or length of the path,  $w$  denotes the width of the strip,  $h$  signifies the thickness of the strip, and  $r$  stands for the radius of the rod material. We equate formula (1) with formula (2), and formula (3) is used to calculate the rod radius.

$$V_s = L * w * h \quad (1)$$

$$V_r = \pi * r^2 * L \quad (2)$$

$$r = \sqrt{\frac{w * h}{\pi}} \quad (3)$$

Changing the pattern, as well as the Eulerian path, alters not only the overall shape (size and curvature) but also other characteristics of the form, including density, transparency, and structural displacements. This effect is more noticeable with strip material.

By utilizing the Eulerian paths with a flexible material, we can transition from a one-dimensional element, representing the yarn, to a two-dimensional pattern, which subsequently transforms into a three-dimensional form. To illustrate the 3D forms generated from the 2D patterns, we depict the bounding box of each individual form.

Figure 3 shows all possible 44 Eulerian paths of the House of Nikolaus, all of them first inscribed into a 10 cm x 15 cm rectangle therefore with a continuous strip length of 82.46 cm. The resulting relaxed equilibrium shapes are depending on the individual path. The overall sizes of the generated forms are shown as volumes, represented by the length, width, and height of their bounding boxes. The outmost gray box displays the bounding box encompassing all 44 paths as a reference, while the black box represents the bounding box specific to the individual displayed path.

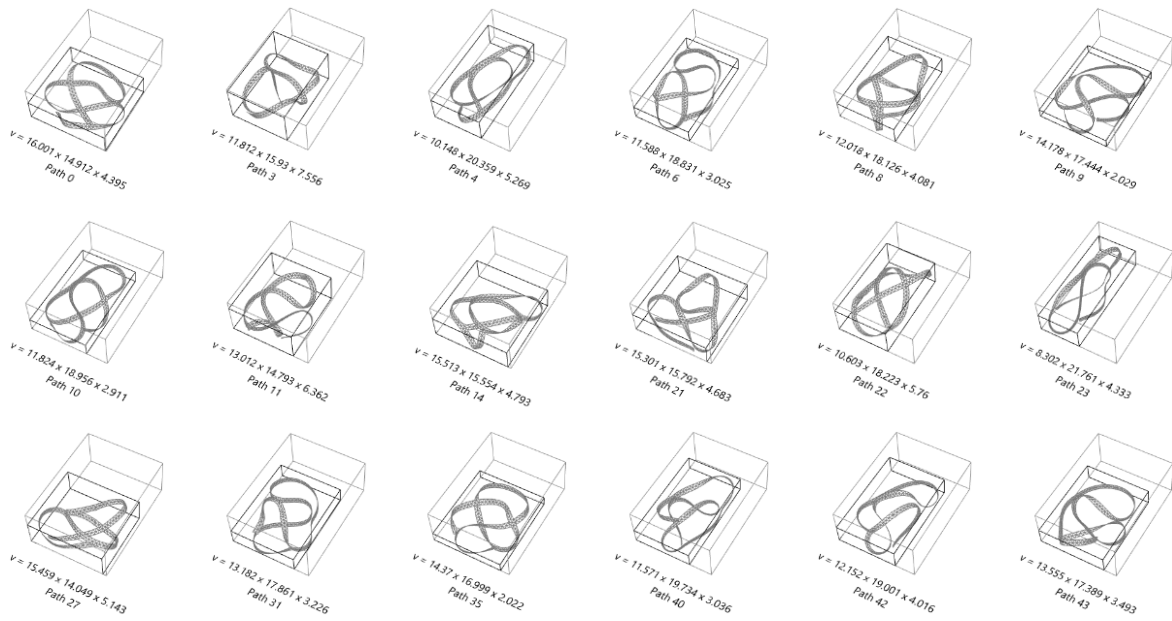


Figure 3: The overall size change of 44 paths of the House of Nikolaus with strip cross-section. The bounding boxes encompassing a selection of the 44 paths shall serve as a reference for the comparison in size and volume of the various configurations.

Overall, the size change of the House of Nikolaus paths with strip cross-section differs in a range of 8.302 cm to 16.001 cm in width, 14.049 cm to 21.761 cm in length, and 1.783 cm to 8.081 cm in height. The volume of the bounding box changes from 380.137 cm<sup>3</sup> to 1498.653 cm<sup>3</sup>, which means that depending on the path we can reach 3.9 times volumetric change between the paths. Figure 4 shows the 44 Eulerian paths with circular cross-section following the same logic as explained above. Similar to the strip cross-section, the gray box shows the bounding box containing all 44 paths, whereas the black box indicates the bounding box for the currently displayed path. As illustrated, the overall size exhibits less noticeable variation compared to those with a strip cross-section.

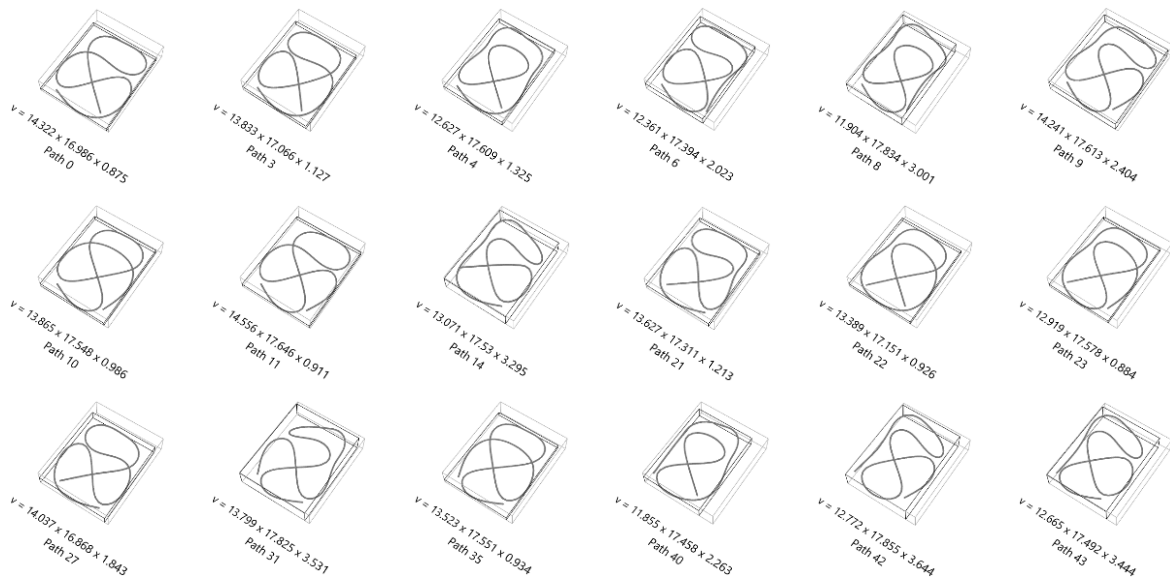


Figure 4: The overall size change of 44 paths of the House of Nikolaus with circular cross-section. The bounding boxes encompassing a selection of the 44 paths shall serve as a reference for the comparison in size and volume of the various configurations.

The size change of the House of Nikolaus paths with circular cross-section differs in a range of 11.273 cm to 14.556 cm in width, 16.802 cm to 18.127 cm in length, and 0.875 cm to 3.644 cm in height. The

volume differs from 195.586 cm<sup>3</sup> to 868.51 cm<sup>3</sup>, which means that depending on the path we can reach 4.4 times volumetric change between the paths. Figure 5 shows a comparison of generated volume for a strip and a circular cross-section for 44 different paths of the House of Nikolaus. In almost all Eulerian paths of this pattern, the strip cross-section yields a larger volume compared to the circular cross-section, with the difference reaching a maximum of 1170 cm<sup>3</sup>.

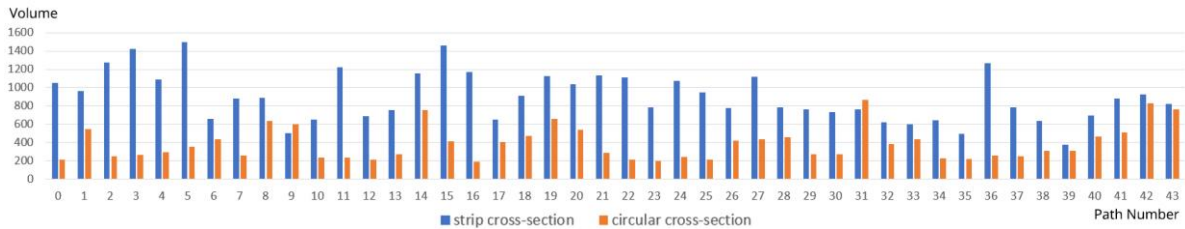


Figure 5: Comparison of the overall volume generated for all possible 44 Eulerian paths of the House of Nikolaus. The blue and orange bars are showing the volumes of strip and circular cross-sections respectively.

In two special cases, paths 9 and 31, the circular cross-section exhibits a larger volume. This is attributed to the circular cross-section maintaining the pattern shape more effectively, whereas the strip cross-section may shrink depending on the applied twist to the profile. These two paths for both strip and circular cross-sections are shown in Figures 3 and 4. To enhance the visibility of the 3dimensionality of the generated forms, we generate a patch surface from the points of the simulated mesh. We then compare the surfaces by two values of surface deformation ( $dv$ ) that represents the difference between the minimum and maximum  $z$  value of all the points on the mesh and the Mean Gaussian curvature (MGC). Figure 6 shows 4 Eulerian paths for the House of Nikolaus and the surface deformation, the regions of zero (shown in green), negative (represented in blue), and positive (shown in red) Gaussian curvature, and also the mean value for the Gaussian curvature of the generated surfaces for strip and circular cross-sections. The gray box is used as a reference to enable the comparison between the paths and cross-sections.

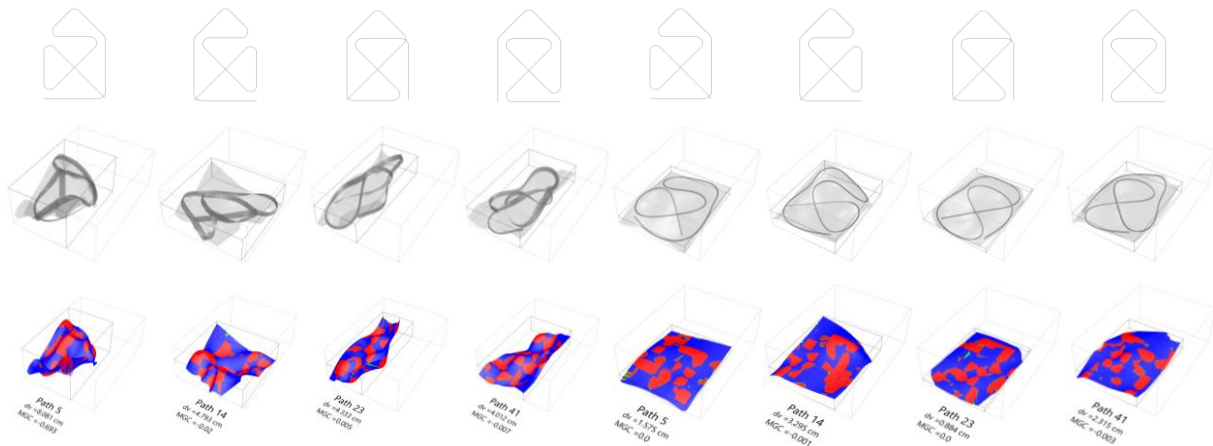


Figure 6: Path 5, 14, 23, and 41 of the House of Nikolaus and the corresponding surface deformation ( $dv$ ) and surface Mean Gaussian Curvature (MGC) for two cross-sections of strip and circular.

As illustrated in Figure 6, the  $dv$  values in the strip cross-section are higher than those in the circular cross-section. This discrepancy arises from the applied twist on the strip cross-section, resulting in distinct strong and weak axes. Conversely, in the circular cross-section, twist is typically disregarded, leading to less pronounced curvature changes. The curvature is compared by measuring the Gaussian curvature of the surface, using a mesh to identify areas with zero, negative, or positive Gaussian curvature. In the circular cross-section, the mean Gaussian curvature is almost zero, whereas in the strip cross-section, the mean curvature can reach higher values. Through these comparisons, we observe the impact of each path within the same pattern on the overall shape of the structure. To further relate it to architectural applications, the following case study not only delves into shape, size, and curvature

analysis but also explores density, transparency, shadow, and structural performance of the generated grid structures in an architectural scale.

### 3.2. A 5 x 5 grid

As an architectural design example, we explore multiple Eulerian paths for an initially planar 5 x 5 grid. Different paths or routes are possible for this pattern. Unlike the House of Nikolaus case study, for which we explored all possible 44 paths, here we only examine a selection of them. Figure 7 illustrates three potential paths of a 5 x 5 grid positioned on an initial boundary rectangle measuring 500cm x 500cm. The simulation results, including surface deformation (dv) and mean Gaussian curvature (MGC) values, for both strip and circular cross-sections, are presented.

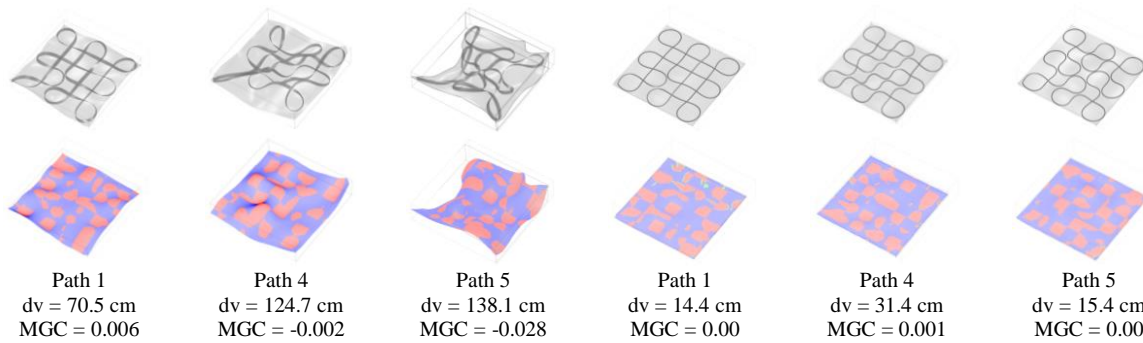


Figure 7: Comparison of paths 1, 4, and 5 of a 5x5 grid using strip and circular cross-sections, along with their corresponding surface deformation and Gaussian curvature.

As Figure 7 shows, the surface curvature varies up to 9 times higher for strip configurations compared to configurations with circular cross-section. The circular cross-section, on the other hand, is more suitable for being applied on a curved surface as it can follow the curvature of the surface more easily and does not show excessive out-of-plane behavior. The circular cross-section tends to form flatter overall shapes and to keep the size of the found grid closer to the initial uniform 5x5 grid.

The twist applied to the strip profile significantly alters the transparency of the resulting structure as the profile rotates along its longitudinal axis. For the circular cross-section, the effect of the twist is less pronounced, resulting in minimal change in transparency. Only the distances between the initially uniform square grid marginal changes. Figure 8 illustrates how the strip cross-section produces varying forms and shadows depending on the path. However, a targeted use of twist and its associated interplay of density, transparency and shadow can significantly influence architectural atmospheres and functional performance.



Figure 8: Comparison of shadows for paths 1, 4, and 5 of the 5 x 5 grid using strip cross-sections (top) and circular cross-sections (bottom).

Clearly, our research focuses on examining the feasibility of Eulerian paths and their influence on selforganized 3d formation. Still, we conduct mesh structural analysis on the generated forms resulting from various Eulerian paths applied to an initial flat 5 x 5 m grid to gain preliminary structural insight into the found designs. Since the circular cross-section primarily yields planar grids that operate predominantly in bending, such configurations seem less relevant for the focus of our research at this point. The strip cross-section, in contrast, yields a 3D form emerges from the 2D grid due to the flexibility of the material, the strip cross-section and from the selected Eulerian path, albeit not as a conventional gridshell.

In our simulation, the strip is assumed flexible plywood material with dimensions of 20cm in width, 5200cm in length, and 0.9cm in thickness. As a basic rule for all cases, four support points at a distance

of 4 m in square arrangement, represented as small black spheres in Figure 9, are considered for the 5 x 5 m grid. Additionally, universal joints that connect the strips are taken into account to ensure structural integrity. Loading conditions remain consistent, based solely on self-weight. Figure 9 illustrates the investigation of maximum displacements for four Euler paths with a strip cross-section. Accordingly, the displacements range between approximately 11 cm to 28 cm for the shown configurations. As a general observation, the displacements increase with increasing irregularity of the found grid forms.

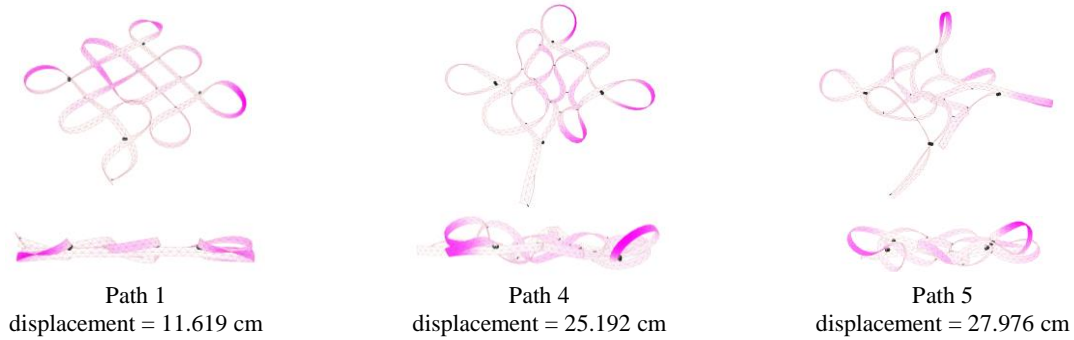


Figure 9: Comparison of path 1, 4, and 5 of a 5x5 grid with a strip cross-section and the corresponding maximum displacement.

Figure 9 illustrates how changes in surface curvature, depending on the paths, lead to variations in structural performance and resulting maximum displacement. The applied twist induces additional stress to the elements; however, the structures geometrically respond to the applied forces by shifting along the profile axis. Of course, factors such as the location of supports and additional stiffening effects, such as reinforcing the structure with cables, can improve structural performance. However, when comparing to traditional gridshells, it's important to consider different objectives from both architectural and structural perspectives.

While circular cross-sections or rods may not be suitable for planar grids, they can be effectively utilized in scenarios where a 3D shape is predefined and the aim is to find a continuous path for the continuous, flexible material. Again, compatibility with surface curvature is achieved depending on the chosen pattern and path. Figure 10 illustrates an example of using rods to realize 3D surfaces. The captivating formal characteristics, coupled with structural performance and the potential for reconfigurability and material reuse in different arrangements, make these structures truly unique. Euler path structures can also be implemented in multiple layers to improve the shape-keeping potential of the resulting form. Figure 11 illustrates the process of generating a double-layer Euler path structure from an initial 3D surface (Figure 11-left). Various Eulerian paths are simulated and analyzed based on their compatibility with the surface curvature. The pattern selected for the physical model is depicted in Figure 11-center and applied at a medium-scale (Figure 11-right).

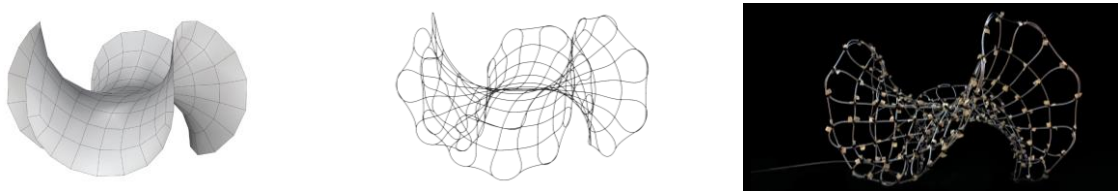


Figure 10: The surface (left), simulation (center), and physical prototype of a 3D surface realized by a continuous rod.



Figure 11: The surface (left), simulation (center), and the physical prototype (right) of a double-layer 3D surface realized by a continuous rod.



Realizing a 3D surface with a strip cross-section may pose additional challenges, primarily due to the curvature change inherent in the pattern itself. As such, experiments with strip cross-sections on predefined surfaces will be further explored.

#### **4. Results**

The simulation of 44 different Euler paths for the House of Nikolaus demonstrates that varying paths and cross-sections of the flexible material can result in changes in volume. In circular cross-sections, the overall volume changes is up to 4.4 times between the paths, which rises to a maximum of 868.51 cm<sup>3</sup>. For the strip cross-section, the process is the same, and the overall volume changes up to 3.9 times between the paths up to a maximum volume of 1498.653 cm<sup>3</sup>. Comparing the two cross-sections, the volumetric change in the strip cross-section is 1.7 times greater than in the circular cross-section. With a strip cross-section, the length and width of the overall shape can even be smaller than those of the original grid, as indicated by negative numbers. This implies that forms generated with strip cross-sections can undergo both volumetric condensation and expansion. The comparison of the two cross-sections in the experiments with a 5x5 grid reveals more significant changes. For instance, the surface deformation in the strip cross-section is 9 times higher than that in the circular cross-section. Structural performance varies depending on the paths, with displacements ranging from 3.8% to 9.3% of the span.

#### **5. Discussion and Conclusion**

This paper presented a novel approach for circularity in design of grid structures using a hypothetically continuous element: Euler path spatial structures. By eliminating or reducing the discretizing of the material into several pieces, the proposed structural forms are reusable and reconfigurable. Presently, the concept of a continuous material is theoretical and may not fully match real-world applications. Yet the idea of endless elements, material, and spaces has been used in the work of architects and researchers [21, 22]. It emphasizes the notion of material continuity rather than separate building elements connected to form larger structures. While kit-of-parts systems build on modules and regular geometries, along with the fact that the main functions of (re-)assembly and transformability are largely dependent on the joints [23–25], Wachsmann’s continuous structures aimed to “create a completely open space, neither disturbed nor penetrated by any kind of column or support either on the inside or at the perimeter of the building” [26]. Inspired by Wachsmann's concept [27], we envision constructing structures from a single continuous material, similar to the technique used in knitting.

We focus on two primary parameters that influence the final self-formed structure for a given graph and material: the path and the cross-section. Both exhibit significant effects on the end result. The circular cross-section, exemplified by both the House of Nikolaus and the 5x5 grid, offers a more accurate replication of the original contours compared to the strip cross-section. Overall, across all paths, it appears flatter (since the original graph is flat) with smaller surface deformation ( $dv$ ) values. Additionally, the differences between different paths with the same cross-section seem to be less pronounced when utilizing the circular cross-section, while ensuring higher degree of structural performance. Conversely, the strip cross-section tends to twist when the path crosses itself, resulting in a notable impact on the outcome. This twisting increases the  $dv$  and introduces complexity, offering potential for intriguing outcomes. When analyzing the path, it becomes evident that the degree of resemblance to the original shape correlates with the nature of the segments traversed. Straight segments, characterized by fewer turns, tend to closely mirror the contours of the original graph. Conversely, segments resembling loops, featuring multiple turns in the same direction, demonstrate a moderate level of distortion from the original contours. Finally, segments resembling a slalom, alternating turns in opposite directions at each node, exhibit the most pronounced deviation from the original contours. When using the strip cross-section and incorporating twist in each of the aforementioned options, the results are influenced by both parameters, making it challenging to differentiate the effects caused by the path or the cross-section alone. This study examines a system that may seem simple at first glance, consisting of a single element with distinct characteristics. The continuous element, bent and interconnected at various points, creates diverse geometric shapes. However, its simplicity challenges the ability to predict changes and deformations in the system’s self-forming since any small change at

one point can have far-reaching effects on the whole system. The complexity escalates further when considering the proliferation of Eulerian paths as the graph expands. In fact, quantifying the abundance of Eulerian paths in undirected graphs presents a formidable obstacle, falling within the computational complexity class labeled #P-complete [28]. It becomes evident that exhaustively assessing all available options to determine the optimal path for a particular design aspiration is impractical. This challenge underscores the difficulty in ensuring that the selected path is indeed the most suitable among the myriad alternatives. Looking ahead, integrating a machine reinforcement learning algorithm into the research framework could prove beneficial. Such an algorithm could leverage the path identified through our approach (or any other) and endeavor to minimize the disparity between the original positions of the nodes within the mesh and their subsequent positions post-simulation under physical relaxation.

## Acknowledgements

This research was funded by the University of Innsbruck as an early-stage funding 2022.

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