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Assessment of the structural, buildability and sustainability performances of the FreeGrid Design Baseline Gridshells

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Abstract

Gridshell design is an intrinsically multidisciplinary activity that potentially involves architects, engineers, builders, and experts in mathematics and computer graphics. Correspondingly, the design of gridshell structures should comply with multiple design issue requiring different competences, mainly referring to structural mechanics, construction and sustainability. This makes gridshell design an intricate task, if compared to conventional structures. This study provides a deep insight into a methodological framework for their holistic performance assessment taking into account the intrinsic multidisciplinarity required when conceiving gridshell structures. The overall performance results from the linear combination of three partial performance metrics, namely structural response, buildability and sustainability. Each performance metric is defined by combining different design goals, identified on the basis of the current state of the art. The outlined methodology is then applied to three free-edge gridshells adopted by the FreeGrid benchmark and their fully-constrained counterparts. This application demonstrates how the adopted methodology is able to effectively reflect the performance of the examined gridshells, it allows to shed some light on the scarcely investigated mechanical behavior of free-edge gridshells, and potentially inspires future design solutions improving their performances within the FreeGrid benchmark.

Keywords: FreeGrid benchmark, gridshells, performance assessment, stability, sustainability, buildability

1. Introduction

Gridshells are lightweight form-resistant structures given by the discretization of a continuous doubly curved surface into a grid of line-like structural members. Their design can be quite an intricate process due to the significant number of variables affecting the performance of the built structure. The design and optimization of gridshells is naturally turned into a multidisciplinary activity jointly carried out by experts in mathematics, computer graphics, mechanics, structural engineering and architecture. Schematically, when dealing with gridshells, three intricately linked design aspects bear significant importance, specifically pertaining to structural, construction, and sustainability domains. Regardless, gridshells are usually designed and optimized by separately referring to a specific performance. Even recently, a huge number of studies focus on structural performances only, e.g. [1, 2]. Others studies are specifically addressed to simplify the gridshell geometry, and its construction in turn, e.g. [3, 4]. To the Authors' best knowledge, no studies specifically address the sustainability issue in proper terms, while others [5] are simply intended to reduce the consumption of material rather than to consider the overall environmental impact. Conversely, only two recent studies authored by the same authoritative research team have attempted to simultaneously consider both structural and fabrication issues [6, 7]. The present study is intended to offer a new methodological framework for holistic performance assessment of gridshells in order to make a synthesis of the three design issues discussed above. The methodological framework is tested on three different single-layer gridshells. The performance assessment is carried out considering two different boundary conditions: gridshells fully hinged along their spring lines, and gridshells with a "free-edge", namely having their spring line partially not constrained. This study is carried out in the near wake of the newborn FreeGrid benchmark [8, 9]. In the FreeGrid perspective and for future reference, the outcomes of the study are intended to provide to the wide scientific and technical community an in-depth, solid and shared background knowledge about the three baseline gridshells adopted in the benchmark.

2. Methodological framework

The proposed framework is graphically schematized in Figure 1. For each conceptual part of the scheme, the nomenclature of the corresponding quantitative metric is given in rounded boxes.

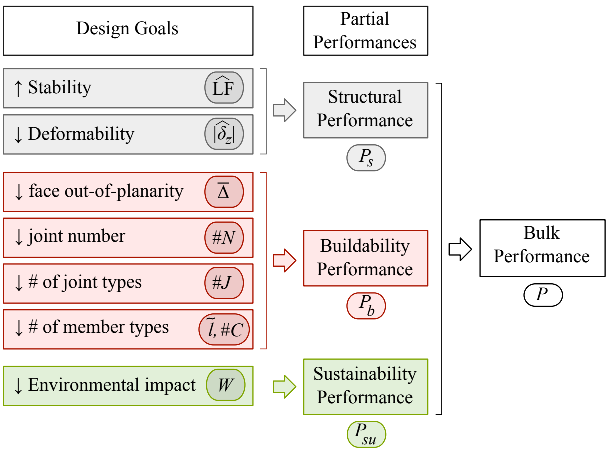

Figure 1: Scheme of the methodological framework: from Design Goals to Partial Performances and Bulk Performance. After [10].

The scheme retraces the usual design workflow, i.e. from setting the goals of the design to the synthetic performance assessment of the solution. The approach moves from 7 selected Design Goals (DG). For each of them, a quantitative Goal Metric (GM) to be increased or decreased is defined (↑ and ↓ in Fig. 1, respectively). Each GM is *normalized* with respect to a relevant gridshell *characteristic quantity* (e.g. the span length, the surface area, or a mesh measure) or to a design target (e.g. a limit value of the structural response), to allow the direct comparison among multiple solutions to a single or different design problems. In the following, the normalized GMs are generally marked by the superscript "$*$". DG and GMs can be ascribed to 3 categories, namely *structural*, *buildability* and *sustainability*. GMs belonging to the same category are combined in a Partial Performance (PP) metric. Finally, the 3 PP metrics are combined in turn in a single Bulk Performance (BP) one to concisely quantify the overall performance of the gridshell. For the sake of clarity, the framework is analytically described in reversed order in the following, that is from BP to GMs nested expressions. The BP metric P is expressed as a linear combination of the PP ones as:

$$P = \gamma_s P_s + \gamma_b P_b + \gamma_{su} P_{su}, \tag{1}$$

where P_s, P_b, P_{su} are the PP metrics related to structural response, buildability and sustainability, respectively, and γ_s, γ_b, γ_{su} are the corresponding dimensionless weighting factors. In the following, we set $\gamma_s = \gamma_b = \gamma_{su} = 1/3$. The adopted additive form (1) necessarily requires the PP metrics are homogeneous quantities. Conversely, structural, buildability and sustainability PP metrics and related GMs are arguably different, so that ensuring the same unit of measure for all of them is hardly viable. A *dimensionless form* of the PP or GM metrics allows fulfilling the requirement above and can be obtained by relating such values to the metrics of a *baseline solution*. In the following, these latter are marked by the subscript "0". In such a way, the PP and BP metrics express the relative performance change of a solution with respect to the initial one. In the next three subsections, each PP metric is expressed through the corresponding GMs.

2.1. Structural Performance

Deformability at SLS. At SLS, gridshells structural performances are driven by their deformability. The magnitude of the maximum vertical displacement over the whole gridshell $\hat{\delta}_{z,k}$ under the k-th Load Condition LC$_k$ is retained as the DG quantitative metric to be reduced. The DG metric is normalized with respect to the upper limit of deformability for gridshells $\hat{\delta}_{z,l}$, in formulas $\hat{\delta}^*_{z,k} = \hat{\delta}_{z,k}/\hat{\delta}_{z,l}$. In the present study, $\hat{\delta}_{z,l} = B/200$ is conventionally set.

Behaviour at ULS. At ULS, gridshells structural performances are affected by several modes of failure involving their stability and strength. In the present study, the critical Load Factor $\widehat{\text{LF}}$ is retained as the DG quantitative metric to be increased. It is classically defined as $\widehat{\text{LF}} = \mathcal{Q}_u/\mathcal{Q}$, where $\mathcal{Q}_u$ is the magnitude of the ultimate load and $\mathcal{Q}$ is the one of the design load defined for the generic Load Condition LC. In the proposed methodological framework we rigorously accounts for global, local and member instability, and cross-section plasticization, if any. The load factor is then analytically defined as

$$\widehat{\text{LF}} = \min\left(\text{LF}_I, \text{LF}_P\right), \tag{2}$$

i.e. as the minimum load multiplier among the LF$_I$ inducing the instability of the gridshell, and the LF$_P$ corresponding to the full plasticization of at least a single cross section of one structural member (Fig. 2a). In the following, each load multiplier is analytically defined.

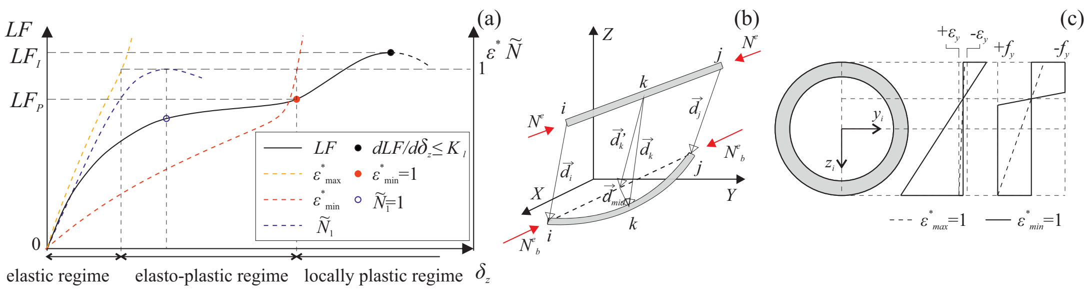

Figure 2: Structural performance: Conceptual graphical scheme exemplifying the variables used for establishing the critical load factor $\widehat{\text{LF}}$ as the minimum between LF$_I$ and LF$_P$ (a), the member buckling (b, after [11]), and the member cross-section yielding (c). After [10].

Gridshell instability is usually categorized according to the amount of structural joints involved in the collapse. "Global", "local" or "member" instabilities are conventionally referred to. Global instability involves the structure buckling as a whole, while local instability involves the snap-through of single or multiple adjacent joints, and member instability refers to the buckling of a single structural member [12]. Gridshell overall instability can be detected by the singularity of the global tangent stiffness matrix. Hence, the single Load Factor LF_I is defined by referring to the load-displacement curve (see Fig. 2a) as the condition

$$\mathrm{LF}_I \mid \frac{d\mathrm{LF}}{d\delta_z} \leq \mathcal{K}_l, \tag{3}$$

where the limit value $\mathcal{K}_l$ of the stiffness is ideally null, and numerically set as $\mathcal{K}_l = 0.02\mathcal{K}$, i.e. the 2% of the stiffness at origin.

Besides or jointly with instability issues, members may also be subjected to *cross-section yielding* [12], especially in the case of free-edge gridshells. The member cross-section may partially or totally plasticize depending on the distribution of cross-section internal stresses $\sigma(z)$ and strains $\varepsilon(z)$ (see Fig. 2c). In order to provide a quantitative observable of the degree of plasticization of the most stressed member in the gridshell, two new indices are proposed $\varepsilon^*_{\text{min}} = \max_{i=1:M}\left[\frac{\min_z \varepsilon(z)}{\varepsilon_y}\right]_i$ and $\varepsilon^*_{\text{max}} = \max_{i=1:M}\left[\frac{\max_z \varepsilon(z)}{\varepsilon_y}\right]_i$. where subscript i refers to a generic member, M is the total number of members, $\min_z \varepsilon(z)$ and $\max_z \varepsilon(z)$ stand for minimum and maximum value of the strain along the local axis z, and ε_y is the material yield strain. $\varepsilon^*_{\text{max}} = 1$ determines the plasticization of the first fiber in the most stressed beam, i.e. the occurrence of the gridshell elasto-plastic regime, while $\varepsilon^*_{\text{min}} = 1$ determines the full plasticization of the member cross-section, i.e. the occurrence of a plastic hinge and the onset of the gridshell plastic regime. The Load Factor LF_P inducing the onset of the first plastic hinge is defined by referring to the $\varepsilon^*_{\text{min}}$ - $\hat{\delta}_z$ curve and the $\varepsilon^*_{\text{min}} = 1$ point (see Fig. 2a) as the condition

$$\mathrm{LF}_P \mid \varepsilon^*_{\text{min}} = 1. \tag{4}$$

2.2. Buildability Performance

The buildability performance metric P_B is analytically defined as the arithmetic average of five GMs as

$$P_b = \frac{1}{\frac{1}{5}\left[\frac{1+\bar{\Delta}}{1+\bar{\Delta}_0} + \frac{\#(N^*)}{\#(N^*_0)} + \frac{\#(J^*)}{\#(J^*_0)} + \frac{1+\tilde{l}}{1+\tilde{l}_0} + \frac{\#(C^*)}{\#(C^*_0)}\right]}. \tag{5}$$

In the following, each proposed GM is defined and discussed.

Face out-of-planarity. Face planarity is a preferred requirement for double curved gridshells, as a sufficient even if not necessary condition to adopt flat cladding panels. In what follows, the out-of-planarity Δ_f of a single generic polygonal face is expressed in merely geometric terms as the average over the n_f incident vertices of the distance d_j between the j-th vertex and the best fitting plane π, normalized by the face half perimeter p_f [13, 14] $\Delta_f = \frac{\sum_{j=1}^{n_f} |d_j|/n_f}{0.5p_f}$. The global metric of the out-of-planarity $\bar{\Delta}$ is defined as average over the whole number of the gridshell faces F of the f-th face out-of-planarity metric Δ_f, or in formulas:

$$\bar{\Delta} = \frac{\sum_{f=1}^{F} \Delta_f}{F} \tag{6}$$

Joint number. The number of structural joints largely affects the overall cost and buildability. The adopted metric to be decreased is the cardinality of the ensemble of the structural joints normalized to the gridshell surface $\breve{S}$, i.e. in formulas $\#(N^*) = \#(N)/\breve{S}$.

Uniformity of structural joints. The DG is aimed at shortening the joints chart, but also increases the visual uniformity of the gridshell topology. The adopted metric to be decreased is the cardinality $\#(J)$ of the ensemble of the joint types, or in geometrical terms the number of clusters of congruent vertices, normalized to the overall number of joints, i.e. in formulas $\#(J^*) = \#(J)/N$. Two vertices are congruent if they have the same valence v (the number of edges incident to a vertex) and similar shape. The shape similarity algorithm to define clusters of congruent joints follows from [4].

Uniformity of structural members. The DG is aimed at shortening the members chart, but also affects the visual homogeneity of the gridshell structural components. In what follows the member type results from its length and cross section. The corresponding metrics to be decreased are the coefficient of variation of member lengths $\tilde{l}$ [15], and the cardinality of the ensemble of the member cross-sections $\#(C)$ normalized to the overall number of structural members M, i.e. in formulas $\#(C^*) = \#(C)/M$.

2.3. Sustainability Performance

The sustainability performance metric P_{su} compactly reads as

$$P_{su} = \frac{1}{W^*/W_0^*}. \tag{7}$$

where W^* is the adopted quantitative GM to be decreased, i.e. the equivalent weight normalized by the gridshell surface $\breve{S}$, analytically expressed as $W^* = W/\breve{S} = \sum_{i=1}^{M} g_i l_i \alpha_i / \breve{S}$, where the summation over the M structural members includes for the i-th of them the material weight per unit length g_i, the length l_i, and the new "environmental impact correction coefficient" α_i. They are extrapolated over the range of products in order to have a homogeneous and consistent approach, and normalized with respect to hollow sections made of S355. The resulting linear fitting law of α takes the form $\alpha = a + 0.0002\ f_y$ where f_y is expressed in [MPa], and a=0.475, 0.641, 0.792, 0.939 for I/H/C/L sections, round bars and rods, plates and flats, hollow and welded sections, respectively (Fig. 3).

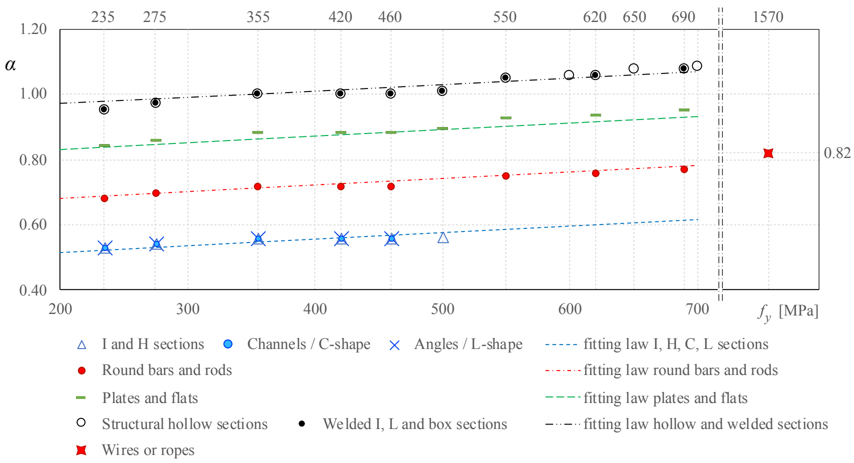

Figure 3: Environmental impact correction coefficient α versus steel grade and cross section type.After [10].

3. Description of the case studies

The methodological framework outlined in the previous section is tested on three case studies, which are the gridshells adopted by the FreeGrid benchmark (Fig. 4). Specifically, three types of single-layer

Table 1: Specific geometrical features of each baseline design solution

	Barrel vault	Parabolic dome	Hyperbolic paraboloid
Directrix equation	$z = f$	$z = -\dfrac{y^2}{2B} + f$	$z = \dfrac{y^2}{2B} + f$
h	$B/8$	$B/8$	$B/4$
L	A	$2/3\pi B$	$3/2B$
L^*	L	$L/2$	L
S	BL	$\pi B^2/4$	$B^2/2$

gridshell geometries are considered: i. a barrel vault, with simple curvature; ii. a parabolic dome, with double Gaussian positive curvature; iii. a hyperbolic paraboloid, with double Gaussian negative curvature. For each case study, the performance assessment is discussed considering two different boundary conditions: (i) the gridshells with their spring line partially not constrained along what is called a "free-edge" (referred to as "Design Baseline Gridshells" in the FreeGrid nomenclature) are retained as design baseline solutions, and their GM/PP metrics labelled by the subscript "0"; (ii) the corresponding structures fully hinged along their spring lines (in the following "fully-constrained", "Background Gridshells" in the FreeGrid nomenclature) are adopted herein as dummy design solutions whose relative performances are to be evaluated with respect to the free-edge counterparts.

3.1. Geometrical setups

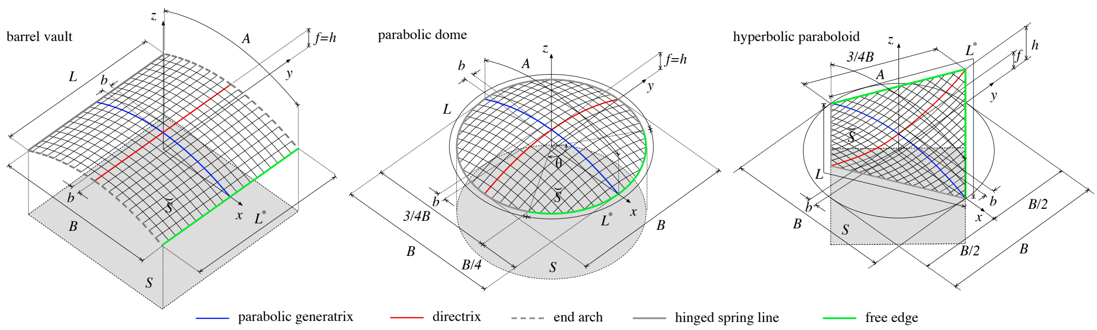

Figure 4: Baseline design solutions: barrel vault, parabolic dome and hyperbolic paraboloid. After [10].

The main features of the gridshell geometries are illustrated in Figure 4. All the gridshells share the same parabolic generatrix, described by the equation $z = -\dfrac{x^2}{2B} + f$, in $\left\{-\dfrac{B}{2} \leq x \leq \dfrac{B}{2}, y = 0\right\}$. For all gridshells, B = 30 m is the span length, $f = B/8$ the rise and its arc length A is $A = B\left[\frac{\sqrt{5}}{4} + \ln\left(\frac{1+\sqrt{5}}{2}\right)\right] \approx 1.04B$. Conversely, the three gridshells differ in their directrix, whose equations are defined in the domain $\{-B/2 \leq y \leq \ B/2, x = 0\}$, and summarized in Table 1 together with other characteristic dimensions (refer to Fig. 4 for nomenclature): h is the maximum height above the horizontal reference plane, i.e., the horizontal plane $z = 0$; L and L^* are the lengths of the continuous spring line and free edge, respectively; the surface extent S corresponds to the area encircled by the projection of the continuous spring line, free edge and end arches, if any, on the horizontal reference plane. Both directrix and generatrix are divided into 20 edges of constant length $b = A/20 \approx 1.56$ m. The three gridshells share the same kind of homogeneous grid made by $b \times b$ planar quadrangular square faces, except for the faces along the spring line, where the grid intersects the boundary.

3.2. Structural setups

3.2.1. Structural members

The structural members of all gridshells are made of steel with a bilinear elastic-perfect plastic constitutive law. Specifically, steel S355 is adopted, with density $\rho = 7850$ kg/m^3, Young's Modulus $E = 210000$ MPa, Poisson's ratio $\nu = 0.3$ and yield strength $f_y = 355$ MPa. All structural members of each baseline geometry have the same circular hollow section. In particular, the external radius and thickness are respectively equal to 139.7 mm and 14.2 mm for the barrel vault, while they are respectively equal to 101.6 mm and 10 mm for the parabolic dome and hyperbolic paraboloid.

3.2.2. External and internal constraints

External constraints at the structural joints along the spring lines L are perfect hinges, except for the head arches of the barrel vault. The latter are allowed to move in y direction in order to avoid non-linear stiffening induced by the y-wise members. All the internal structural joints are rigid. The structural joints along the free-edge length L^* in free-edge gridshells are not constrained.

3.2.3. Load conditions

The considered Load Conditions are ideal and simplified. Their moduli represent standardized design loads on gridshells. The first symmetric Load condition LC$_1$ includes the self-weight of structural members and point loads $Q_{1,j} = q_1 s_j$ applied at all structural joints, where s_j is the projection on the horizontal reference plane of the tributary area of the j-th joint. The distributed load q_1 accounts for both the weight of glass glazing and the snow load. The second asymmetric Load condition LC$_2$ includes the self-weight of structural members and point loads $Q_{2,1,j} = q_1 s_j$ and $Q_{2,2,j} = q_2 s_j$ applied according to the following rule: (i) $Q_{2,1,j}$ is applied on the joints with $x > 0$ for barrel vault and parabolic dome, and with $y > 0$ for hyperbolic paraboloid; (ii) $Q_{2,2,j}$ is applied on the joints with $x < 0$ for barrel vault and parabolic dome, and with $y < 0$ for hyperbolic paraboloid. The distributed load q_2 accounts for the weight of glass glazing only. Structural performance at ULS are evaluated by setting $q_1 = 1800$ N/m^2 and $q_2 = 600$ N/m^2, while SLS performances are assessed with $q_1 = 1200$ N/m^2 and $q_2 = 400$ N/m^2.

4. Results

Figure 5 summarizes the structural Goal Metrics for all the gridshells. The following synthetic comments can be outlined: (i) all free-edge gridshells do not satisfy performance levels at both SLS ($|\hat{\delta}_z|/\hat{\delta}_{z,l} > 1$) and ULS ($\widehat{\mathrm{LF}} < 1$), while fully-constrained gridshells do; (ii) fully-constrained gridshells dramatically increase the structural performances of all geometries at both ULS and SLS: the critical load factors of the fully-constrained gridshells are from 2 to 20 times higher than the ones of the corresponding free-edge ones; the maximum vertical displacements of the fully-constrained gridshells are from 13 to 440 times lower than the ones of the corresponding free-edge ones. In particular, the hyperbolic paraboloid is the gridshell with the highest difference in terms of $\widehat{\mathrm{LF}}$ under both load conditions, while the barrel vault is the most sensitive in terms of $|\hat{\delta}_z|$, namely under uniform LC$_1$; (iii) $\widehat{\mathrm{LF}}$ approximately takes the same value for all free-edge gridshells; (iv) conversely, the free-edge gridshells perform differently at SLS, in the light of their geometrical (i.e., simple or double curvature) and mechanical (i.e., structural members in tension or compression) specific features; (v) the structural performances of both free-edge and fully-constrained gridshells at both SLS and ULS are analogous under the two different load conditions. The only exception is the barrel vault fully-constrained case, where the asymmetric LC$_2$ induces more critical serviceability and ultimate behavior. Such a difference does not hold for the corresponding free-edge barrel vault, where the free-edge effects largely prevail over the ones of the load condition.
According to the features of the adopted case studies, both buildability and sustainability Goal Metrics

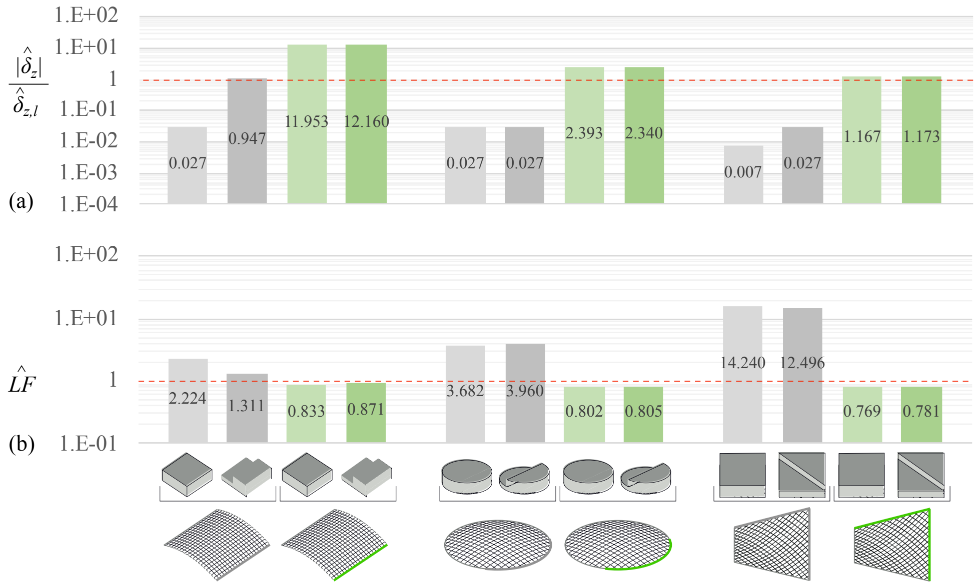

Figure 5: Structural Goal Metrics: Dimensionless vertical displacement (a) and critical Load Factor (b). After [10].

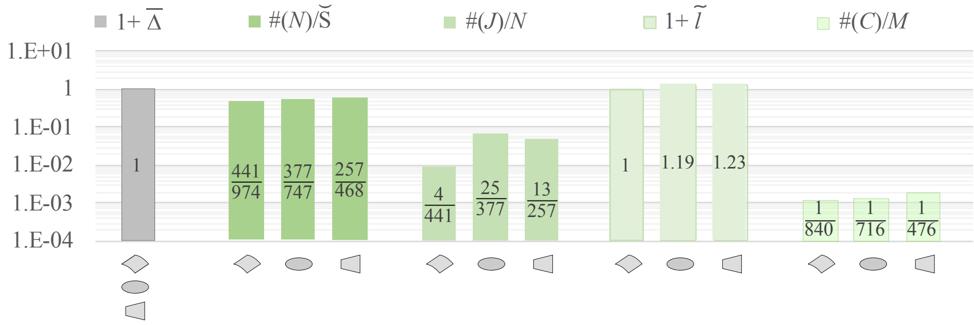

Figure 6: Buildability Goal Metrics: normalized values of out-of-planarity of faces $\bar{\Delta}$, joint number $\#(N)$, cardinality of the ensembles of joint types $\#(J)$, coefficient of variation of members length $\tilde{l}$, and cardinality of the ensembles of members cross sections types $\#(C)$. After [10].

have the same values for free-edge and fully-constrained gridshells. Figure 6 summarizes the buildability Goal Metrics for all the gridshells. The following synthetic comments can be outlined: (i) out-of-planarity of the gridshell faces is nil since all quadrangular square faces are planar, resulting from discrete translational surfaces, i.e., obtained from the translation of a discrete parabolic generatrix over a discrete directrix. Both the generatrix and the directrix are homogeneously decomposed into all-equal segments; (ii) the barrel vault contains the largest cardinality of structural joints while the hyperbolic paraboloid contains the lowest one. This is mainly related to the gridshell surface $\breve{S}$ of each gridshell that is maximum for the barrel vault and minimum for the hyperbolic paraboloid. However, surface density of the joints is not constant, despite the mesh uniformity. It ranges within $\#(N^{*}) \in [0.45, 0.55]$, with the hyperbolic paraboloid scoring the highest value, because of the high surface density of joints resulting from the intersection of the gridshell internal members with the members along the spring line; (iii) uniformity in structural joints varies sensibly among the three gridshells, even if they all share the same positive x-wise curvature and the parabolic dome and hyperbolic paraboloid also share the same

absolute value of the y-wise curvature. Non-uniformity in structural joints rises because of the intersection of internal members with members arranged along the spring line; (iv) uniformity in members length slightly varies among the three gridshells, despite their uniform quadrangular square faces. The barrel vault is perfectly uniform, having all members with the same length $l = b$; (b) non-uniformity in member cross-sections is similar for the barrel vault and parabolic dome, since they have similar number of structural members. Conversely, it almost doubles for the hyperbolic paraboloid since the number of members M is almost halved.
In this case study, sustainability performance is directly related to the grid density and the gridshell total weight, since the steel grade and the type of member cross-section are the same among the three design solutions. On the one hand, gridshell weight is directly proportional to the cross-section area of the structural members and the gridshell surface. On the other hand, grid density is almost constant among the three gridshells and slightly increases approaching the boundaries for the parabolic dome and hyperbolic paraboloid. The barrel vault scores the highest equivalent weight $W = 57556$ kg among the three solutions and about equal to two times the value of the parabolic dome ($W = 23564$ kg) and more than three times the value of the hyperbolic paraboloid ($W = 15589$ kg). This is because of the double cross-section area of its members with respect to the other design solutions.
Finally, the Bulk Performance metrics of the fully constrained gridshells are evaluated with respect to the baseline free-edge ones. BP metrics are directly related to structural Partial Performance ones, since buildability and sustainability ones result equal to unit. The fully-constrained hyperbolic paraboloid scores the highest BP, i.e. $P = 658$. This is induced by (i) the extent of the free edge, i.e. the largest one among the three geometries together with the barrel vault, and (ii) the inborn high structural performance of the hyperbolic paraboloid geometry, taking advantage of arches under compression and rods under tensions arranged along the two principal directions. Conversely, the parabolic dome scores the lowest BP, i.e. $P = 141$, followed by the barrel vault, $P = 203$. On the one side, the extent of the free edge of the parabolic dome is the shortest among the baseline design solutions, and its restraining does not dramatically change its overall structural performance. On the other side, the double curvature allow the free-edge parabolic dome to perform significantly better than the free-edge barrel vault at SLS.

5. Conclusion

This study provides a detailed insight on a methodological framework for the holistic performance assessment of steel, single-layer gridshells. The performance assessment is intended to: i. be conceptual design oriented and supportive by referring to design goals; ii. analytically consider multiple design goals, each expressed by a quantitative goal metric; iii. secure the direct comparison in absolute terms of each goal metric among multiple design solutions to different design problems thanks to their suitable normalization; iv. consider three different types of performances, namely the structural, buildability and sustainability ones; v. make synthesis of them in a bulk performance metric by making the partial ones homogeneous and dimensionless; vi. allow the comparison of the bulk performance of several design solutions even if all referred to a single design problem by necessarily making reference to a guessed baseline design solution.

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