Proceedings of the IASS 2024 Symposium
Redefining the Art of Structural Design
August 26-30, 2024, Zurich, Switzerland
Philippe Block, Giulia Boller, Catherine DeWolf,
Jacqueline Pauli, Walter Kaufmann (eds.)

Design solutions for the barrel vault FreeGrid Design Baseline Gridshell

Luca BRUNO[a], Lorenzo RAFFAELE[a], Fiammetta VENUTI[*]

[*] Politecnico di Torino, Viale Mattioli 39, 10125 Torino, Italy
fiammetta.venuti@polito.it

[a] Politecnico di Torino, Viale Mattioli 39, 10125 Torino, Italy

Abstract

This paper presents a contribution in the framework of FreeGrid, a benchmark on design and optimization of free-edge steel gridshells. The present study focuses on the barrel vault gridshell and proposes three Design Solution Gridshells (DSG), whose performances are evaluated with respect to the Design Baseline Gridshell (DBG). The proposed DSGs share the same geometry, mesh pattern, type of cross-section and material of the DBG. Enhancement of the performances is obtained through a design-from-precedent approach, i.e. taking inspiration from previous design solutions. In particular: the free-edge flexural stiffness is increased by simply changing its cross-section; the free-edge is stiffened through a truss-girder laying on the vault surface, in analogy e.g. with Nervi's hangars; an unprecedented solution for gridshells is borrowed by masonry barrel vaults, based on the deviation of thrusts from free-edge to lateral constraints by means of a relieving arch laying on the vault surface. The structural performances of DSGs are evaluated through the metrics adopted in the FreeGrid benchmark. Comparison with the DBG structural metrics allows to discuss pros and cons of each solution and to rank them.

Keywords: FreeGrid benchmark, gridshells, barrel vault, design solutions, performance assessment

1. Introduction

This paper presents a contribution in the framework of FreeGrid, a benchmark on design and optimization of free-edge steel gridshells [1], launched at the IASS Annual Symposium 2023 [2]. The aim of the FreeGrid benchmark is to test and compare different approaches to the design and optimization of steel gridshells on the basis of ad-hoc defined performance metrics. To this aim, FreeGrid proposes three Design Baseline Gridshells (DBGs): a barrel vault, a parabolic dome, and a hyperbolic paraboloid, having free-edges and subjected to uniform and piecewise uniform load conditions. Participants are called to modify the DBGs in order to improve their structural, buildability, and sustainability performances through the maximization of a bulk quantitative performance metric.

The present study focuses on the barrel vault DBG, since it represents the most challenging structure among the three, having single curvature and the free-edge along the direction of zero curvature (i.e., the free-edge runs along a horizontal straight line). Actually, the study reported in [1] shows that the barrel vault has the lowest structural performance in terms of serviceability behaviour, with a maximum displacement at Serviceability Limit State (SLS), which is about twelve times greater that the limit displacement.

In order to improve the barrel vault structural performances, and in particular the SLS ones, three Design Solution Gridshells (DSG) are proposed. In the following sections, both DBG and proposed DSGs are described; then, some preliminary results are presented and the pros and cons of each proposed solution are discussed.

2. Description of the DBG

The barrel vault DBG has a span B of 30 m, a length $L \approx 1.04B$ and a parabolic generatrix of rise $f = B/8$ (Figure 1). The gridshell is characterized by a quad mesh obtained by dividing both directrix and generatrix into 20 edges of constant length $b = A/20 \approx 1.56$ m. The grid structural members are made of steel with a bilinear elastic-perfect plastic constitutive law. Specifically, steel S355 is adopted, with density $\rho = 7850$ kg/m^3, Young's Modulus $E = 210000$ MPa, Poisson's ratio $\nu = 0.3$ and yield strength $f_y = 355$ MPa. All the structural members have the same cross section, i.e., a circular hollow section with diameter of 139.7 mm and thickness equal to 14.2 mm.

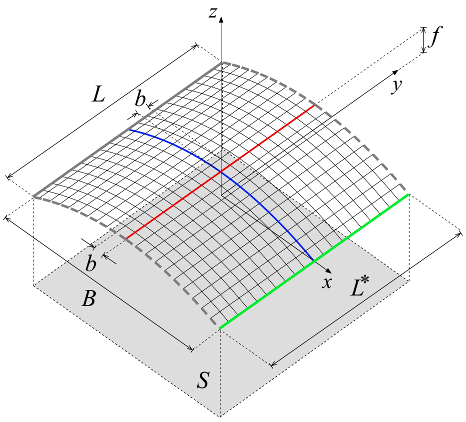

Figure 1: Geometry of the barrel vault DBG

All the joints along the boundaries, except for those along the free-edge L^*, are externally constrained by perfect hinges. Moreover, the joints along the head arches are allowed to move in y direction. All the internal structural joints are rigid.

In the present study only the symmetrical load condition is considered, where all structural joints are subjected to point loads $Q_{1,j} = q_1 s_j$, where s_j is the projection on the horizontal reference plane of the tributary area of the j-th joint and $q_1 = 1800$ N/m^2 for Ultimate Limit State (ULS) and 1200 N/m^2 for SLS.

The full detailed DBG description is available in [1].

3. Description of the DSGs

Three different design solutions are explored to enhance the structural performances through a design-from-precedent approach, i.e. taking inspiration from previous design solutions. All the proposed DSGs share the common objective of introducing minimal modifications to the original DBG, by simply adding or modifying a number as smallest as possible of structural members: in other words, the shape of the DBG is unchanged in terms of joint numbers and their coordinates, and added structural members have the same cross section as the grid ones. In particular:

DSG1 the free-edge flexural and torsional stiffness is increased by simply changing the size of its O-type cross-section according to the criteria proposed in [3] (Fig. 2a, Cabot Circus, Bristol [4]). Specifically, the members of the free-edge (red line in Fig. 2d) are assigned a circular hollow section with diameter of 610 mm and thickness of 16 mm, having about 120 times the inertia of the grid current structural members;

DSG2 the free-edge is stiffened through a truss-girder, in analogy e.g. with Nervi's hangars [5] (Fig. 2b). It is worth pointing out that the FreeGrid design constraints do not allow to fully exploit Nervi's solution, which involved a spatial truss-girder. Actually, in order to maintain a single-layer gridshell (i.e. non-manifold vertices and edges are not permitted), the truss-girder should lay on the vault surface. To this aim, the red diagonal members in Fig. 2e) with the same cross section as the grid members are added to the DBG;

DSG3 an unprecedented solution for gridshells is borrowed by masonry barrel vaults (Fig. 2c), based on the deviation of thrusts from free-edge to lateral constraints by means of a relieving arch laying on the vault surface. The red diagonal members in Fig. 2f) with the same cross section as the grid members are added to the DBG to form an arch-like trussed, non planar structure.

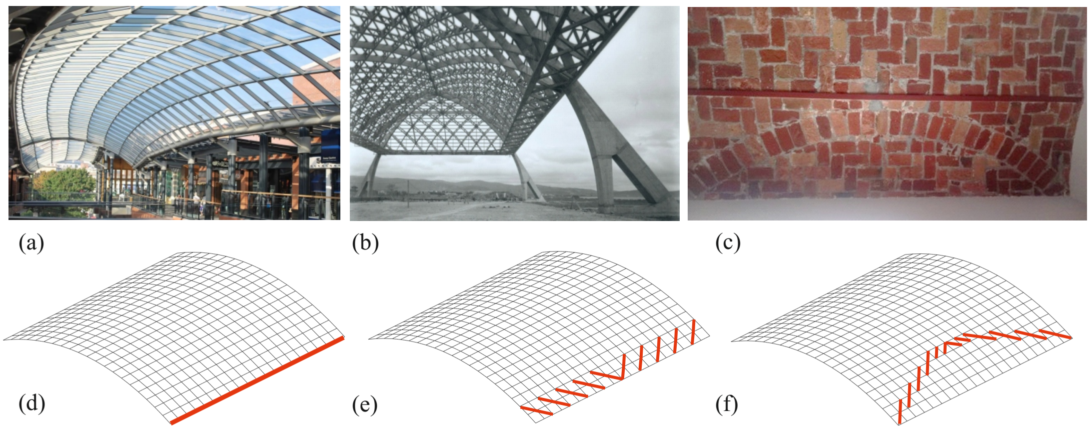

Figure 2: Reference structures (a-c) and schemes of the adopted DSGs (d-f)

4. Analysis and results

First, 2D simplified reference structural models of the central arch only ($y = 0$) are defined to portray its structural behaviour, depending on the adopted design solution of the free-edge (see Fig. 3a). The DBG central arch can be interpreted as supported by a Winkler's bed of discrete elastic springs, where each couple of spring takes the place of the structural members orthogonal to the arch and is aligned along the two principal axis of the members' cross sections. The DSG1 has an elastically-restrained edge with even flexural stiffness in both normal and tangential directions, and significant torsional stiffness because of the adopted O-type cross-section of the edge beam. The DSG2 has an elastically-restrained edge with uneven flexural stiffness, i.e. the tangential one is larger because of the in-plane truss. Finally, the DSG3 can be interpreted as restrained by a roller (or more precisely by an axial spring with very high stiffness) with sliding plane orthogonal to the central arch in correspondence of the top of the relieving arch, and by an elastically-restrained edge with even flexural stiffness in both normal and tangential directions because of the nonlinear tension stiffening effect induced on the directrix-wise beam at the free edge, i.e. the chain of the relieving arch with eliminated thrust.
The structural behaviour of the DBG and DSG gridshells is evaluated through 3D FEM models according to the technical specifications provided in [6]. Geometrically and Materially Non-Linear Analysis is performed through the software Ansys® Mechanical APDL. Figure 3b plots the deformed shapes at

SLS for a vertical section of the vault at $y = 0$, where maximum displacements occur. Circles on the deformed arches highlight a change in the direction of vertical displacements (from upwards to downwards).

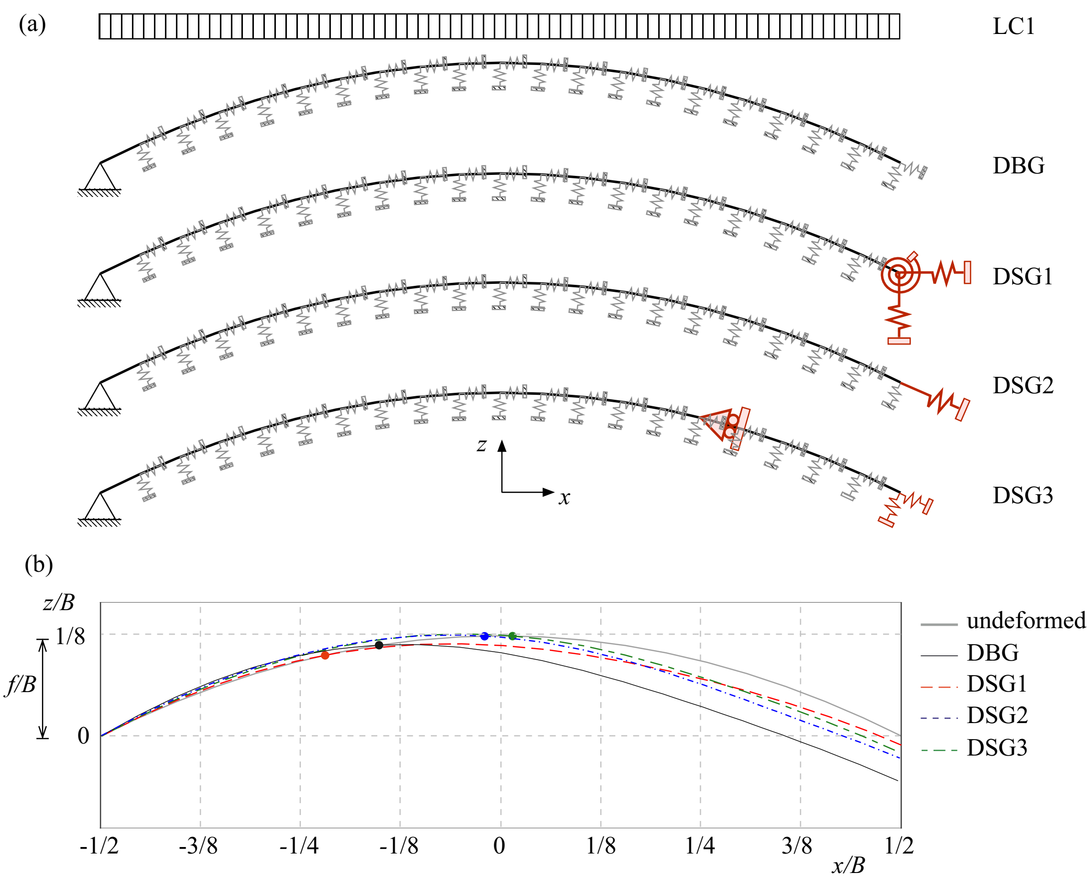

Figure 3: 2D simplified reference structural models of the central arch ($y/B = 0$) for each gridshell (a), and deformed shapes of the same arch at SLS resulting from the full 3D nonlinear model (b)

The vault deformed profiles at the symmetry $x - z$ plane obtained by 3D FEM model are qualitatively consistent with the 2D simplified reference models in Figure 3a. All DSGs reduce the vertical displacement with respect to DBG. In particular, DSG1 is the most efficient solution in reducing vertical displacements, because of the large normal stiffness of the directrix-wise beam at the free edge. Downwards displacements induced by kinematic rotation arise for $x/B > -1/4$. Conversely, DSG2 results the worst performing one, because of the negligible normal stiffness at the free edge. The deformed shape of DSG3 follows DSG2 up to the $x = 0$, then it deviates because of the absorption of tangential displacement in correspondence of the top of the relieving arch and tension stiffening of the directrix-wise beam at the free edge. In both cases, downward displacements induced by kinematic rotation arise for $x/B > 0$.

A deeper insight into the mechanical behaviour of DSGs at ULS is provided in Figure 4. For each analyzed gridshell, the figure reports the curves of LF, ϵ^*_{max} and ϵ^*_{min} versus the normalized displacement (Fig. 4a-d-g-l). The last two quantities have been defined in [1]: specifically, $\epsilon^*_{min} = 1$ refers to the condition of full plasticization of at least one cross section, while $\epsilon^*_{max} = 1$ refers to the plasticization of at least one fiber of a cross section. Moreover, for each gridshell, members in tension ($N^e \geq 1$) and in compression ($N^e \leq 1$) (red and blue lines in Fig. 4b-e-h-m) and yielded Finite Elements (Fig. 4c-f-i-n)

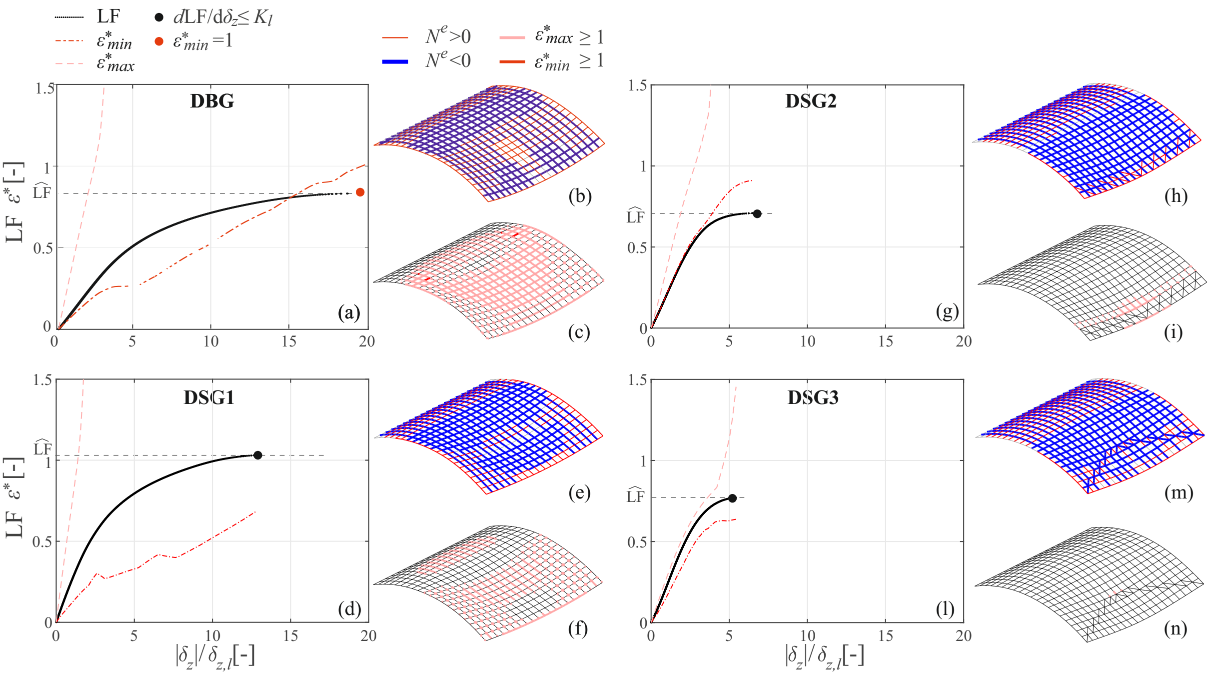

Figure 4: LF, ϵ^*_{max} and ϵ^*_{min} versus normalized displacement (a,d,g,l); members under tension and compression (b,e,h,m); yielded FE (c,f,i,n) for DBG and DSGs

are plotted on the deformed gridshells. It is worth pointing out that the slopes of the LF curves at the origin denote an increase in stiffness of all DSGs with respect to DBG. Despite this common feature, the four gridshells show quite different failure modes:

- in DBG, characterized by the highest values of vertical displacement, the critical LF occurs due to the formation of plastic hinges (Fig. 4a), after a great number of members are partially plasticized (Fig. 4c);
- in DSG1, critical LF occurs in the elasto-plastic regime, with a significant number of elements partially plasticized (Fig. 4f);
- in DSG2 and even more in DSG3 instability occurs in an almost elastic regime, where only a few members (only three in DSG3) have some yielded fibers;
- buckling in DSG2 locally takes place at the upper (inner) chord of the trussed edge beam along the out of surface direction, inducing the gridshell global collapse. In the perspective of the final sizing of the members of such a chord, a section with different radii of inertia is more advisable than the current O-type section.

The effects of each proposed solution on the gridshell displacements at SLS are highlighted in Figure 5, which plots the normal and tangential displacements scaled with respect to the maximum absolute value from the four cases, both occurring on the DBG ($|\widehat{\delta}_{\perp}| = 1.82$ m, $|\widehat{\delta}_{||}| = 0.5$ m). DSG1 significantly reduces both components of displacement keeping similar spatial patterns as DBG; the maximum normal displacement moves from the free-edge to the core. DSG2 and DSG3 share similar spatial distributions of both normal and tangential displacements: in both cases, normal diplacements have their maximum at around $x = L/4$ and a change of sign at around $x = 0$; the relieving arch in DSG3 is more effective than the truss girder in DSG2 in reducing the tangential components of displacement.

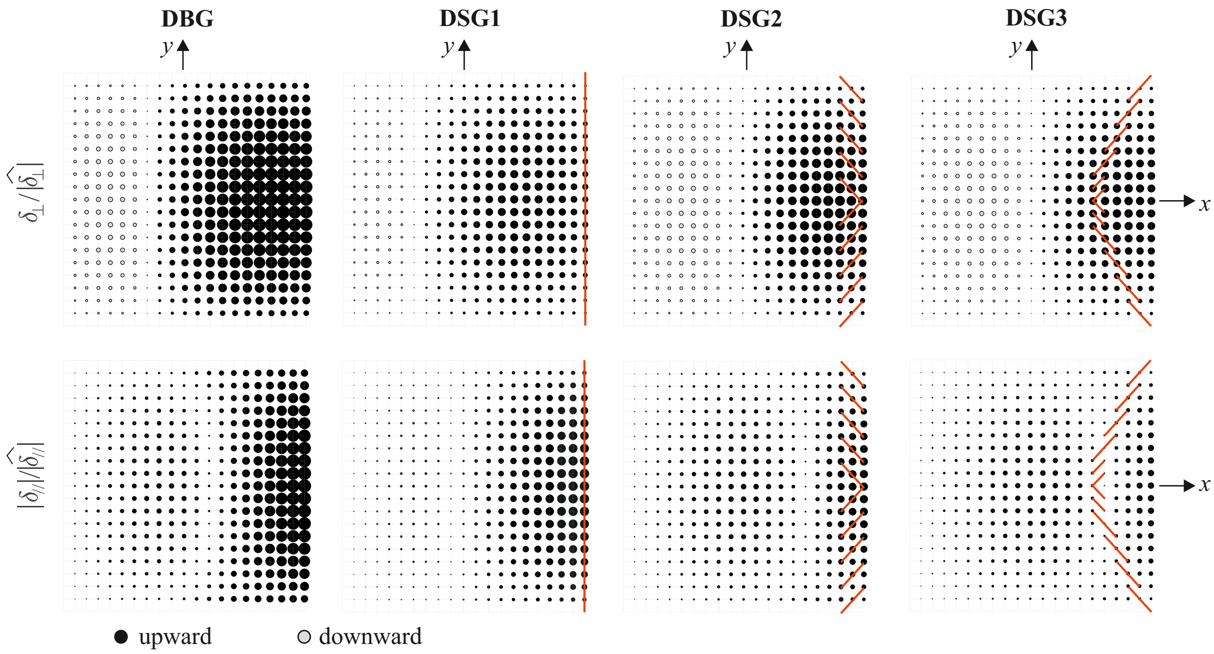

Figure 5: Normal and tangential displacements

The effects of each proposed solution on the gridshell deformability at SLS are put in evidence in Figure 7, which plots the out-of-plane and in-plane nodal deformations, according to the definition provided in [3]. Specifically, out-of-plane deformations are evaluated through the difference in discrete gaussian curvature $\Delta\kappa = \kappa - \kappa_0$ between the deformed and undeformed gridshell, where the discrete gaussian curvature in the node p is $\kappa(p) = 2\pi - \mathcal{E}\widehat{p}\mathcal{N} - \mathcal{N}\widehat{p}\mathcal{W} - \mathcal{W}\widehat{p}\mathcal{S} - \mathcal{S}\widehat{p}\mathcal{E}$ (Fig. 6a); in-plane deformations are evaluated in terms of equivalent shear deformations $\Gamma = \gamma_{xy}(p\mathcal{N}) + \gamma_{yx}(p\mathcal{E})$ of each quadrangular face, where $\gamma_{xy}(p\mathcal{N}) = (|\delta_{x,\mathcal{N}} - \delta_{x,p}|)/b$ and $\gamma_{yx}(p\mathcal{E}) = (|\delta_{y,\mathcal{E}} - \delta_{y,p}|)/b$ (Fig. 6b). Both metrics are normalized with respect to the maximum absolute value among the four cases ($|\overline{\Delta\kappa}|$ and $\bar{\Gamma}$) and plotted over the gridshell plan view with circles, whose dimensions are proportional to the normalized values of each metric. $|\overline{\Delta\kappa}|$ occurs on the DBG, while $\bar{\Gamma}$ occurs on DBG and DSG2. The following considerations

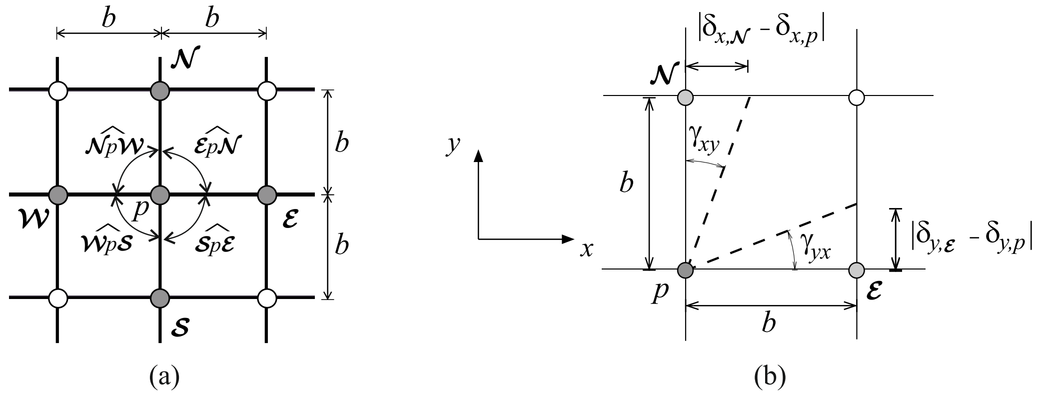

Figure 6: Schemes for the definition of discrete gaussian curvature (a) and equivalent shear deformation (b) [3]

can be outlined:

- DSG1, which is the most widely adopted solution in the design practice, is the one that minimizes both out-of-plane and in-plane deformations; out-of-plane deformations have a pattern similar to DBG, with a change in the sign of $\Delta\kappa$ at around $x = -B/4$; in-plane deformations are almost negligible with respect to DBG and other DSGs because of the impressive in-plane and out-of-plane inertia of the free-edge members;
- DSG2 allows to concentrate out-of-plane deformations in the core of the gridshell, giving rise to an arch-like pattern of nil nodal deformations, with a change in the sign of $\Delta\kappa$ at about $x = -B/10$ for $y = 0$. In-plane deformations are generally lower than in DBG.
- DSG3 allows to reduce both out-of-plane and in-plane deformations with respect to DBG, and further localize out-of plane deformations in the core of the gridshell, with a change in the sign of $\Delta\kappa$ at about $x = 0$ for $y = 0$. However, in-plane deformations remain localized close to the lateral support of the arch laying on the vault surface.

The non negligible in-plane deformations close to the tips of both the trussed edge beam (DSG2) and the trussed relieving arch (DSG3) clearly indicate that their shear stiffness is too small, and that the structural elements that constitute them cannot share the same cross section of the current members of the gridshell.

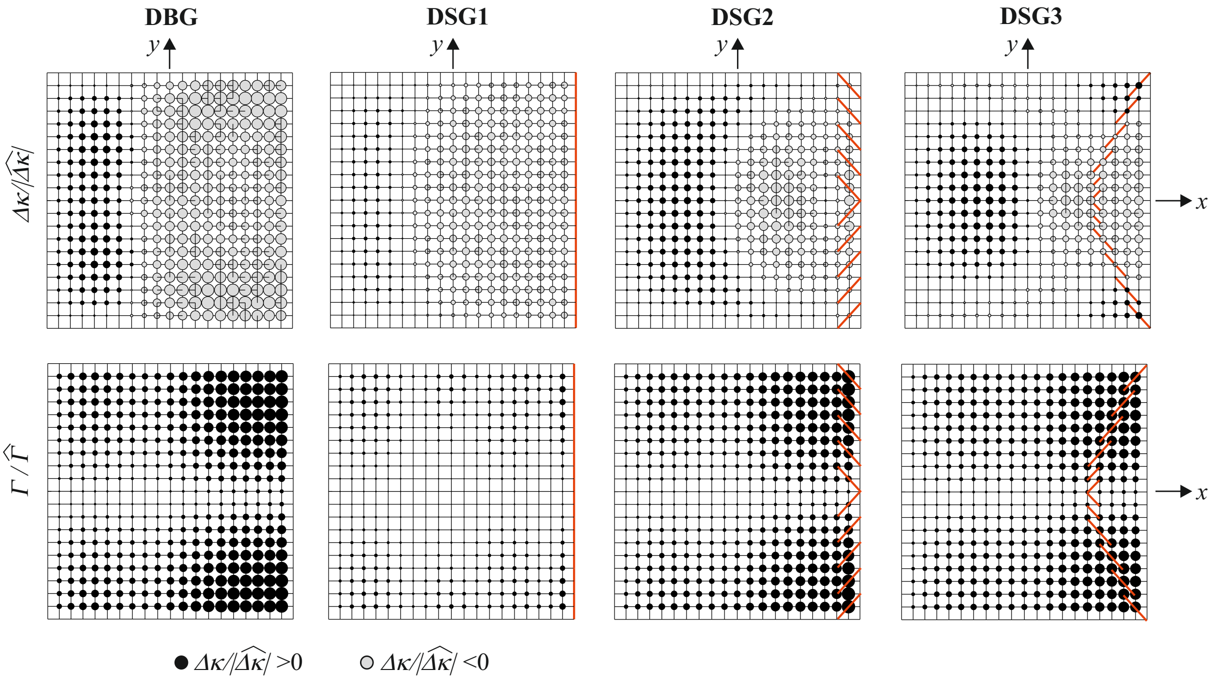

Figure 7: Out-of-plane and in-plane deformations

According to the FreeGrid benchmark technical specifications [7], the structural performances should be evaluated through two bulk Design Goal (DG) metrics: the critical Load Factor $\widehat{\mathrm{LF}}$ at ULS and the modulus of the maximum vertical displacement $|\hat{\delta}_z|$ at SLS. Structural performances should be improved by solutions that allow to obtain $\widehat{\mathrm{LF}} \geq 1$ and $|\hat{\delta}_z| \leq \hat{\delta}_{z,l}$. For the present case study, $\hat{\delta}_{z,l} = B/200 = 0.15$ m. Figure 8 summarizes the structural DG metrics obtained for each DSG in comparison with the DBG ones. The maximum displacement is normalized with respect to the limit value. Furthermore, Figure 8 reports the percentage differences Δ of both metrics with respect to the DBG values. DSG1 is the only

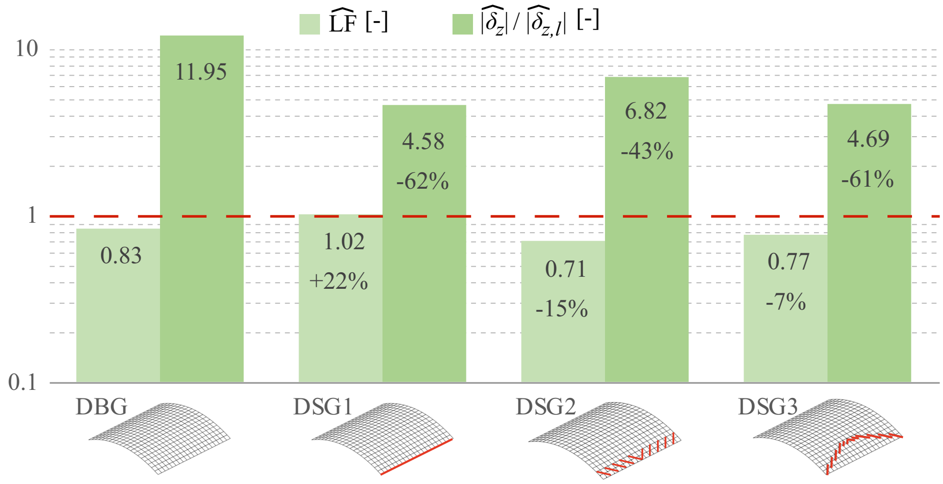

Figure 8: Structural metrics for DBG and DSGs

solution that fulfils the stability performance. None of the DSGs satisfies the serviceability performance, even though DSG1 and DSG3 are highly effective in reducing the maximum vertical displacement of about 60 %. DSG2 underperforms with respect to the other solutions.

5. Conclusion

This paper has presented and discussed some preliminary results concerning the proposal of three design solutions to address one of the FreeGrid design challenges. Three DSGs are thought with the aim of improving the structural performances of the barrel vault DBG by introducing the least amount of modifications to the reference structure. All the proposed solutions have demonstrated to induce a significant variation in the structural behaviour at both SLS and ULS. Despite none of the proposed solutions have allowed to meet the imposed requirements in terms of both LF and maximum displacement, they all look promising in terms of conceptual design and are expected to reach the structural performances by proper sizing of the added structural members (e.g., through optimization procedures). This will be done in the next steps of the study, together with the quantification of the DSGs buildability and sustainability performances according to the FreeGrid specifications.

Acknowledgments

The conceptual design and performance assessment have been developed to answer the call of "FreeGrid: a benchmark on design and optimisation of free-edge gridshells" (`https://sites.google.com/view/freegrid`). The FreeGrid benchmark is supported by the Italian Council for Steel Structures (CTA), under the umbrella of the International Association for Shell and Spatial Structures (IASS), and in partnership with ArcelorMittal Steligence.

References

[1] L. Bruno *et al.*, "Exploring new frontiers in gridshell design: The freegrid benchmark," *Structures*, vol. 58, p. 105 678, 2024.

[2] L. Bruno *et al.*, "Freegrid:a benchmark on design and optimisation of free-edge gridshells," in *Proceedings of the IASS Annual Symposium 2023, Melbourne*, 2023.

[3] F. Venuti and L. Bruno, "Influence of in-plane and out-of-plane stiffness on the stability of free-edge gridshells: A parametric analysis," *Thin-Walled Structures*, vol. 131, pp. 755–768, 2018.

[4] H. Schober and S. Justiz, "Cabot Circus, Bristol Ebene Vierecknetze für freigeformte Glasdächer," *Stahlbau*, vol. 81, pp. 28–42, 2012.

[5] S. Kimura and T. Taji, "Pier Luigi Nervi's design process of hangars," *Japan Architectural Review*, vol. 4, pp. 455–468, 3 2021.

[6] L. Bruno *et al.*, "Ts03: Finite element model," FreeGrid benchmark, Tech. Rep., 2023. [Online]. Available: `https://sites.google.com/view/freegrid/docs?authuser=0`.

[7] L. Bruno *et al.*, "Ts04: Performance metrics," FreeGrid benchmark, Tech. Rep., 2023. [Online]. Available: `https://sites.google.com/view/freegrid/docs?authuser=0`.