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Pre-rational approach for bamboo construction from a Dini surface

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Abstract

This paper is focused on the development of a geometric approach that provides favorable constructive properties for the low-tech construction of doubly curved bamboo structures. The approach was developed for a structure that was built in connection with the exhibition *Plant Fever* at Pillnitz Palace and Park near Dresden. Due to site and project constraints, a specific fabrication and assembly approach is employed, involving curved bamboo poles arranged in planes, orthogonal intersection of the primary and secondary structure while bisecting thebracing structure from straight, flat bamboo splits. These fabrication and assembly concepts can be translated into geometry properties of at least one direction of its coordinate curves planar (PU-patch) and correspondence between principal and asymptotic patch. Using a pre-rationalization approach, we define the design domain of negative constant Gaussian curvature (nCGC) surfaces to meet the geometric conditions exploring the Dini surface because of its resemblance to an unrolling leaf. Following form definition, the used fabrication techniques are traditional bamboo methods, namely ”Rup-rup” and ”Lidi-bundles”. The structure was completed in seven days with 20 participants of the ”Architectural Math for Bamboo Structures” summer school organized by the GMV research group at TU Dresden.

Keywords: Bamboo structures, Architectural geometry, Low-tech construction, Planar curvature lines, Dini surface, Principal and asymptotic patch correspondence, nCGC surfaces, Pre-rationalization approach, Differential geometry

Figure 1: The realized bamboo structure

1. Introduction

The structure's conception integrates contextual, historical, technical, and logistical considerations. To address these diverse factors at the start of the design process, we adopted a pre-rationalization approach. This approach involved translating project/site considerations into design, fabrication, and construction requirements, as shown in Table (1). Based on these requirements, we targeted specific geometric properties to achieve our design goals.

Considerations	**Impact on design, fabrication, and construction aspects**
The theme of the Plant fever exhibition	The shape of the structure (floral) and the choice of the building material (bamboo)
The garden as a heritage site	The off-site fabrication of structural components (prefab), the chosen type of foundation (above ground base with anchors so as to not damage the grass, and the minimizing of the structure's footprint, etc.)
The narrow access passages to the site	The division of the prefab structural components into partitions, Falsework-free construction, etc.
The interdiction of cranes, falsework, etc.	The partitioned prefab structural components sized to be handled easily by max. two people, simple assembly methods, etc.

Table 1: The impact of the considerations on the design and construction aspects of the bamboo structure

As indicated in Table (1), the first consideration was the theme of the *Plant fever* exhibition, that the project was associated with, at the botanical *Pilnitz* garden near Dresden. To align with this theme, we aimed for a floral aesthetic (cf. Section 3.) using natural and sustainable building materials. Moreover, there were some site-related considerations including the limited accessibility to the site through only four narrow passages of ($\approx 1.3m$) wide and the historical significance and heritage status of the garden, which restricted the use of larger transport vehicle as well as "invasive" foundations, and the number of working hours. The absence of supplementary construction tools and machinery that are not handheld influenced the decision to prefabricate the structural elements and ensure their dimensions were small and light enough to be carried, moved, and installed manually by a maximum of two people. Given the diversity of these considerations, we opted for traditional low-tech bamboo fabrication techniques and more importantly, a falsework-free assembly strategy in which the primary structure can be fixed into the foundation (anchored to the ground to counter balance the weight of the structure against wind forces) and in that manner create a stable enough "falsework" upon which the remaining structural components are assembled. This combination of low-tech fabrication and falsework-free assembly proved to be far from a trivial task and required an intensive level of interdisciplinary work as will be explained below. To address the defined task, we employed a pre-rationalization approach as a design methods allowing us to link certain fabrication and assembly requirements to precise geometric properties that are provided by both the choice of the surface and the choice of its patches available in differential geometry [1][2][3].

Since this approach provides the surface in a particular parametrization within its whole domain. The type of parametrization enables us to control the necessary geometric properties, their domain, and a large search space of shapes. The result is that it allows us to work in a continuous parametric manner and to establish a continuous flow between the shape, its variants or iterations that are optimized to respond to site requirements, and fabrication and assembly techniques. Hence, we used a parameterization (Principal curvature lines) on a doubly-curved surface (Dini) that yields one direction of its coordinate curves being planar (u) direction (PU-patch). While, the other direction (v) generating developable cladding strips ($\mathcal{V}$-strip) the joins of two neighboring v-curves that can be unrolled without distortion on the ground. Combining these two together yields the condition for our surface patch to be a conjugate PU-patch. It's important to note that the two directions here constitute the primary and secondary elements of the structure with the former realized in bent bamboo poles using a parameterized version of the traditional Rup-rup technique (cf. Section 4.) and the latter in bamboo Lidi-bundles of splits. Finally, to stiffen the doubly-curved structure, bracing is generated in the two diagonal directions. To ensure that the straight bamboo strips can be used through as braces (through cold bending), we know that strips generated by sweeping the normal along asymptotic curves possess that geometric property. Hence, the condition that we require of our principal PU-patch is that it satisfies the so-called Principal-Asymptotic networks correspondence [4]. After considering surfaces combining floral aesthetic and the two previously-mentioned conditions, we are led to use Tcheb-02 Iso-conjugate patch on the Dini surface (cf. Section 2.).

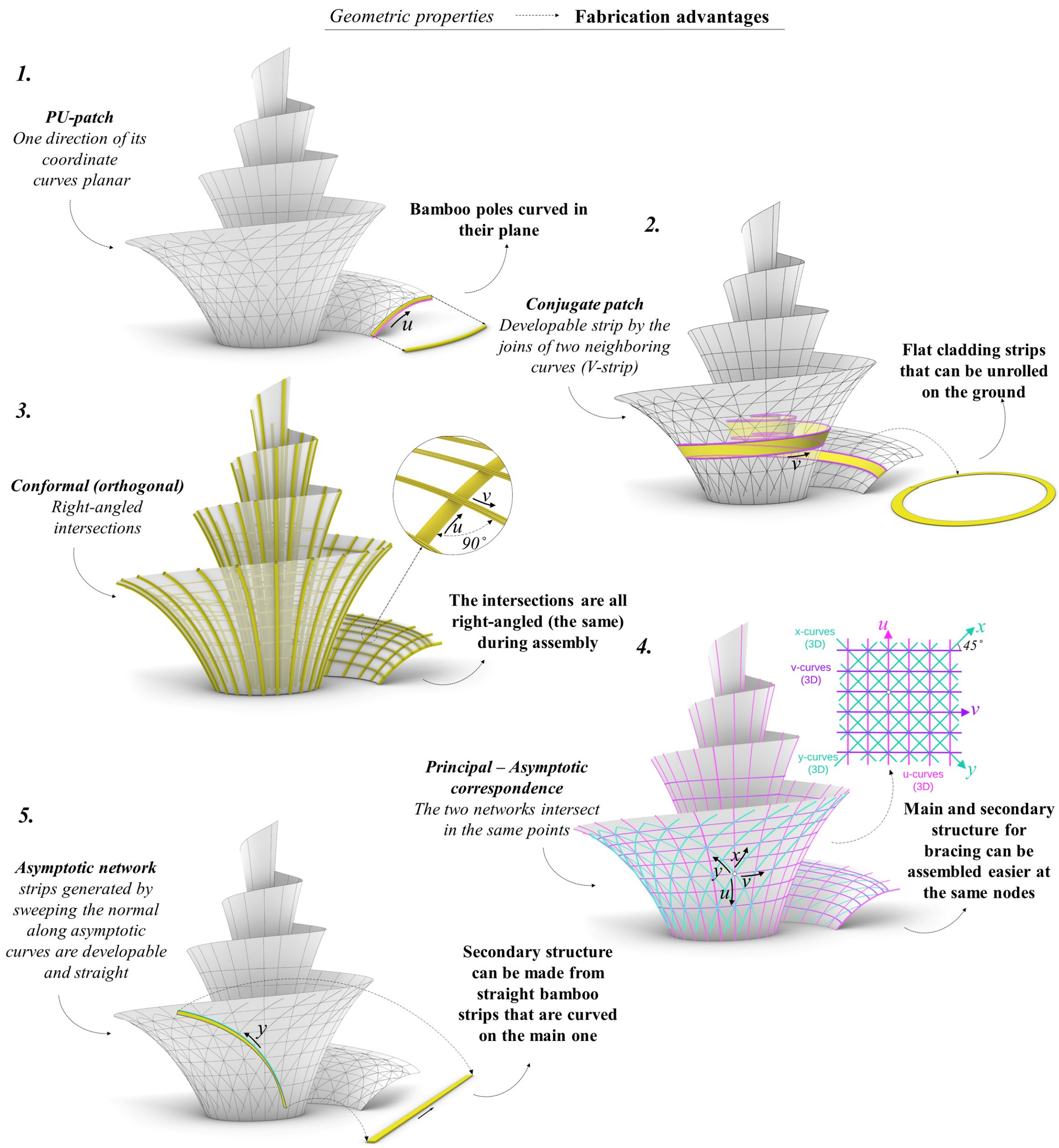

Figure 2: Linking fabrication and assembly requirements to geometric properties of the surface

2. Geometry of the Dini surface

To begin our description of the paramterization of the surface satisfying the geometric conditions highlighted in the introduction, we will recall some basic concepts from differential geometry, as follows.

2.1. Conjugate PU-patches and Principal-Asymptotic correspondence

Let $\langle \cdot, \cdot \rangle$ denote the standard Euclidean scalar product on $\mathbb{R}^3$ and $|\cdot|$ denote its associated norm. For every paramteric (spatial) curve $\alpha(t)$ its torsion is given by $\tau(\alpha) = \langle \alpha' \times \alpha'', \alpha''' \rangle / |\alpha' \times \alpha''|^2$. In particular, the curve α is said to be planar if its torsion $\tau(\alpha) = 0$ this is equivalent to $\langle \alpha' \times \alpha'', \alpha''' \rangle = 0$. In this context, a surface in $\mathbb{R}^3$ is given by a smooth parameterization patch $X(u, v)$ with partial derivatives X_u, X_v and its normal vector field denoted by N. A surface patch X is said to be a PU-patch if for every fixed v_o, the u-curve is planar, that is, $\tau\left(X(u, v_o)\right) = 0$ or equivalently, the patch X satisfies the PU-patch condition

$$\langle X_u \times X_{uu}, X_{uuu} \rangle = 0. \tag{1}$$

Next, let E, F, G, e, f, g denote the coefficients of the first and second fundamental forms respectively, there follows that the Gaussian curvature of the surface (parameterized by X) is given by the expression $\mathcal{K}(X) = (eg - f^2)/(EG - F^2)$. A surface patch X is said to be orthogonal if it satisfies $F = 0$, and iso-speed if $E = G$. Similarly, X is said to be conjugate if $f = 0$ and iso-conjugate if in addition $e = -g$. Moreover, X is called asymptotic if $e = g = 0$ and it is called developable if its Gaussian curvature vanishes, that is $\mathcal{K}(X) = 0$. Observe that if X is conjugate, then the v-strip defined by

$$\mathcal{V}(v, t) = X(u_o, v) + t(X(u_o + \epsilon, v) - X(u_o, v)) \qquad \text{for } t \in [0, 1], \text{ fixed } u_o \text{ and some } \epsilon$$

is "almost" developable, that is $\mathcal{K}(\mathcal{V}) \approx 0$. This is because, a developable strip can be seen as the limit of a 1-dimensional array of planar quad meshes (cf.[5]). From another point of view, it is easily checked that by a simple change of coordinates $u = x + y$, $v = x - y$, an orthogonal iso-conjugate patch $X(u, v)$ gives rise to an iso-speed asymptotic patch $Y(x, y)$ given by

$$Y(x, y) = X(x + y, x - y). \tag{2}$$

It was shown (cf. [4]), that up to appropriate equidistant subdivision (along the two parameters) of the domains of (u, v) and (x, y) respectively, the patches $X(u, v), Y(x, y)$ in question, give rise to corresponding principal and asymptotic networks. We are thus interested in surface patches that combine the properties of being a conjugate PU-patch and giving rise to corresponding principal and asymptotic networks.

2.2. Parametrization of the Dini surface

It turns out that Dini surface admits patches satisfying the above desired geometric properties, due to the fact that, it has negative constant Gaussian curvature (nCGC). Recall that a surface of nCGC $\mathcal{K} = -1/\rho^2$ for some constant ρ, then it admits a Tcheb-2 iso-conjugate patch $X(u, v)$. More precisely, it satisfies

$$\begin{cases} F = 0 \ , \ E = \rho^2 \cos^2\theta \ , \ G = \rho^2 \cos^2\theta \\ f = 0 \ , \ e = -g = \pm\rho \sin\theta \cos\theta \end{cases} \tag{3}$$

with angle function θ satisfying the sine-Gordon equation

$$\theta_{uu} + \theta_{vv} = \sin\theta \cos\theta. \tag{4}$$

Clearly, the Tcheb-2 iso-conjugate nCGC patch $X(u,v)$ is a principal patch, in particular, it is orthogonal iso-conjugate, hence it gives rise to an iso-speed asymptotic patch $Y(x,y)$ (also called Tcheb-1). The resulting corresponding principal and asymptotic networks can be seen in Figure (3). In the following, we will give the explicit parameterization of the Dini surface by the Tcheb-2 iso-conjugate patch $X(u,v)$, as the Bäcklund transformation of the vertical line. It is known (cf. [6],[7] and [8]) that given a Tcheb-2 iso-conjugate patch X of nCGC $-1/\rho^2$ with angle function θ, can give rise to another Tcheb-2 iso-conjugate patch X^* of nCGC $-1/\rho^2$ with angle function θ^* by the Bäcklund transform

$$X^* = X + \left(\frac{\cos\sigma\cos\theta^*}{\cos\theta}\right) X_u + \left(\frac{\cos\sigma\sin\theta^*}{\sin\theta}\right) X_v \tag{5}$$

of inclination σ, where θ^*, θ satisfy the Bäcklund-Darboux Equations

$$\begin{cases} \theta_u^* + \theta_v = \frac{1}{\cos\sigma}\left(\sin\theta^*\cos\theta + \sin\sigma^*\cos\theta^*\sin\theta\right) \\ \theta_v^* + \theta_u = \frac{1}{\cos\sigma}\left(-\cos\theta^*\sin\theta - \sin\sigma\sin\theta^*\cos\theta\right). \end{cases} \tag{6}$$

Now, seeing the vertical line as a degenerate Tcheb-2 iso-conjugate patch $X(u,v) = (0,0,\rho u)$ with angle function $\theta = 0$, solving Equations (6) yields the angle function

$$\theta^* = 2\arctan\left(\exp(u\sec\sigma - v\tan\sigma + w)\right) \qquad \text{with } w \text{ integration constant.} \tag{7}$$

which upon substitution of θ^* in Expression (5), we obtain the Tcheb-2 iso-conjugate patch

$$X^* = \rho\left(\frac{\cos\sigma\cos v}{\cosh(w + u\,\mathrm{sech}\,\sigma - v\tan\sigma)}, \frac{\sin v\cos\sigma}{\cosh(w + u\,\mathrm{sech}\,\sigma - v\tan\sigma)}, u - \cos\sigma\tanh(w + u\,\mathrm{sech}\,\sigma - v\tan\sigma)\right). \tag{8}$$

By varying the inclination σ and the constant of integration w, different Dini surfaces are obtained as seen in Figure (3), while varying the parameter ρ just produce uniform scaling of the surface. Finally, it can be directly verified that the Dini surface patch X^* given by Expression (8) satisfies the PU-patch Condition (1)

$$\langle X_u^* \times X_{uu}^*, X_{uuu}^* \rangle = 0.$$

Hence, all the u-curves of $X^*(u,v)$ and all the $\mathcal{V}$-strips are "almost" developable as seen in the Figure (3).

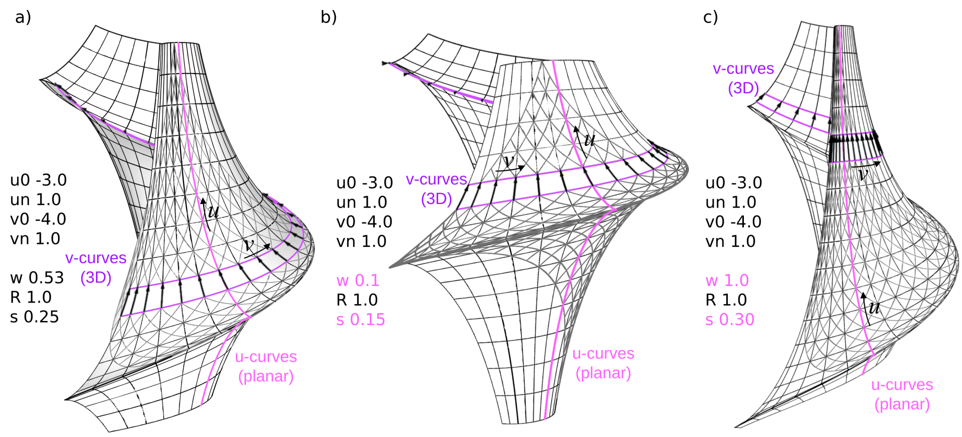

Figure 3: Different forms of Dini surface based on their input parameters.

By using the defined parametrization of the Dini as principal patch, next step is exploration with variety of variants within the Dini surface, addressing form aesthetics, project objectives and construction needs.

3. Form

The formal similarity of the Dini surface to a blossom or an unfolding leaf, as shown in Fig (4), highlights the theme ”nature-inspired art forms”. But instead of making natural forms tangible through geometric abstraction, the design process starts from the abstract mathematical surface. Through manipulation of the domains and variables, it arrives to a shape that when trimmed, gives a the final floral-like structure while maintaining all the constructive properties highlighted in the introduction.

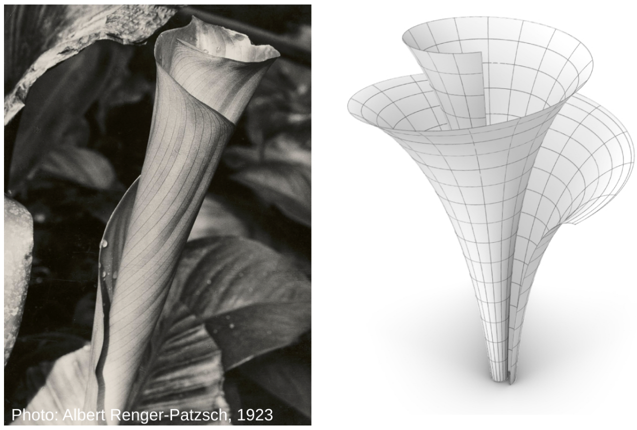

Figure 4: Resemblance of the Dini surface to a picture of an unfolding of a leaf

In Fig (5), we can see how the u, v-domains of the Dini surface were expanded to create a shape that rolls about itself while ascending, which in return creates a floral like composition with the spiral footprint and the spiral-like ridge emulating the petals. This shape is then trimmed by a horizontal plane (ground) and a spatial spiral curve to further enhance the floral shape of the structure. And as explained in Section 2., the Dini is a particular surface as it allows for principal and asymptotic patches to correspond. Thanks to that correspondence, each patch could be used to generate certain structural components, the principal network generating the primary and secondary structures as well as the cladding and the asymptotic network generating the bracing elements that bisect the main structural elements at the intersecting points.

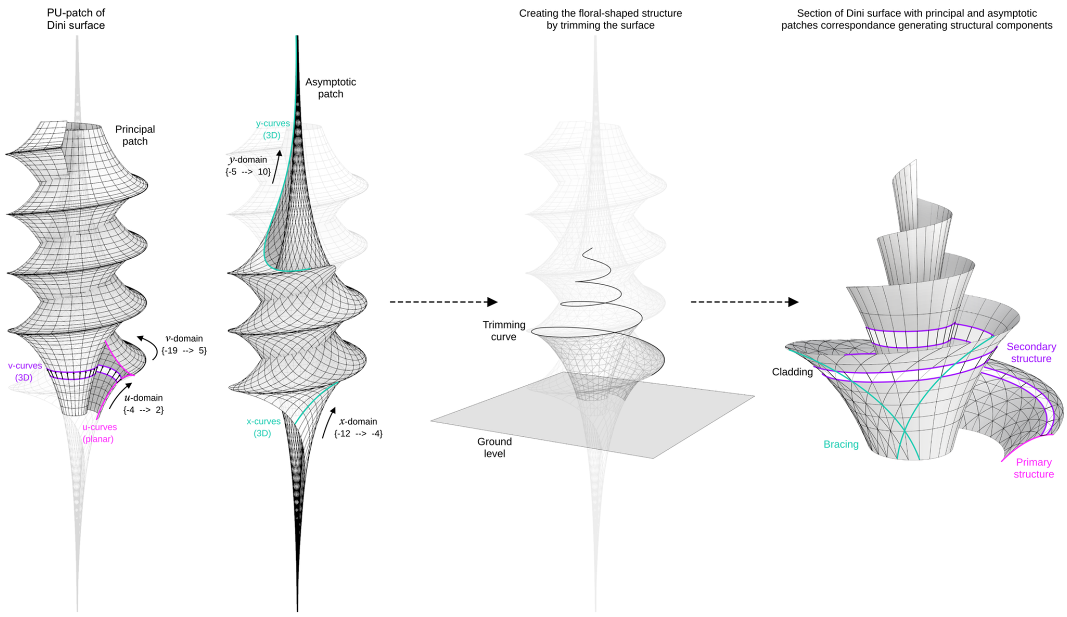

Figure 5: Geometric generation of the floral-shaped structure

4. Structural components

The bamboo structure could be broken down into 5 structural components, as shown in Fig (6). Those components were fabricated using traditional techniques that were geometrically-formulated to create a strong link between design and fabrication while optimizing its precision and fabrication time as explained below on one of the components.

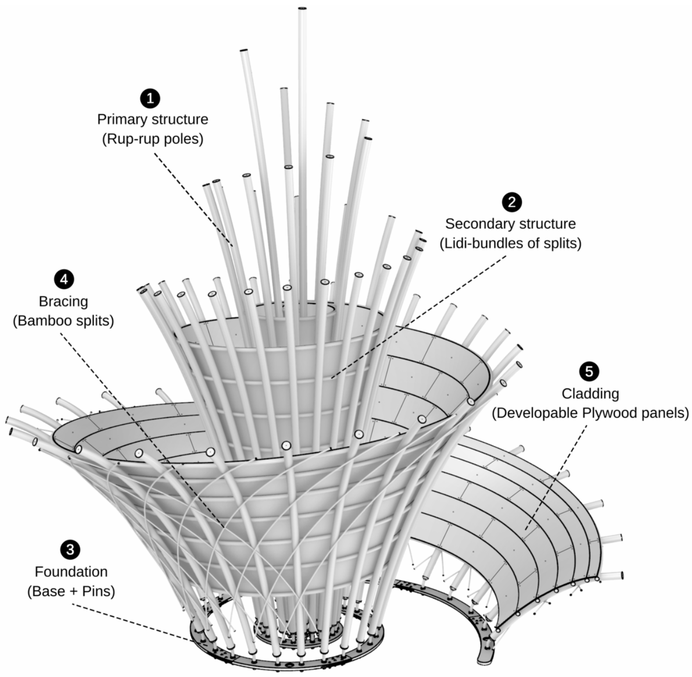

Figure 6: Structural components

Component 01: Primary structure → Rup-rup poles :

The traditional Rup-rup technique, as per [9], implies making V-shaped cuts (spanning $\approx 3/4$ of the width of the pole depending on the species) at certain intervals along the length of the pole. The resulting pole should follow the desired curve once pressure is applied at its extremities till the two walls of each cut meet. To note that even though increasing the number of cuts will better approximate the curved pole to the target curve, it also makes the pole more structurally fragile.

In order to determine precisely the position and angles of the cuts on the straight pole so that curved, it corresponds to the network curve it is tracing, we've reformulated those parameters geometrically by inferring global properties (of the curved shape of the bent bamboo pole) into local properties such as the change of curvature and torsion at specific points and it is as follows:

Since the planar curve C contains no torsion. It then follows that the shape of the curve C is determined by how it curves in the plane, as explained in the following. Let $(p_i) = (p_1, ..., p_n)$ be a list of discrete points on the planar curve with s_i the length of the piece of curve (p_i, p_{i+1}). Next, let $(v_i) = (v_1, ..., v_n)$ be the tangent vectors at the points (p_i), with θ_i the angle between (v_i, v_{i+1}), also known as the turning angle, as shown in Fig (7) (Step 01). We will also refer to the turning angle θ_i as the (average) curvature angle, since the actual (signed) curvature κ of the curve C at p_i is given by

$$\kappa(C)(p_i) = \lim_{s_i \to 0} \frac{\theta_i}{s_i}.$$

Now, to carry on the (local) geometric method of bending the straight pole, we need to collect two further lists of lengths, $(a_i) = (a_1, ..., a_{n-1})$ and $(b_i) = (b_1, ..., b_{n-1})$. These are lengths of the segments between points (p_i) and points $(q_i) = (q_1, ..., q_{n-1})$ where each point q_i is obtained by intersecting the line $\mathbb{R}v_i$ with the line $\mathbb{R}v_{i+1}$, giving us the following data list:

$$\begin{cases} \text{Lengths of segments: } (a_i), (b_i) \\ \text{Angles of curvature: } (\theta_i). \end{cases}$$

Next, we mark on the straight pole, the appropriate lengths $(a_i), (b_i)$, and finally use the angles (θ_i) to create incisions in the pole (V-shaped cuts). There follows that, by simply bending the pole to close these cuts, we obtain the desired curved pole, as seen in the Fig (7) (Step 02 - 03). In closing this part, it is important to mention that the choice of the position (not only the number) of the discrete points (p_i) plays a role in approximating the curve C by segments. Indeed, even for the same number of points, we can have points closer together in high curvature zones than in low curvature zones. Hence, by using a non-uniform distribution of the points we are able to get a better approximation with the same number of points.

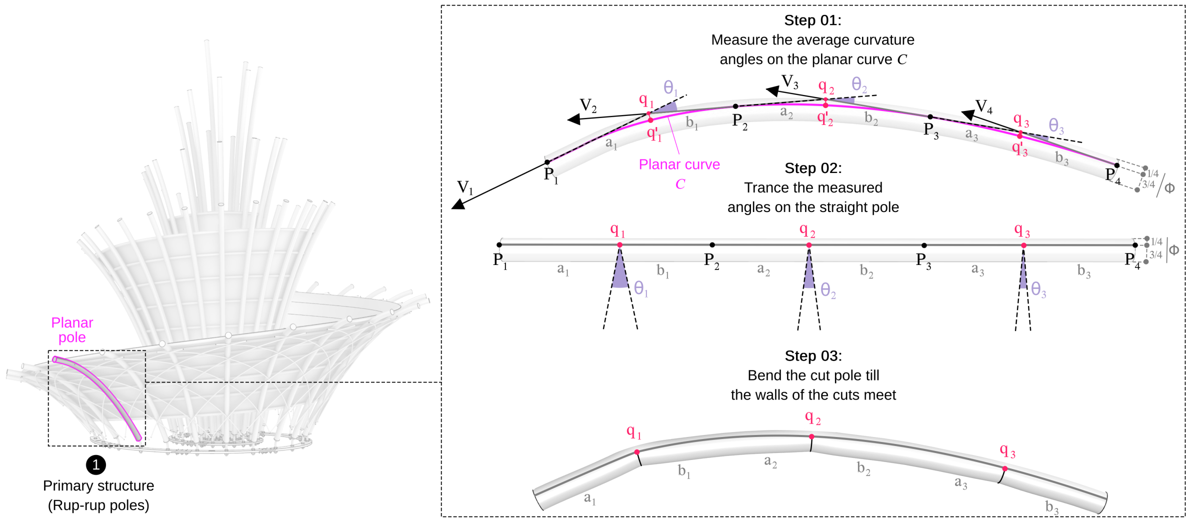

Figure 7: Geometrically-formulated Rup-rup technique used for the fabrication of the primary structure

While the bent pole traces the network curve, it doesn't stay in shape when the pressure on its extremities is released. There are several methods shown to deal with this issue (cf. [9]). Nevertheless, The method that we've used was to screw a longitudinal bamboo split (obtained as described by [10]) along the side of the pole that was cut while bent so it stays in shape when the pressure is released, as shown in Fig (8).

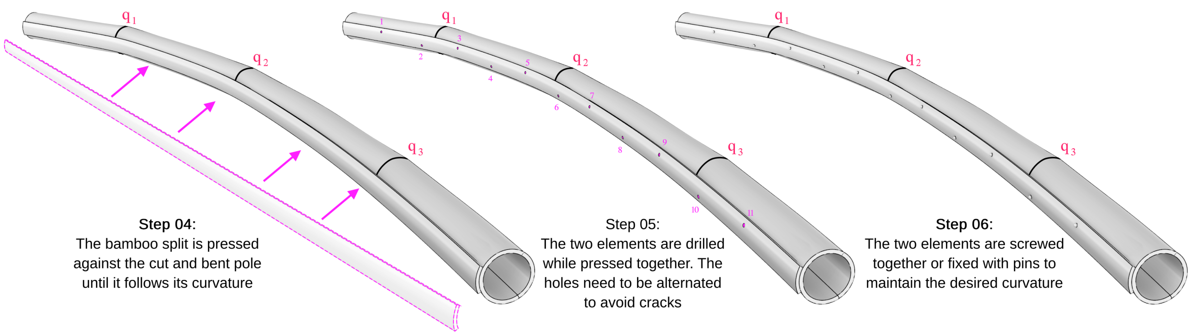

Figure 8: Attaching a bamboo split to the cut pole to maintain its shape

5. Falsework-free assembly

Due to the project constraints highlighted in Table (1) and thanks to having the primary structure be planar elements as shown in Section 4., we were able to assemble the structure on-site and without Falsework. This was achieved through the sequential assembly of the structural components broken down as follows;

First, we place the steel base (that comes in parts) over wooden beams and drill holes in the ground where the anchors will be inserted later. The wooden beams creates enough space so as to have access underneath the steel base to bolt the primary structure to it, as shown in Fig (9).

Figure 9: Installing the steel base

Next, We start alternating between bolting a number of the primary structure (rup-rup poles) to the base and screwing the secondary structure (a Lidi-bundle made out of thin longitudinal bamboo splits) unto the primary structure following markings made earlier on them. The process is repeated till all the components of the primary and secondary structures are assembled, as shown in Fig (10) (A,B,C). This method of assembly was conceived due to the Dini's particular shape that narrows towards the center making it difficult to access the inner parts if we were to install the entirety of the primary structure before we start installing the secondary one. Once, they are both installed, the bracing elements made out of bamboo splits (component 4) are lashed to the outer side of the primary structure using cords, as shown in Fig (10) (D,E,F).

Figure 10: Installing the primary, secondary structural components and the bracing elements

Finally, the wooden beams beneath the steel base are pulled out slowly lowering the base till it rests on the ground and the steel anchors are inserted to fix the structure in place.

6. Conclusion

In this paper, we presented a pre-rationalization approach illustrated in the use of Tcheb-2 iso-conjugate Dini surface patch. This shows how to target and translate certain intrinsic geometric properties into constructive properties adapted to the construction of doubly curved bamboo structures. Our idea was to not just maintain its low-tech nature but also reintegrate the traditional building techniques through mathematical reformulation to expand their versatility and accessibility to designers. Moreover, we showed how through the established approach, a variety of site considerations could be addressed from the point of departure of the design process. Finally, we hope that by show-casing this specific example's entire process (from conception to realization) we demonstrated a potential more general falsework-free approach towards construction of doubly-curved stuructures.

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