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A grammar-based framework for strut-and-tie modelling of reinforced concrete structures

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Abstract

Strut-and-tie models offer a simplified design approach for reinforced concrete structures such as walls or beams and are particularly suitable for static or geometrical discontinuities. They guarantee designs that are safe based on the lower bound theorem of the theory of plasticity. Currently, their manual generation demands significant time and expertise to navigate the solution space for various configurations with different objectives in mind. Automating the generation of strut-and-tie models has faced several challenges, with previous methods such as discrete layout optimisation or topology optimisation struggling to consider either (i) user adjustments, such as changes of nodal coordinates, or (ii) practical aspects of fabrication and constructability.

In response, this work presents a novel grammar-based generative framework that imposes strict and constraining rules, tailored to strut-and-tie models. Unlike previous work, our framework incorporates engineering judgement directly into its rule set, thereby significantly reducing the design space. Furthermore, the sequential application of rules allows for user intervention and thus human-computer interaction in the sense of a design co-pilot. We demonstrate the effectiveness of this framework through its application to two use cases: a cantilevered beam with a point load at its end and a dapped-end beam with an opening and two acting loads. The truss structure is represented as a graph and the rules are applied akin to graph grammar. Compared to optimisation-based methods, the developed models are practical, consider the preference towards orthogonal and distributed reinforcement and are typically preferred by professional structural engineers. This marks a first step towards an AI-assisted, grammar-based generative design approach for strut-and-tie models. The framework offers interpretability that closely mirrors the intuitive decision-making process employed by human engineers in the selection of suitable strut-and-tie models.

Keywords: Reinforced concrete, Membrane structures, Strut-and-tie models, Truss structures, Shape grammars, Generative design

1. Introduction

Strut-and-tie models (STMs) assist structural engineers in the design of reinforced concrete structures, alleviating the dependence on intricate computational models. In addition to being intuitive, they can additionally account for discontinuity regions due to abrupt changes in loads or geometry [1]. They can serve as simple and interpretable hand-calculations to validate numerical models and thereby support the avoidance of design errors. Such modelling errors, if not detected, may result in structural safety concerns, construction delays and increased construction cost. STMs provide safe designs based on the lower bound theorem of the theory of plasticity, as long as they satisfy equilibrium and static boundary conditions, do not infringe the yield condition and fulfil the requirements of the theory of plasticity, such as sufficient ductility by adding minimum reinforcement [2]. Any STM satisfying these conditions provides an *admissible* design ensuring structural safety. As the design space can be explored targeting different objectives, there exists an infinite number of admissible solutions for each structural problem. Not all admissible designs are (equally) *suitable*, as the selected STM significantly influences both the quality (e.g. constructability, behaviour under service loads) and efficiency (e.g. required amount of steel reinforcement) of the design. The search for a suitable STM – essentially a highly constrained truss – is, thus, an iterative multi-objective process, which depends heavily on engineering judgement and expertise.

Classical stress fields methods consider the structure as a continuum and not as discrete entities as in the case of STMs, which typically leads to more refined results including the detailing of nodal zones [3]. Using numerical methods, finite-element analyses based on stress fields have been proposed, such as elasto-plastic stress fields [4] or the compatible stress field method [5], enabling more realistic analyses. Nevertheless, these are more challenging to interpret, susceptible to modelling errors and require the prior definition of the exact locations of the reinforcing steel bars, which is often unknown a priori.

With these challenges in mind, an assisted STM generation tool could save considerable time for structural engineers, while ensuring theoretical soundness and required reliability. Current approaches are typically formulated in terms of structural optimisation [6], initially applied with the motivation of reducing material costs or self-weight (resources) and recently regained interest due to additional considerations on environmental impact. Structural optimisation methods can be classified into discrete layout or continuous topology optimisation, or are a hybrid thereof. In discrete layout optimisation a predefined ground structure, similar to a fully connected graph with nodes in a grid pattern, is optimised under certain structural (such as stress) and geometrical constraints [7]. In turn, topology optimisation redistributes material inside a two- or three-dimensional continuum to find the minimum self-weight or volume under certain constraints [8]. Both methods are generalisable to various structural problems, particularly flexible in geometry. However, all share similar shortcomings: Discrete optimisation cannot easily account for continuous boundary conditions or user adjustments, such as modifying nodal coordinates, since form and force are coupled. Despite the advantageous feature of topology optimisation in admitting high geometrical degrees of freedom, for the case of STMs, they cannot easily accommodate aspects of fabrication and constructability, such as the preference towards orthogonal reinforcement or a minimum length of the ties. Furthermore, it does not directly lead to an axial-force only truss structure as required to design the reinforcement, albeit some recent proposals have been put forth on automated truss generation derived from topology optimisation results [9]. Lastly, both methods do not cover minimum reinforcement, despite the requirement for the safe application of the theory of plasticity and the enforcement in current design standards of engineering practice, such as the Swiss design codes (SIA) [10] and Eurocode 2 (EN 1992) [11].

To address the limitations of current methods, we propose a novel grammar-based generative framework for strut-and-tie modelling. Compared to previous rules-based approaches [12, 13, 14] for generative design, the grammar and rules suggested in our framework are tailored to the problem of STMs and are, thus, more stringent and restrictive. In what follows we elaborate on the following points:

(i) the concept of grammar for strut-and-tie modelling;

(ii) how rules can be applied to generate STMs;

(iii) metrics to assess the performance of STMs; and

(iv) two case studies of STM design demonstrating the parsing into sequential rule applications, highlighting the systematic nature of the design process.

2. Methodology

2.1. Background of grammars for design

Stiny and Gips [15] introduced *shape grammars* as a generative design algorithm by applying the concept of universal grammars to design, inspired by the Chomsky hierarchy [16] of *formal grammars* [17]. Shape grammars define transformation rules which generate geometrical shapes. A rule is made of a left-hand side (LHS) and a right-hand side (RHS), where the RHS shape results from spatially transforming the LHS shape. Notably, Shea and Cagan have coupled shape grammars with a heuristic approach, i.e. simulated annealing, for structural design to optimise trusses, also known as shape annealing [18]. More recently, Lee et al. [12] combined graphic statics with grammars, on the basis of which rules- or policy-based approaches have been applied to reticulated equilibrium shell structures for design exploration [13] and even coupled with machine learning algorithms and neural network architectures, for instance, reinforcement learning and graph neural networks [14].

The generative design process of grammatical itemisation has been widely explored in the discipline of architecture, but not yet for structural engineering applications, particularly for the case of strut-and-tie modelling of reinforced concrete structures. Grammar rules allow to implicitly account for constraints, while the suitability of a suggested STM design should subsequently be further evaluated both based on performance metrics and the structural engineer's expertise. As STMs are highly constrained trusses, it is both challenging to define these constraints mathematically and, thus, also to implement them in conventional optimisation schemes.

2.2. Representation and strut-and-tie model grammar

STMs can be represented as a graph by directly taking the nodes of the STM as nodes of the graph and the bars as edges, see Figure 1. Hence, the connectivities of a truss and of its corresponding graph are the same. The geometry, material properties and boundary conditions such as forces or supports are either incorporated as edge attributes (geometry and material properties) or as node attributes (boundary conditions).

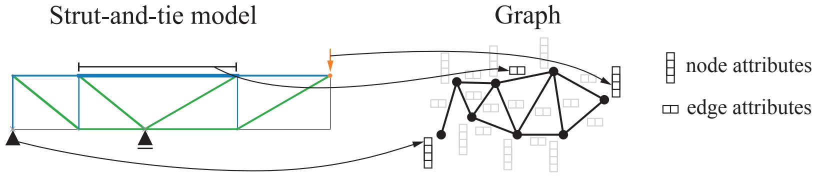


Figure 1: Representation of a STM as a graph with node and edge attributes.

Figure 2 shows the framework and concept of grammar for STMs, where the rules are applied sequentially and transformatively. That means that an initial truss state, represented by a graph, is derived from the given structural problem and transformed through the iterative application of rules. Each rule has a LHS and a RHS, where the LHS needs to be matched to a subgraph of the current truss using graph matching algorithms, in order to be deemed as applicable. Once a subgraph is found, it can be replaced with the RHS that is suggested by the applicable rule. From all applicable combinations of rules and subgraphs, one is selected (by engineering judgement) and modifies the truss with the goal of first attain-

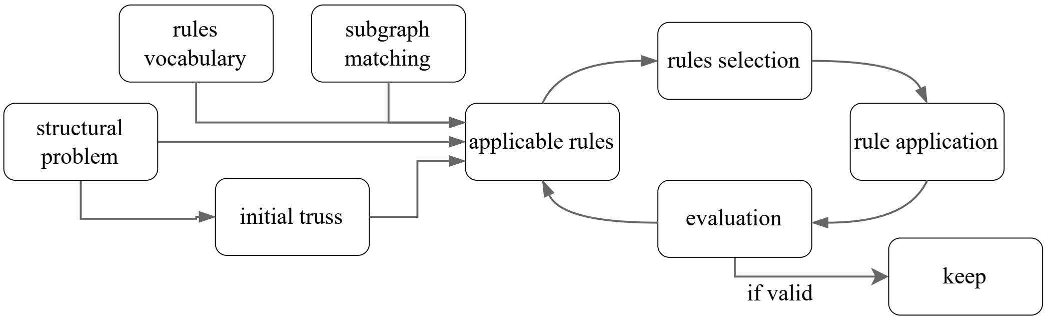


Figure 2: Concept of grammar for STMs.

ing a valid design and then of improving its suitability. The new truss is then assessed using established performance metrics and kept in a pool of STMs, if deemed valid.

Table 1: Basic rules of graph grammar for STMs.

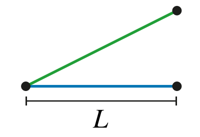
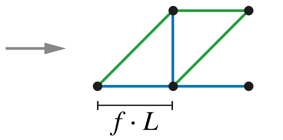
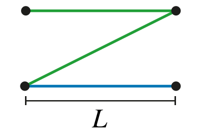
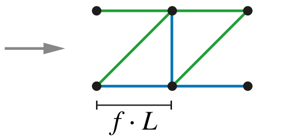
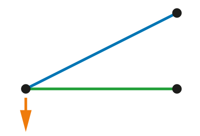
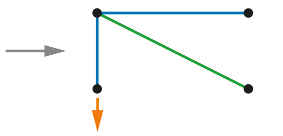
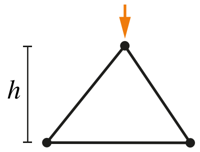
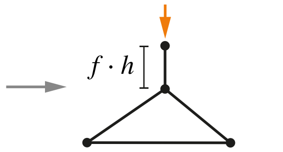
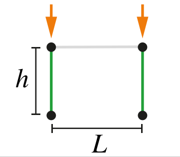
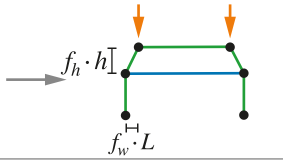
Basic rules	LHS	→	RHS
A: add node			•
B: add edge	• •		•—•
C: remove node	•		
D: remove edge	•—•		• •

As a starting point, four basic rules of the graph grammar are defined: adding and removing a node or an edge, see Table 1. Furthermore, edge connections can be easily modified by changing the indices of connected nodes. More complex and restrictive rules are established as sequences of these basic rules. As these rules preserve equilibrium, just as in rules-based graphic statics, form and forces are coupled. This set of rules can be extensive, and Table 2 shows exemplary five such rules which are also used in the case studies of this paper. Adopting the convention of Marti et al. [2], green is used for struts (compressive force), blue for ties (tensile force) and black if the force can reflect both tension and compression. Following engineering judgement, Rules 1 and 2 can be applied if the strut angle (angle between strut and tie) is too small and transverse reinforcement, usually stirrups, should be placed to avoid shear failure; note that design codes typically recommend a minimum strut angle of 25°or 30°, see e.g. [10]. The rule essentially adds two nodes, modifies two edges and adds four. Per default, the parameter f is set to 0.5 but other values can also be chosen. Rule 3 suspends the applied load and therefore incorporates the preference towards orthogonal over inclined reinforcement. The parameterised Rule 4 can be employed to both struts and ties. Lastly, Rule 5 is more complex, since it simultaneously modifies the nodal locations and spreads the force, which is pertinent if there exists a geometrical obstacle to circumvent. These rules constrain the design space of admissible solutions (see Section 1.) to an infinite subset, which can be refined by introducing more rules.

2.3. Performance metrics to assess validity and suitability of strut-and-tie models

The navigation of the design space by the grammar usually requires an interaction with the engineer to evaluate and guide the design [17]. Yet, with increasing numbers of rules and iterations the design space grows exponentially and it can be challenging to steer the exploration. As this grammar is ambiguous, which means that different parsing trees, i.e. sequences of rules, can lead to the same truss, it is not trivial to apply graph hashing or other approaches needed for Bayesian optimisation or heuristic search methods.

Table 2: Examples of more complex rules for STMs.

	Parameters	LHS	RHS
Rule 1	f		
Rule 2	f		
Rule 3			
Rule 4	f		
Rule 5	f_h, f_w		

In current approaches (as presented in Section 2.1.), this is alleviated through optimising a specific single objective, usually minimising the required reinforcing steel volume. However, as discussed in Section 1., this is not a sufficient criterion for a satisfactorily suitable STM. Hence, we introduce performance metrics which either determine the validity of a STM (mandatory criteria such as geometrical conditions and design code requirements), or quantify its suitability.

Assuming that the STM is admissible (see Section 1.), the following criteria are mandatory for a valid (but not necessarily suitable) STM (linearly interpolated between 0 = fulfilled, 1 = not fulfilled):

G: The STM should stay inside the geometry of the reinforced concrete continuum. This criterion applies strictly to the final configuration, but may be violated during intermediate steps.

A: The strut angle (between a strut and a tie) shall be above the minimum required angle (typically either 25 or 30°, [10]. Here, 25° is chosen).

CE: Only ties are allowed to cross other edges without an intersecting node.

Most optimisation-based methods account for the geometry performance metric **G** by only allowing for connections inside the continuum during the optimisation process. In this grammar-based approach, however, geometrical infringements will be tolerated for intermediate steps.

For assessing performance, i.e. quantifying suitability, and potentially serving as objectives in optimisation methods, the following metrics are established:

C: Constructability: Takes into account fabrication aspects by penalising inclined reinforcement.

S: Steel consumption: Accounts for steel reinforcement quantity and minimum reinforcement of 0.2% (and here for B500B steel).

3. Case studies

In order to showcase the approach, this section presents the parsing of STMs into sequences of rules for two exemplary STM design problems common in engineering practice: a cantilevered beam and a dapped-end beam.

3.1. Cantilevered beam

Figure 3a shows the structural problem of a cantilevered beam with a concentrated load of 1 MN applied at the cantilevered end. The starting graph, respectively the STM, is simply initialised by connecting the three prescribed nodes: one force and two support nodes (see Figure 3b). Subsequently, since the left subgraph can be matched with the LHS of Rule 3, Rule 3 substitutes this subgraph, thereby avoiding inclined reinforcement. Rule 1 is then employed twice until the STM in Step 3 is obtained. At this iteration, the STM is valid according to the performance metrics established in Section 2.3. The steel consumption metric **S** has also decreased from 8.22 at initialisation (Step 0) to 3.04. However, deriving the associated stress fields of this STM uses only concentrated struts and ties (see left side of Figure 3c). Thus, no distributed minimum reinforcement can be activated and concrete stresses are modelled as concentrated deviating significantly from the real material behaviour. In typical automated STM generation approaches, stress fields are ignored as it is not straightforward to formulate them as constraints and, hence, incorporate them into the mathematical optimisation scheme. For illustration, Figure 3d shows a truss generated using an exemplary discrete layout optimisation method, namely the adaptive layout optimisation for trusses as implemented in [19] (which is not the most recent development in

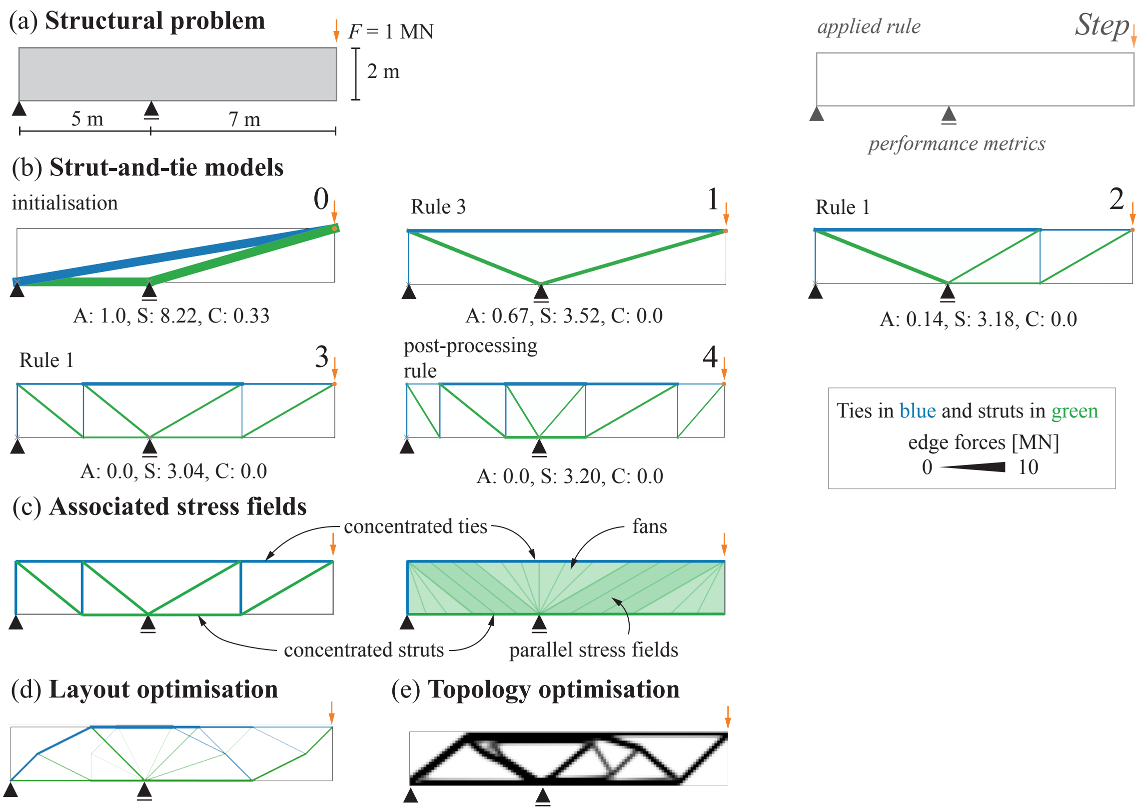


Figure 3: Cantilevered beam: (a) structural problem, (b) STMs illustrating the iterative applications of rules, (c) the associated stress fields for Steps 3 and 4, (d) the results from an adaptive layout optimisation method and (e) from a topology optimisation approach (SIMP).

the field but used here for simplicity). For comparison, the results of a common topology optimisation method – the SIMP (Solid Isotropic Material with Penalisation) approach [8, 20] – are also shown in Figure 3e. Yet, this method does not directly lead to an axial-force only truss. Note that both exemplarily illustrated methods neither consider fabrication and constructability aspects nor account for distributed reinforcement and concrete stresses. The latter can be accounted for in our method via consideration of associated stress fields by applying a post-processing rule, which accounts for concentrated boundary conditions such as point loads or supports as fans, to a pertinent STM. Differing from the local Rules 1 and 2, this rule essentially detects the locations of concentrated boundary conditions, checks whether the STM can be modified to consider fans for the introduction of concentrated forces, and if so it adjusts the nodal locations and strut angles accordingly. In this example, the corresponding stress fields consist of both fans and parallel stress fields, considering the structure as a continuum. This leads to a more realistic representation of the material behaviour of a reinforced concrete beam and a design commonly preferred by practising structural engineers. Due to the consideration of fans and additions of vertical ties, the steel consumption metric increases by 5%, which may appear unfavourable. However, now the transverse reinforcement does not need to be placed as concentrated reinforcement as in Step 3, but can be distributed, such that it is (partly) covered by the minimum reinforcement. This requirement would have to be fulfilled in the STM of Step 3 as well, in order to comply with design codes and to ensure robustness. As the minimum reinforcement can be more meaningfully exploited, the resulting design is thus more efficient, which is often ignored in STMs.

3.2. Dapped-end beam

The second case study presents the application of the STM grammar to a dapped-end beam with two point loads and an opening in Figure 4, a more complex example than the first one; note that no other methods are illustrated here for conciseness. The STM is initialised with connections between each point load and the boundary conditions. However, the edge connecting the point loads (in grey) experiences no force until Step 4. Rules 3 and 4 presented in Table 2 can then be applied to the initial truss. Rule 5 transforms the STM of Step 3, resulting in circumventing the opening. Subsequently, the related Rules 1 and 2 can be used to ensure validity by avoiding low strut angles. Finally, the post-processing rule can be applied to consider both parallel stress fields and fans, see for comparison Figure 4c, similar to the previous example in Section 3.1. Again, the metric for steel consumption **S** increases from before to after the application of the post-processing rule. If this particular sequence of rules is selected to generate a STM, the objective or performance metrics do not decrease monotonically during the iterative application of said rules.

4. Conclusion and outlook

Common methods for automated STM generation, such as discrete layout optimisation or topology optimisation, fail to account for user adaptions and human-computer-interaction, fabrication and constructability aspects and the association of STMs with stress fields. As an alternative, we present a novel grammar-based generative framework, which intrinsically includes the restrictions of highly constrained trusses, such as STMs, into the definition of the grammars and views the restrictions as prior knowledge. Therefore, the theoretically infinite number of admissible trusses is vastly reduced.

Two case studies show the successful parsing of trusses into sequences of rule applications. Employing an additional post-processing rule takes into account the correspondence between stress fields and STMs for fans and parallel stress fields. For the first example of a cantilevered beam, the generated STM is compared with results of common optimisation-based methods. Their stark differences show that single-objective optimisation with the common goal of minimising the steel volume does not lead to practical

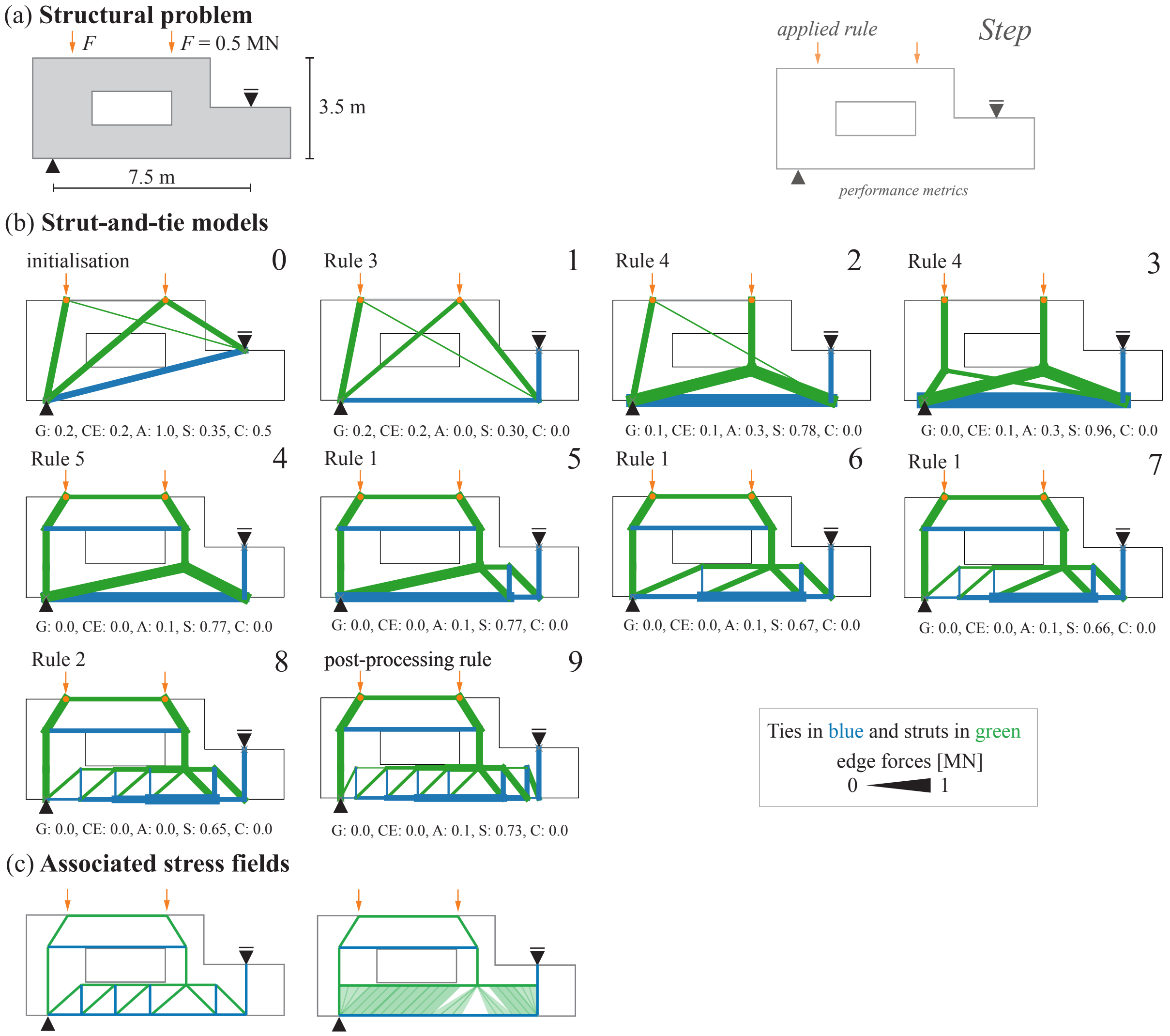


Figure 4: Dapped-end beam: (a) structural problem, (b) STMs with iterative rule applications, and (c) the associated stress fields for Steps 8 and 9.

STMs which can be fabricated. In comparison, this grammar-based framework leads to more practice-usable STMs but also illustrates that the automated generation of STMs is not simply an optimisation but rather a guided exploration problem.

With this work we introduce the first steps towards an AI-assisted grammar-based generative framework for STM generation. The framework at its current stage lacks the consideration of continuous boundary conditions, which could easily be included via specific rules and an automated navigation scheme. Both the choice of the values for parameterised rules and the selection of the next rule depends currently on human and computer interactions, although the framework suggests applicable rules for related subgraphs. This selection process may become overwhelming for engineers, creating the need for an assisted search process, where employing AI methods such as reinforcement learning, capable of minimising a reward function, could be beneficial. For instance, a guided navigation could propose the next iterations of rules applications to the user but at any iteration the user could intercept, modify the parameters or change the selected rules.

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