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Preliminary research on the shape determination approach of tubular curved origami structures

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Abstract

Curved origami has potential to be applied in the design of architectural geometry owing to not only the unique configuration, but also the mechanical behavior. Especially, the tubular origami structures show distinguished stiffness owing to the closed cross sections. This paper is focused on the shape determination to design tubular curved origami structures. We define the fully developed flat situation as the original state. The state in three-dimensional configuration is formed with folding and bending deformation. Since the curved origami is formed by developable surfaces, the surface can be described by the principal curvature. The origami model has tendency to morph back to deployed shape, which is stopped by tubular configurations. Thus, tubular curved origami forms a sort of self-stressed state. As the first step of designing the tubular curved origami structure, this paper discusses the shape determination of a simple model based on discretized differential geometry.

Keywords: curved origami, tubular structure, deployable surfaces, architectural geometry, self-stressed states.

1. Introduction

Origami, with its roots in foldability and mechanical properties, has become a significant area of interest for researchers and designers in structural engineering Tachi [1], Demaine et al. [2]. The emergence of curved origami, characterized by curved creases, presents opportunities to create curved developable surfaces, departing from traditional flat plates. This deviation towards curved forms has attracted designers aiming to realize organic and freeform structures.

Previous studies have made strides in designing curved origami, focusing on rulings and folding angles to represent surface configurations. However, most of them often lack in elucidating the intricate relationship between mathematically derived surfaces and their mechanical behavior as structures. Concerning thin plate materials, the curved origami made of flat sheet can be seen as a sort of thin-walled structure. One alternative solution is to utilize the benefit of tubular characteristics of thin sheet materials.

Tubular origami structure has 2 main advantages. Firstly, tubular origami are elements with closed cross-sections. Comparing against elements in the same shape with opened cross-sections, closed ones are generally much stronger in tortional deformation. Secondly, the tubular origami can be deployed into flat configuration. Consequently, a useful element can be dent and folded from a flat sheet material. This approach can be realized with less cost and difficulty than directly fabricating a curved configuration.

Therefore, this study investigates the design methodology of tubular curved origami. We start the study from discretized ruling equations. The three-dimensional curved shape is approximated as a series of quadrilateral planes, where the normal directions of the quadrilaterals describe the bending energy stored in thin plate materials. A simple design example is then explained based on the method illustrated in this paper. Focused on its application in structural engineering, this research aims to determine the shape of tubular curved origami elements, thereby exploring the potential application of curved origami in building structures.

2. Geometric Design

The fundamental equations are derived from discretized rulings, with both continuous and discretized cases validated in previous references Tachi [1], Watanabe and Mitani [3]. This enables the derivation of crease patterns and three-dimensional shapes without iterative calculations.

The ruling model method describes two developable surfaces using two sets of rulings for an arbitrary curve in three-dimensional space (Figure 1). The curve is discretized into n points $\boldsymbol{x}_i$, where $i = 1,2,\dots,n$ denotes the index of individual rulings. The rulings are defined as follows:

$$\boldsymbol{r}_i^{\,-} = cos\beta_i^-\boldsymbol{T}_i - sin\,\beta_i^- cos\alpha_i\boldsymbol{N}_i + sin\beta_i^- sin\alpha_i\boldsymbol{B}_i \quad (1)$$

$$\boldsymbol{r}_i^{\,+} = cos\beta_i^+\boldsymbol{T}_i + sin\,\beta_i^+ cos\alpha_i\boldsymbol{N}_i + sin\beta_i^+ sin\alpha_i\boldsymbol{B}_i \quad (2)$$

Here, $\boldsymbol{T}_i, \boldsymbol{N}_i,$ and $\boldsymbol{B}_i$ denote the tangent, normal, and binormal vectors of the curve, respectively.

β_i^- and β_i^+ represent the angles formed by the tangent vector of the crease curve when each ruling is expanded in two-dimensional space, determined by:

$$cot\beta_i^- = \frac{-\alpha_i{}' + \tau_i}{\kappa_i \cdot sin\alpha_i} \quad (3)$$

$$cot\beta_i^+ = \frac{\alpha_i{}' + \tau_i}{\kappa_i \cdot sin\alpha_i} \quad (4)$$

Here, α_i denotes the angle formed by the contact plane between the developable surface and the curve, while κ_i and τ_i represent the curvature and torsion of the curve, respectively.

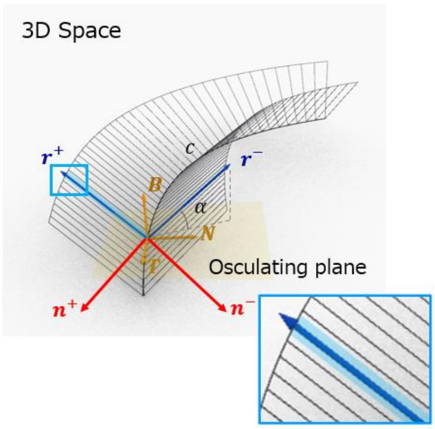


Figure 1: Ruling model for one particular discretized curve

Since the normal vectors of the surfaces are perpendicular to the tangent vectors and rulings, the normal vectors can be calculated as:

$$\boldsymbol{n}_i^{\,-} = -\boldsymbol{T}_i \times \boldsymbol{r}_i^{\,-} \quad (5)$$

$$\boldsymbol{n}_i^{\,+} = \boldsymbol{T}_i \times \boldsymbol{r}_i^{\,+} \quad (6)$$

Substituting Eq. (5) and (6) yields:

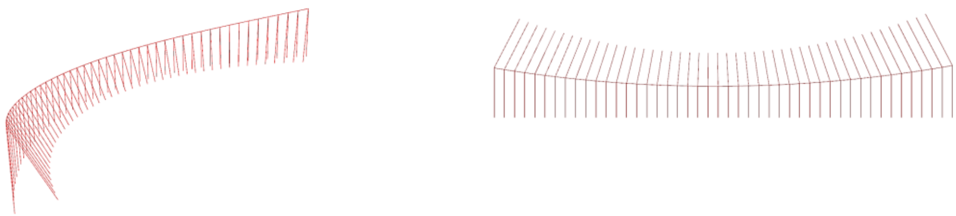
$$\boldsymbol{n}_i^{\,-} = sin\beta_i^- sin\alpha_i\boldsymbol{N}_i + sin\,\beta_i^- cos\alpha_i\boldsymbol{B}_i \quad (7)$$

$$\boldsymbol{n}_i^{\,+} = -sin\beta_i^+ sin\alpha_i\boldsymbol{N}_i + sin\,\beta_i^+ cos\alpha_i\boldsymbol{B}_i \quad (8)$$

3. Numerical example

According to the modelling methodology proposed in this paper, we present a numerical example of tubular origami. For simplicity, this example is focused on a single creased case. We defined a three-dimensional Non-Uniform Rational B-Spline (NURBS) curve with 6 control points using Rhinoceros 6 software. This curve serves as the initial model, with the knot vector of the initial state automatically updated by the software. It is discretized into n segments of equal lengths to obtain the discretized version. It is notable that this pre-designed curve generates parallel rulings on one side of the curved crease. Utilizing this characteristic, a tubed configuration is realized with mirroring process. The detailed process is illustrated as following.

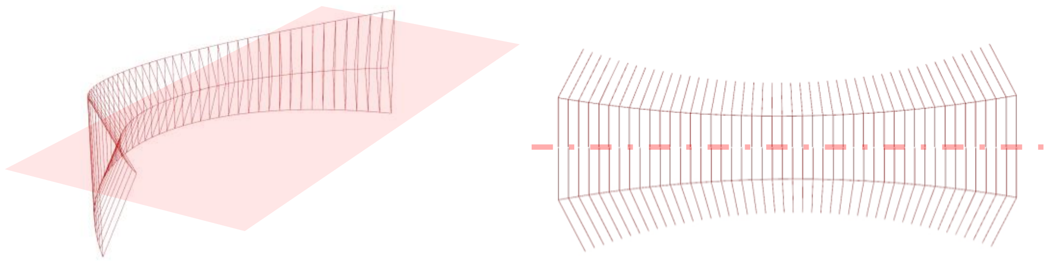
Utilizing Equations (1) and (2), the three-dimensional configuration is derived, as depicted in Figure 2(a). Simultaneously, the two-dimensional crease and the corresponding rulings are computed using Equations (3) and (4), illustrated in Figure 2(b). This numerical example showcases the application of the proposed methodology in generating the three-dimensional configuration and determining the two-dimensional crease pattern for a given NURBS curve.



(a) **(b)**

Figure 2: Example of a single-crease model: (a) three-dimensional configuration, (b) two-dimensional crease pattern.

Owing to the parallel rulings in two-dimensional space and the three-dimensional space, tubular models can be generated using mirroring symmetricity as show in in Figure 3. The three-dimensional configuration is symmetric to the mirror plane, while the two-dimensional crease pattern is symmetric to the dash dot line.



(a) **(b)**

Figure 3: Tubular origami generated from a single-crease model: (a) three-dimensional configuration, (b) two-dimensional crease pattern.

The three-dimensional configuration can be analyzed using the curvature of the surface. Because the tubular curved origami is a sort of assembly of deployable surfaces, the non-zero curvature can describe the shape of the corresponding surface. For example, the top surface of the curved beam show in Figure 4 can be described by the gradation diagram of the curvature. Therefore, the structure configuration can be further optimized using the methods proposed in the previous research by the authors [4]. From the view of practical usage, the foldability and material can also be concerned with our research [5-6].

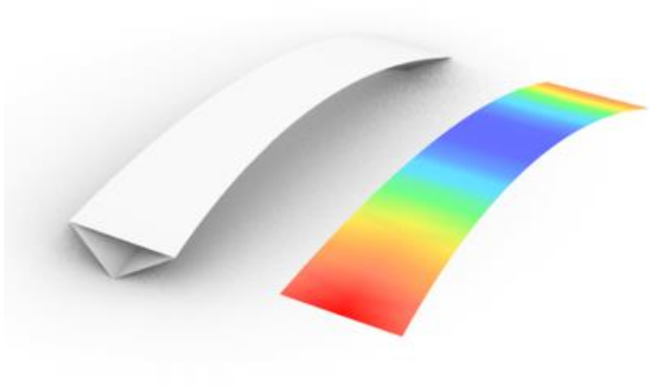


Figure 4: Tubular curved origami model and surface curvature

4. Conclusion

This paper presents an investigation into the shape determination method of tubular curved origami. Based on the discretized differential geometry, the tubular models can be generated using symmetricity. Owing to the single curvature characteristic of the deployable surface, the tubular curved origami model can be described using surface curvature. Since the tubular origami can be realized from a flat shape using folding and bending deformation, it shows potential in cutting cost of fabricating curved surfaces. However, the research on this stage is limited in symmetric models. Also, the investigation of structural behavior for structural engineering needs to be clarified in the future. We believe this research can bridge the gap between mathematical geometry and structural configuration, paving the way for innovative design approaches in structural engineering.

Acknowledgements

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