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Stiff and soft deformation of a 3D-printable tensegrity-inspired metamaterial based on expanded octahedron

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Abstract

This paper presents a computational and experimental study on tensegrity-inspired lattices based on expanded octahedron modules. First, a continuum analysis of tensegrity modules is shown, and soft and stiff deformation modes are discussed. Then, the results of laboratory tests performed on 3D-printed tensegrity-inspired lattices are presented. Two configurations of the module are considered: A – basic module, B – rotated module, and two configurations of the eight-module lattices are subjected to experimental testing: basic supercell (based on A module), modified supercell (based on B module). Laboratory tests confirm the results of computational analysis, which is a stiff behaviour of the basic lattice (A) and a soft behaviour of the modified lattice (B) under vertical compression load.

Keywords: tensegrity, spatial lattices, continuum model, additive manufacturing, experimental testing.

1. Introduction

Recently, a fast development of innovative systems for engineering applications has been observed. Such systems include: artificial materials with untypical behaviour, adjustable structures with controllable mechanical properties, smart materials and structures. This study explores the idea of one of such novel structural systems, namely tensegrity-inspired lattices in various scales (including cellular metamaterials) with extremal mechanical properties. Tensegrity structures [1] have many advantageous features: they have a high stiffness-to-mass ratio, it is possible to control in an internal way their mechanical behaviour, they can be used in smart systems, and it is easy to shape their geometry.

In this work, a computational and experimental research on 3D-printed tensegrity-inspired lattice based on expanded octahedron cell is presented. The main aim of this study is the search for module layout configurations, which ensure soft or stiff elastic properties [2]. It is limited to free-standing configurations adaptable for experimental tests. Proposed systems have many possible application areas, which depend on the considered scale. Metamaterials are usually applied in various vibration and shock energy damping systems, including seismic protection systems, resilient mats, etc. Medium- and macro-scale lattices could be used as lightweight structural elements with enhanced mechanical properties, as 3D fillings of hollow structural elements with high energy absorption capability, or as reinforcing systems with controllable mechanical properties aimed at applications in old buildings.

The first step of the research was the analysis of expanded octahedron cells (single modules) and supercells (sets of eight modules) using a continuum model [3], and the identification of soft and stiff deformation modes corresponding to compression or shear, following [2]. In the second step, the identified extremal mechanical properties were verified in the laboratory on 3D-printed samples of tensegrity-inspired cells and supercells. The specimens were prepared by the authors using a 3D printer based on SLA (Stereolithography) technique [4]. As a result of the research, two configurations of supercells were indicated, showing stiff and soft mechanical behaviour under vertical load.

2. Extremal mechanical properties of expanded octahedron

The basic configuration (A) of expanded octahedron module is presented in Figure 1a. It is possible to obtain other configurations by rotating the basic module around axis x, y, z by angles α, β, γ. One of possible free-standing configuration (B) is presented in Figure 1b. Configurations A and B can be used to create supercells (e.g. eight-module lattices presented in Figures 1c and 1d).

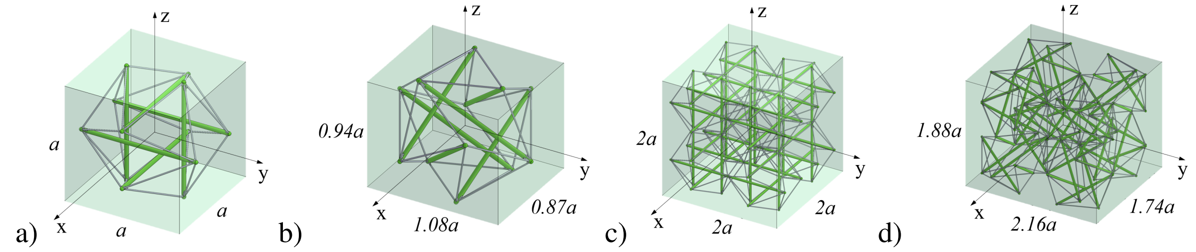


Figure 1: Expanded octahedron inscribed into a cube: a) basic module A; b) rotated module B; c) basic supercell (based on configuration A); d) modified supercell (based on configuration B).

Mechanical properties of the modules and supercells can be described using a continuum model, which leads to defining the elasticity tensor D_{ijkl}. Rotation of the module by angles α, β, γ changes its mechanical properties according to the transformation rule for a fourth-order tensor. In the continuum model each module/lattice configuration is represented by the coefficients, which can be expressed in Voigt notation by a square symmetric matrix 21 coefficients.

For both free-standing modules presented in Figures 1a and 1b: $D_{1123} = D_{2223} = D_{3323} = D_{3313} = = D_{3312} = D_{1312} = 0$. Basic module A is represented by: $D_{1111} = D_{2222} = D_{3333} = = 2EA/a^2(0.0367423k + 0.00474342\sigma + 0.03)$, $D_{1122} = D_{1133} = D_{2233} = D_{2323} = D_{1313} = = D_{1212} = EA/a^2(0.0367423k - 0.00474342\sigma)$, $D_{1113} = D_{1112} = D_{2213} = D_{2212} = D_{2313} = = D_{2312} = 0$. In the rotated module B one can identify: $D_{1111} = D_{2222} = 2EA/a^2(0.0525491k + -0.00135681\sigma + 0.0171625)$, $D_{3333} = 2EA/a^2(0.0560524k - 0.00361817\sigma + 0.0114416)$, $D_{1122} = = D_{2323} = EA/a^2(0.0350327k - 0.000904543\sigma + 0.0114416)$, $D_{1133} = D_{2233} = D_{1313} = D_{1212} = = EA/a^2(0.0280262k + 0.00361817\sigma + 0.0228833)$, $D_{1113} = -D_{2213} = -D_{2312} = = EA/a^2(-0.00834017k + 0.00538356\sigma + 0.0136195)$, $D_{1112} = D_{2313} = -D_{2212} = = EA/a^2(0.00535023k - 0.00345356\sigma - 0.00873688)$, where: $k = (EA)_{\text{cable}}/(EA)_{\text{strut}}$ is the cable-to-strut stiffness ratio, E is the elastic modulus of the parent material, A is the cross-sectional area of the member, and $\sigma = S_0/EA$ is the self-stress level ratio, S_0 is the level of introduced prestressing forces.

The continuum description indicates close to orthotropic properties of the basic module A and more anisotropic properties of the rotated module B. Extremal properties of the modules can be identified using the procedure described in [2], by finding eigenvalues and eigenvectors of the elasticity tensor for the selected parameters k_{extr} and σ_{extr}.

The results are as follows. For module A, the parameters $k_{\text{extr}} = 0.1$, $\sigma_{\text{extr}} = 0.7745$, stiff modes: $\lambda_1 = \lambda_2 = \lambda_3 = 0.0746969EA/a^2$, $\mathbf{w}_{1,\text{O}} = [1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0]^{\text{T}}$, $\mathbf{w}_{2,\text{O}} = [1 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0]^{\text{T}}$, $\mathbf{w}_{3,\text{O}} = [-0.5 \quad -0.5 \quad 1 \quad 0 \quad 0 \quad 0]^{\text{T}}$, soft modes: $\lambda_4 = \lambda_5 = \lambda_6 = 0$, $\mathbf{w}_{4,\text{O}} = [0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]^{\text{T}}$,

$\mathbf{w}_{5,\mathrm{O}} = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0]^{\mathrm{T}}$, $\mathbf{w}_{6,\mathrm{O}} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1]^{\mathrm{T}}$.

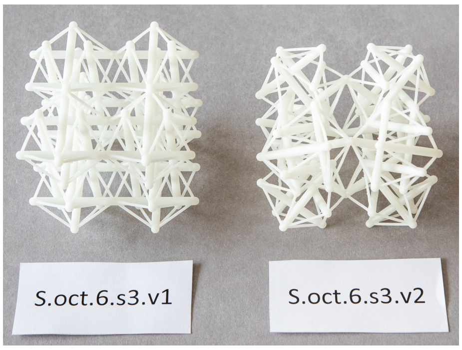
For module B, the parameters $k_{\text{extr}} = 0.1$, $\sigma_{\text{extr}} = 0.7745$, stiff mode: $\lambda_1 = 0.0854656EA/a^2$, $\mathbf{w}_{1,\mathrm{O}} = [1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0]^{\mathrm{T}}$, medium modes: $\lambda_2 = 0.0569761EA/a^2$, $\mathbf{w}_{2,\mathrm{O}} = [0.8401 \quad -0.8401 \quad 0 \quad 0 \quad 1 \quad -0.6415]^{\mathrm{T}}$, $\lambda_3 = 0.042732EA/a^2$, $\mathbf{w}_{3,\mathrm{O}} = [0 \quad 0 \quad 0 \quad -0.8401 \quad 0.6415 \quad 1]^{\mathrm{T}}$, soft modes: $\lambda_4 = \lambda_5 = \lambda_6 = 0$, $\mathbf{w}_{4,\mathrm{O}} = [0.5 \quad 0.5 \quad -1 \quad 0 \quad 0 \quad 0]^{\mathrm{T}}$, $\mathbf{w}_{5,\mathrm{O}} = [-0.8401 \quad 0.8401 \quad 0 \quad 0 \quad 1 \quad -0.6415]^{\mathrm{T}}$, $\mathbf{w}_{6,\mathrm{O}} = [0 \quad 0 \quad 0 \quad 1 \quad 0.3818 \quad 0.5952]^{\mathrm{T}}$.

It can be noticed that in configuration A, all three stiff modes of deformation are volumetric, while three soft modes are represented by shear deformations. In configuration B, there is one stiff mode of deformation (λ_1), which is volumetric (equal extensions in three directions), and three soft modes – one of which (λ_4) is represented by a volumetric deformation (contraction in the vertical direction z with extensions along x and y axes). Therefore, in view of compression tests planned, the two configurations were used as bases for creating tensegrity supercells – lattices of eight modules: configuration A as an example of a stiff system, and configuration B as a soft structure under compressive stresses applied in vertical direction.

3. Laboratory tests

Two configurations of tensegrity lattices were additively manufactured at the Faculty of Civil Engineering of the Warsaw University of Technology using the SLA technique. For each configuration, eight specimens were printed from a resilient resin with the following properties declared by the producer: ultimate tensile strength of 40 MPa, elastic modulus of 1.0 GPa, elongation at break of 40%, flexural strength of 55 MPa. Photos of the specimens and the test stand are presented in Figure 2. Compression tests were carried out in the universal testing machine INSTRON 5567, with the measurement accuracy of 0.5 N, using the displacement control method, with the velocity of 2 mm/min. The results (force–displacement in time) were registered directly from the universal testing machine.

The results presented in Figure 3 allow us to conclude that the lattice based on the basic module A is much stiffer under compression in the vertical direction z and has 3.45 times bigger load-bearing capacity (497 N for A as opposed to 144 N for B) than the lattice constructed from rotated modules B, which exhibits soft behaviour under the compression in the vertical direction z. Results of experimental tests confirm thereby the theoretical considerations presented in Section 2. Moreover, for configuration B, a smaller spread of results was obtained – standard deviation values are smaller than in configuration A, within which the tested specimens started to behave differently after reaching the end of the elastic part of the curve.

a) 

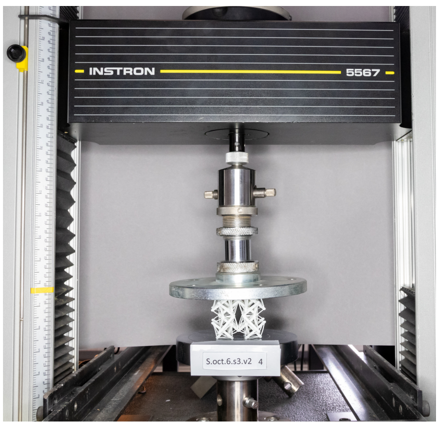
b) 

Figure 2: Tensegrity-inspired lattices: a) 3D-printed supercells based on configuration A (left) and B (right); b) specimen of the lattice in the test stand.

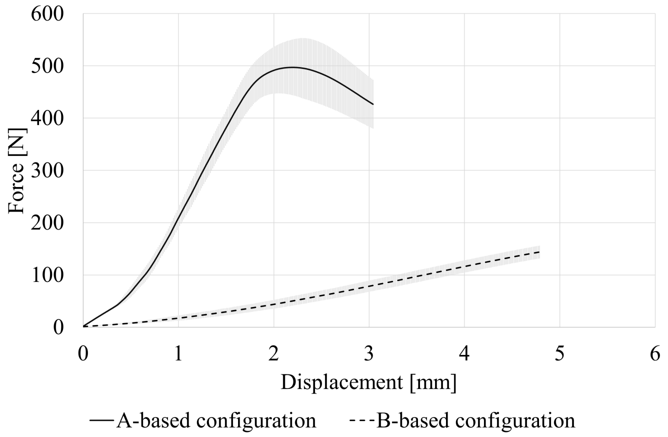


Figure 3: Force-displacement curves obtained for two configurations of expanded octahedron lattices under vertical compression (mean values with standard deviation).

4. Conclusions

In this study, a computational and experimental research on additively manufactured tensegrity-inspired lattices based on expanded octahedron cells was presented.

Two free-standing configurations were considered: A – basic module, B – rotated module. The continuum analysis revealed that the basic module has a volumetric stiff mode of deformation corresponding to the compression in the vertical direction z, while in the rotated module there is a volumetric soft mode under such compression deformation.

The results of computational analysis were confirmed in laboratory tests performed on two configurations of 3D-printed tensegrity-inspired lattices: eight-module supercells based on modules A and B. For the lattice based on configuration A, 3.45 times bigger load-bearing capacity was obtained, but at the same time, a bigger scatter of results was observed in the non-elastic part. The lattice based on configuration B exhibited a soft behaviour under the applied vertical compression load.

The experimental tests presented in this paper were carried out on medium-scale specimens. Further research shall be directed towards micro-scale (metamaterials) or macro-scale (civil engineering spatial structures). Computational simulations are recommended before experimental testing.

Acknowledgments

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