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Equilateral Constant Normal Curvature (ECNC) Gridshell

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Abstract

This paper explores the design and manufacturing of doubly-curved equilateral constant normal curvature (ECNC) gridshell structures using standardized, straight, flat components. The kit-of-parts are connected to form multiple designs for deployable freeform gridshells. The parts have adjustable connection angles to adapt to multiple curvature values and create design shapes with positive and negative Gaussian curvature. Once deployed, the grid is triangulated and fixed as rigid rotational gridshell. This method enables the reuse across multiple designs and life cycles, contributing to energy conservation and carbon reduction [1].

An extended version of this paper has been published in Design Modelling Symposium Kassel 2024, titled *Kit-of-Parts Equilateral Constant Normal Curvature (ECNC) Gridshell*.

Keywords: Equilateral grids, Constant normal curvature, Low-cost construction, Kit-of Parts, Repetitive structure.

1 Introduction

Architectural Geometry [2] research has provided insights into topology optimization for curved structural grids [3] [4] and curved building skins [5], aiming to simplify fabrication and use planar or developable building elements. Gridshell structures are of particular interest due to their high structural efficiency, light-weight construction, and design versatility.. Despite these benefits, the use of gridshells remains limited due to high fabrication costs, need for specialized knowledge, and complex components [6].

Current research focuses on simplifying gridshell components. The use of Asymptotic curves, which follow the path of vanishing normal curvature, has been proposed for low-cost, low-tech gridshell construction. These curves facilitate straight, flat elements with repetitive nodes [7]. Asymptotic lamella networks allow for a self-forming erection process, offering high resilience against external loads [8].

Pellis et all [9] recently explored the use of asymptotic meshes with constant edge length, known as equilateral constant normal curvature (ECNC) nets. These networks offer the use of repetitive, prefabricated flat components. This paper focuses on the design and construction of ECNC gridshells, which can be elastically deformed into doubly-curved shapes without compromising load-bearing capabilities (Figure 1).

The paper is structured as follows: Section 2 introduces the geometrical background of ECNC networks and their application for a Kit-of-Parts system; Section 3 describes the Approach and presents four methods to design ECNC networks; and Section 4 presents the architectural implementation of ECNC gridshells in Hong Kong.

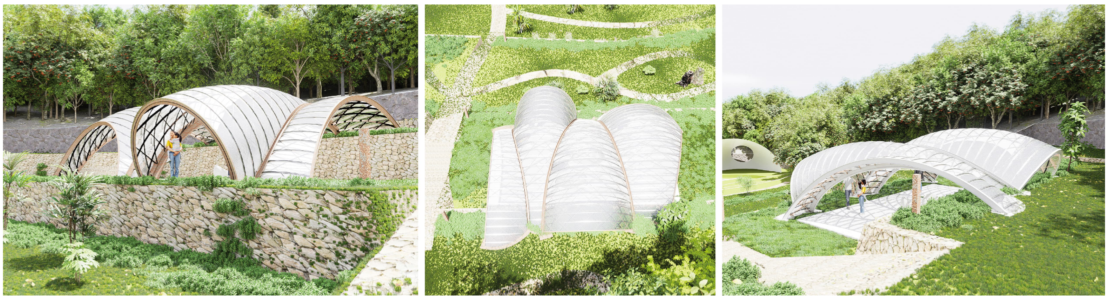

Figure 1: ECNC recyclable Kit-of-Parts structure, a shading pavilion proposal in Hong Kong.

2 Geometrical Background and Kit-of Parts System

2.1 Geometrical Background

This research is based on the theory of differential geometry, to examine three specific curvature values of curves on a surface using the “Darboux frame” (Figure 2): the normal curvature k_n, geodesic curvature k_g, and geodesic torsion τ_g. These measurements allow for a thorough understanding of gridshell geometry and efficient construction with straight or circular lamellas.

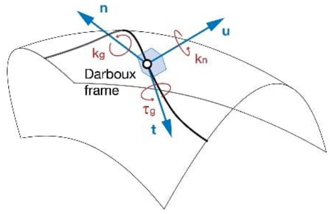

Figure 2: the “Darboux frame” [10].

Curves can be formed by assigning specific normal curvature values along a path on a surface. We are interested in curves with constant normal curvature $(k_n = a)$ (CNC). If $(k_n=0)$ we speak of an asymptotic curve. A circular strip following a CNC curve on the surface experiences lateral bending and twisting to adjust to the inherent geodesic curvature while maintaining a constant normal curvature along its path (Figure 3a).

When unrolled into a flat surface, the curved lamella keeps its normal curvature, transforming into a planar circular arc. The arc's radius can be calculated using the equation $k_n = 1/r$. This characteristic can be employed in constructing complex curved structures (Figure 3b).

Building a gridshell structure using a network of CNC-curves allows every lamella to be unrolled into identical circular arc strips [11]. Using a Chebyshev net with constant normal curvature further simplifies building parts with equal length. It is these nets that are explored here forequilateral constant normal curvature (ECNC) gridshell, where all lamellas form parts of the same circular strip and joints maintain even distances. ECNC networks exist only on specific surfaces known as “linear Weingarten surfaces of hyperbolic type” (HLW-surfaces), which exhibit a hyperbolic linear relationship between Gaussian and mean curvature [12].

2.2 Kit-of Part Building System

Based on section 2.1, a modular Kit-of-Parts system was developed. It uses two equal-length lamellas connected by a scissor joint, with sleeves facilitating bolted connections (Figure 3e). This bolt can later add a third element to stabilize the gridshell design. The system can accommodate various normal curvature values by adjusting the connection angle between elements, achieved via different slot combinations at the lamellas' ends, as illustrated in Figure 4. Assembling these lamellas results in a deployable ECNC grid (Figure. 3h).

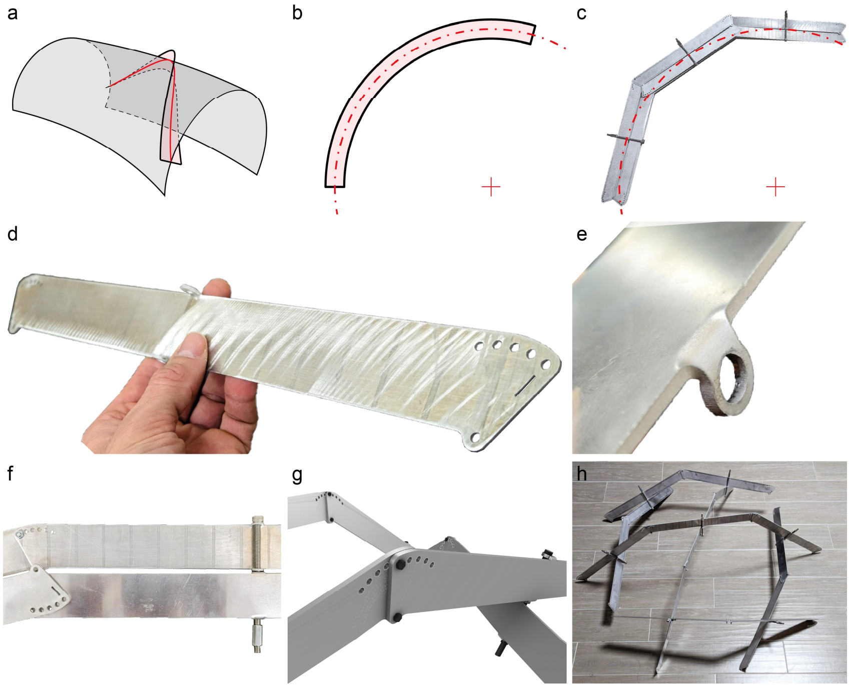

Figure 3: Kit-of Parts system and its application. a) a lamella describing a curve with a constant normal curvature value on a surface; b) the lamella unrolled; c) the collapsed Kit-of Parts system simulating the unrolled lamella; d) Kit-of Parts basic unit; e) folded sleeves to host bolts; f) connected Kit-of Parts in collapsed state; g) connected Kit-of Parts in deployed state (detail); h) connected Kit-of Parts in deployed state [1].

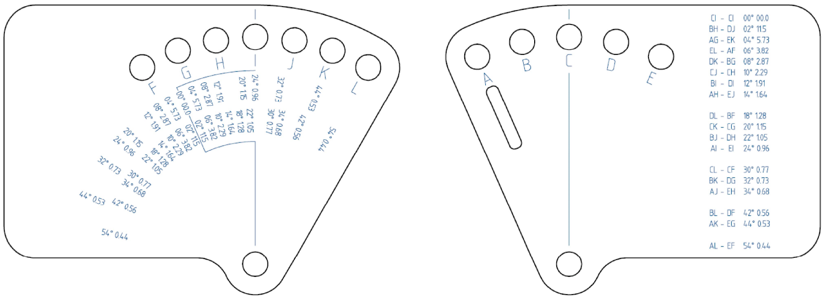

Figure 4 Angle-radius chart at the joint, to indicate the assembly combination of Kit-of Parts and their radius. The value of the angle is calculated by $\theta' = \pi - \theta$ and converted to degrees [1].

3 Digital Modelling

Rhinoceros3D and Grasshopper are used for digital modelling, testing three methods: Bowerbird, Kangaroo, and Python script, to trace constant curvature paths on freeform surfaces.

Bowerbird, a Grasshopper plugin [13] [14], enables users to find curves with constant normal curvature on multi-patch NURBS surfaces. By defining a starting point and normal curvature value, the algorithm traces two paths iteratively until it reaches the surface boundary or the local geometry disallows the designated normal curvature. This approach was initially applied to specific surfaces, such as cylinders, spheres, and pseudo-spheres (Figure 6), to create ECNC networks.

Kangaroo [15] is an interactive particle spring solver and plugin for Rhinoceros3D, Grasshopper. Kangaroo was used to generate ECNC networks following two strategies:

*Spherical Methodolo*gy. The simplest way to generate ECNC networks is to enforce an equilateral network onto a sphere, as here the normal curvature is constant in all directions. Using the "on mesh" function to pull a regular grid onto the spherical surface transfers the sphere's geometric properties onto the network (Figure. 6 right).

Constrained Strategy. This approach utilized length constraints for the gridshell elements, and angle constraints to determine between subsequent edges of the same polyline, to control the normal curvature. the value of the angle is calculated according to the equation $\boldsymbol{\theta = \pi - 2arc\,cos(l\ /\ 2R)}$, where $\boldsymbol{\theta}$ is the angle value measured in radian, l represents arc length, and can be rounded to the rod length, and R denotes the target radius of the unrolled lamella centre line [9]. The solver will adjust the geometry to satisfy the constraints, resulting in the desired ECNC network based on the normal curvature value.

Python Strategy. The method of Pellis et al. [9] was extended to create ECNC networks with a bisecting family of geodesic curves. A Python script for Grasshopper was created using rotational symmetry and HLW-surfaces, defined by the equation $aK + bH + c = 0$, where K and H are the Gaussian and mean curvature respectively. The normal curvature is $-b/2a$. To generate the network, a profile curve of a rotational HLW-surface is created by selecting coefficients a, b, c, a starting point, and a tangent direction. This profile curve is by definition geodesic, and form the third, triangulating family of curves. A complete rotational gridshell can be generated by creating one ECNC on the rotational surface and copying it to form a polar array. The second family of curves is obtained by simple reflection about any plane through the revolving axis. This method allows for intuitive control by specifying desired normal curvature and approximate length between joints.

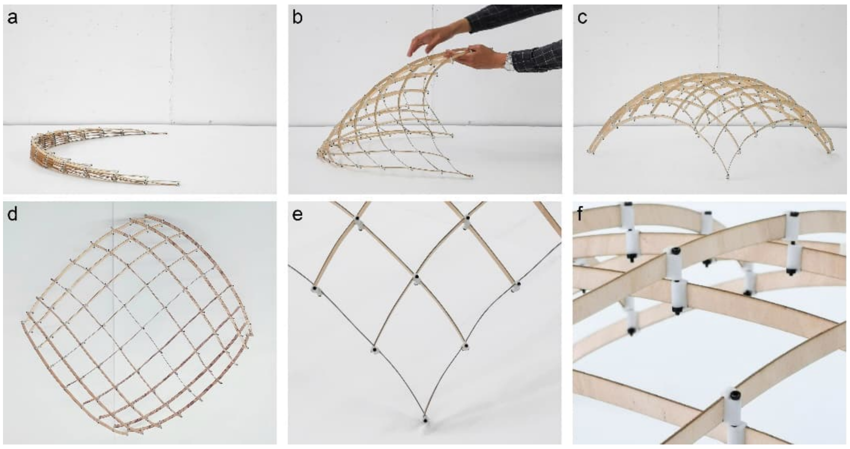

Fig. 5. Physical exploration: a physical model from circular lamellas joined at equal distances to investigate their morphology and kinetic behaviour. a) collapsed state; b) deployment process; c) deployed state; d) top view of the deployed state; e) landing tip detail; f) rotatable connection joints in detail [1].

4 Physical Exploration and Architectural Implication

Initially, a deployable timber model was constructed to examine the basic principles of ECNCs. Subsequently, hands-on physical workshops were conducted to create four models, each representing one of the strategies. Finally, the gathered data informed the design of two larger structures for practical use in Hong Kong, with one discussed below as an architectural implication.

4.1 Physical Exploration

Building an ECNC structure in the initial phase is straightforward due to their repetitive nature of building parts. A physical model of circular lamellas connected at equal intervals was constructed to study their morphology and kinetic behaviour (Figure 5). This model exhibited kinetic properties, enabling collapse into a folded arc-like configuration. When deployed, the model can assume various shapes, including spherical/pseudo-spherical, cylindrical, and rotational surfaces.

To evaluate the digital results, a Kit-of-Parts system was fabricated to construct physical models for each category: cylindrical, dome, and negatively curved surface with a single singularity (Figure 6). The physical experiments correlated with digital simulations, but required additional supports to maintain the desired shape. The slender standardized lamellas are soft and prone to buckling, specifically in the regions of high compression close to the support points, so bracing was introduced, using a third array of aluminium strips to triangulate the structure.

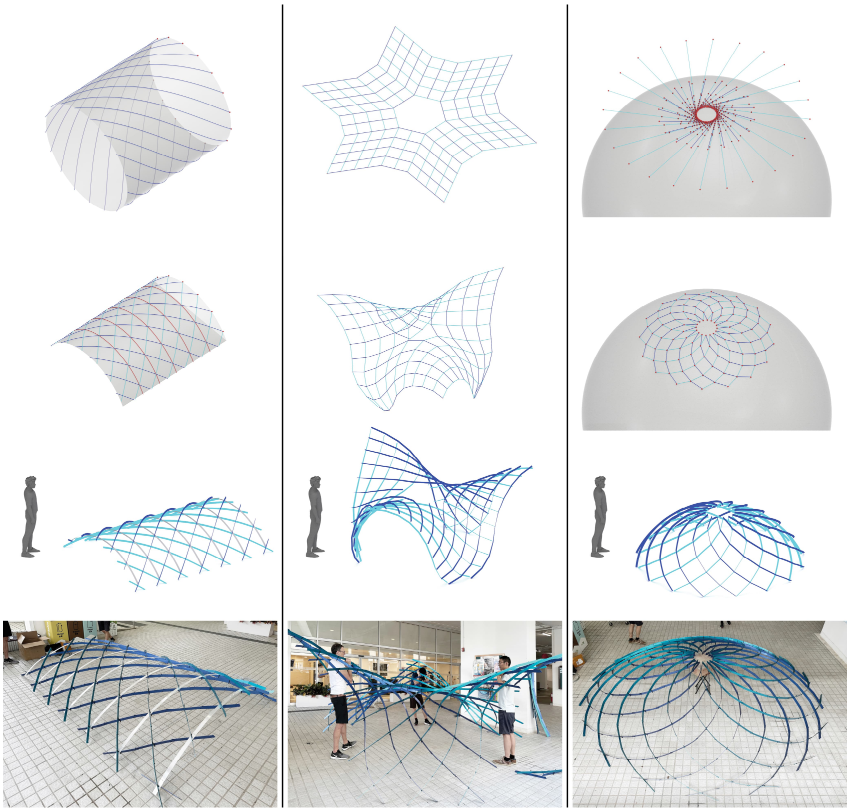

Fig. 6. Geometry workshop using the Kit-of Parts system to simulate multiple geometries. From left to right column: cylindrical shape, negatively curved surface with one singularity, and dome. From top to bottom row: input to grasshopper to find ECNC network, established ECNC network, visualizations, and the Kit-of Parts built models of the three geometries [1]

Next, a 1:1 scale pavilion model was tested using a rotational surface, effectively stiffened by incorporating geodesic planks for triangulation. The primary variable was the distance between bolting slots on the geodesic planks. The installation process was streamlined into two steps: 1) assembling individual Kit-of Parts units into cross-shaped modules, and interconnecting the modules with pre-set angles, and 2) connecting the network, and installing the geodesic planks. The installation finished within 2 hours, resulting in a pavilion measuring approximately 3 by 4 meters. The structure comprised 300 Kit-of Part components and five geodesic planks. Although minor deformation and buckling were spotted on the Kit-of Parts and geodesic planks (Figure 7g), the speed and logistical simplicity highlight the potential of this low-tech approach for reducing complexity in the construction of doubly-curved surfaces (Figure 7e).

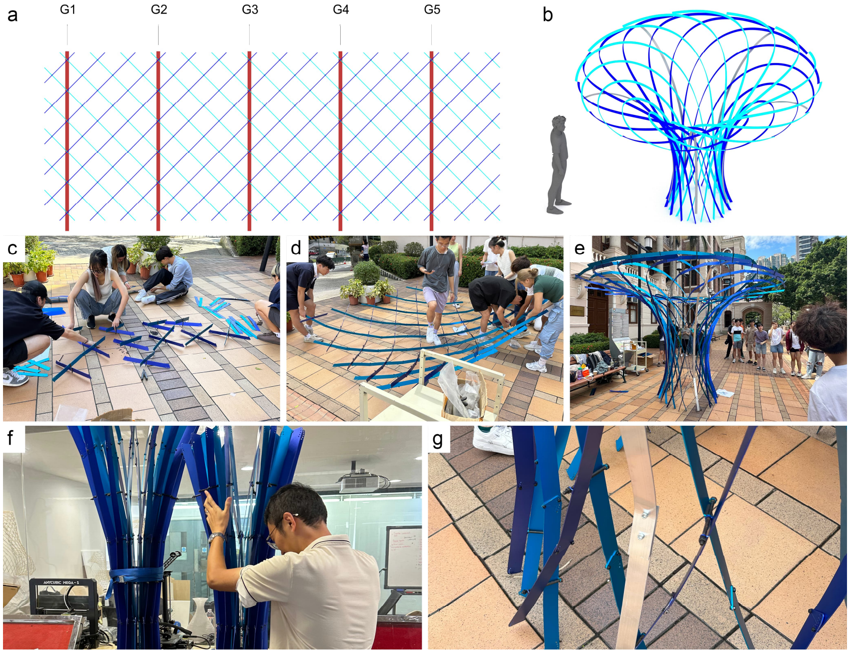

Fig. 7. Kit-of-Parts pavilion: a designed rotational-surface pavilion with students. a) Instruction illustration to students: Kit-of Parts basic unrolled layout; b) visualization the pavilion design; c, d) students assembling individual Kit-of Parts into a network; e) assembled pavilion; f) collapsed and recycled Kit-of Parts system; g) buckling spotted on the structure [1].

4.2 Architectural Implementation

Two ECNC gridshells were designed for a Hong Kong botanic garden: a trellis for plants (Figure 8) and a large weather proof shading device (Figure 1). This section will focus on the small trellis design, discussing the design process and rationalization.

For this scale, cold-bent Aluminium rectangular hollow sections were proposed to ensure stability under high wind forces. Aluminium offers higher flexibility and lower self-weight. To minimize costs and maximize flexibility, the length of lamella segments was set at 1.8 meters, triple the lateral length of the ECNC network. This length was chosen as a compromise between efficient production and easy transportation. A constant curvature radius of 4.5 m was defined, which will be used for multiple future designs within the botanic garden to foster recyclability.

Design-wise, the trellis was founded on an HLW-surface featuring an ECNC network. A rotational surface was created using the Python strategy by adjusting input parameters in Grasshopper, such as ECNC lateral length and constant curvature radius. The extraneous portions of the network were then trimmed to generate a three-leaf trellis (Figure 8).

The trellis aims to create an overhang while maintaining structural integrity. A 2.6 m high rotational surface was designed, balancing the density of lamellas and stability. After multiple iterations, a satisfactory rotational surface with an ECNC curve network was selected, resembling three leaves with downward-facing tips (Figure 9b).

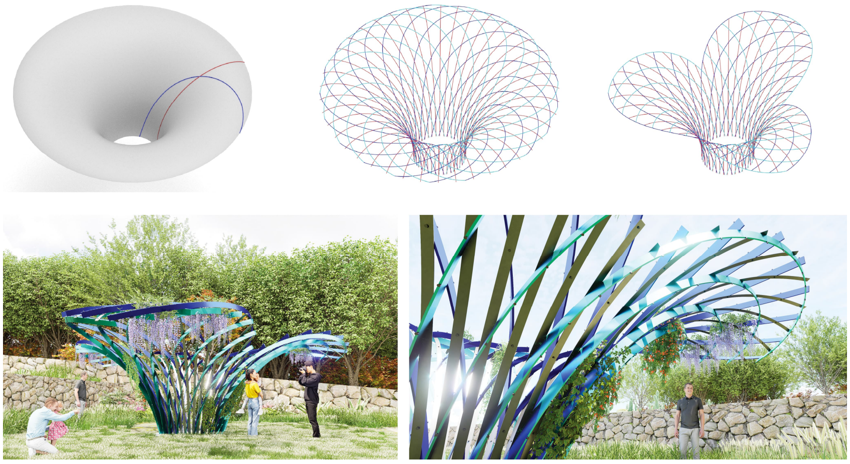

Fig. 8. Above: design process of the Small (S) trellis. Red: geodesic curve; blue and cyan: ECNC curves; below: visualizations [1].

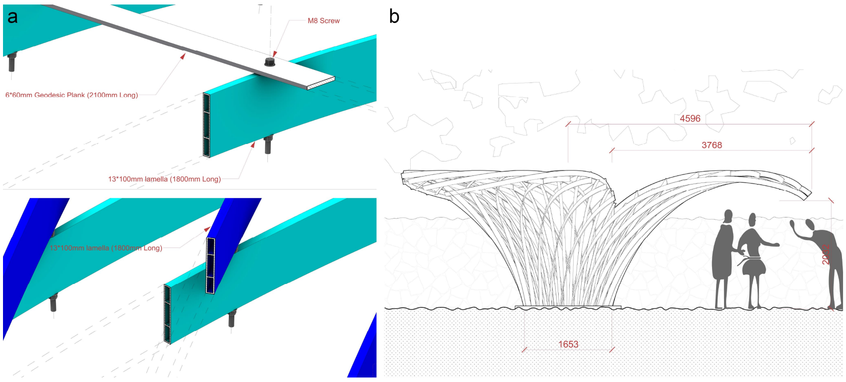

Fig. 9. a) construction details: joints; b) elevation (unit: millimetre) [1].

4.3 Structural Analysis

The dimensioning process for structural planning considers various load combinations, but only self-weight and live loads are discussed here [1]. For the ECNC members, a 100x13x2mm rectangular section was selected, and a 60x6mm solid rectangular section was chosen for geodesic members, with aluminum as the construction material (Figure 9a). A 6.0 mm diameter hoop cable was arranged to enhance structural performance. The analysis revealed maximum displacement of 33 mm at the tips and stress levels lower than the yield strength, with 62 MPa in the cable and 26 MPa in ECNC and geodesic members (Figure 10).

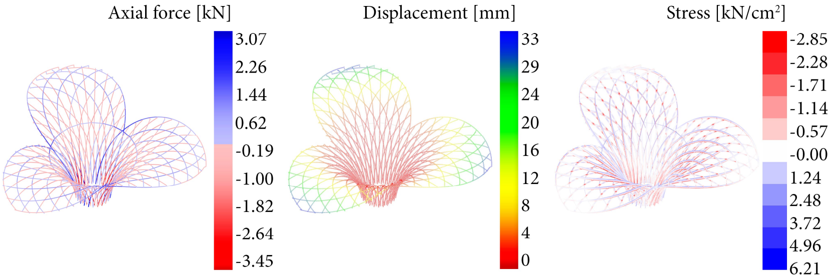

Fig. 10. Structure analysis-lamella layout/axial force (kN)/ displacement (mm)/ stress (kN/cm^2) [1].

5 Conclusion

In conclusion, this paper introduces a Kit-of-Part system for constructing complex gridshells using equilateral constant normal curvature (ECNC) grids triangulated by geodesic planks.. The approach simplifies the construction process, making it accessible to a wide range of builders without the need for specialized knowledge or complex components. It is cost-effective, encourages component reuse in multiple designs, and supports sustainability.

However, challenges remain, particularly in balancing simplified fabrication with individual stress distribution in gridshells. Future research will aim to resolve this, possibly through structural grid densification in high-stress areas or varying element dimensions.

Additionally, future studies may focus on developing computational tools for ECNC network creation and assembly optimization, exploring alternative materials, and investigating cladding strategies. Extending the ECNC system application to free-form surfaces could increase design versatility and broaden architectural applications, contributing to gridshell construction and design advancements for a sustainable future in architecture.

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