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# An Automated Triangular Mesh Partitioning Method for Surfaces Based on Multi-Objective Optimization

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## Abstract
Large-span single-layer grid shell structures are commonly used for the roofs or skylights of large venues and public buildings. These roof surfaces typically exhibit relatively mild undulations but vary greatly in shape. Quickly generating smooth and structurally sound mesh divisions from complex surface geometries presents a significant challenge. Existing mesh division tools and methods often fail to meet the design needs of frontline architects. To address this issue, this paper first introduces a rapid and practical method for generating structured meshes; then, it establishes a multi-objective optimization workflow for surface shape and mesh division, which automatically produces optimal meshes tailored to different performance requirements, forming an easy-to-use, integrated tool designed for engineers. Finally, through an optimization case study, this workflow optimizes a surface model with the dual objectives of minimizing strain energy and achieving uniform mesh distribution, resulting in a series of smooth and rational mesh division schemes.

**Keywords**: single-layer grid shell structures, mesh division, multi-objective optimization, structural optimization, smart construction

## 1. Introduction
In the field of architectural design and engineering, the creation of structured meshes for surfaces is a critical task that bridges conceptual design with practical application[1]. Mesh generation methods are pivotal for modeling, analyzing, and fabricating complex architectural forms, especially those involving roof-like surfaces. These techniques not only need to ensure structural integrity and aesthetic appeal but also adapt to the constraints of modern construction technologies and materials[2].

Recent advancements in computational design tools have enabled architects and engineers to explore more complex geometries and sophisticated design solutions[3][4][5]. However, generating structured meshes that accurately conform to specified geometrical and functional criteria remains a significant challenge, particularly when dealing with non-planar or highly curved surfaces[6][7]. This challenge is compounded by the need for methods that can efficiently handle large-scale computations without sacrificing accuracy or detail.

This paper introduces a novel structured mesh generation method based on the principle of homeomorphism in topology, which simplifies the mesh partitioning task by mapping complex three-dimensional surfaces onto two-dimensional planes. This method facilitates the generation, optimization, and transformation of meshes with enhanced control over geometric parameters, leading to optimized structural and aesthetic properties.

Furthermore, this paper explores a multi-objective optimization workflow that incorporates various architectural and structural parameters to refine the mesh generation process. By optimizing mesh uniformity, structural stability, and material usage, the proposed method aims to offer robust solutions that align with contemporary architectural needs and sustainability standards.

The subsequent sections will detail the mesh generation process, discuss the specific tools and algorithms employed, and present a case study that demonstrates the practical application and effectiveness of the proposed method in optimizing architectural designs. This introduction sets the stage for a comprehensive exploration of a method that not only enhances the functionality and efficiency of architectural modeling but also contributes significantly to the field of computational design in architecture.

## 2. A Method for generating structural meshes for surfaces

### 2.1 Structured Mesh Generation Method Based on Base Mesh Mapping

This section introduces a structured mesh generation method suitable for roof-like surfaces. The workflow of this method is as follows:

*(1) Surface to Mesh Conversion:*

Architects commonly utilize surfaces to draw and adjust roof geometries. Initially, a Surface or Brep (**G1**) is converted into a Mesh (**M1**). Subsequently, Grasshopper's *TriRemesh* component is employed to regenerate the mesh, resulting in a more uniform member length in Mesh (**M2**). The primary parameters of *TriRemesh* include target length and number of iterations.

Figure 1 illustrates this transformation process. It is evident that the member length in **M1** varies significantly, whereas in **M2**, the member lengths are more uniform. However, the mesh in **M2** contains a large number of singular points, which are areas where the mesh does not conform smoothly, potentially leading to issues in both structural integrity and aesthetic quality.

*(2) Mapping the Mesh to a Plane:*

Using the *Squisher* tool, the three-dimensional mesh (Mesh **M2**) is mapped onto a plane, resulting in a flattened mesh (Mesh **Mp**) (shown in Figure 1). The *Squisher* tool effectively transforms complex 3D geometries into 2D representations by preserving the structural integrity and connectivity of the original mesh. This tool is particularly useful for architectural applications where patterns derived from 3D forms need to be manufactured or analyzed in a 2D format. The flattened mesh (Mp) can then be used for further analysis, fabrication processes, or as a base for additional design modifications and optimizations.

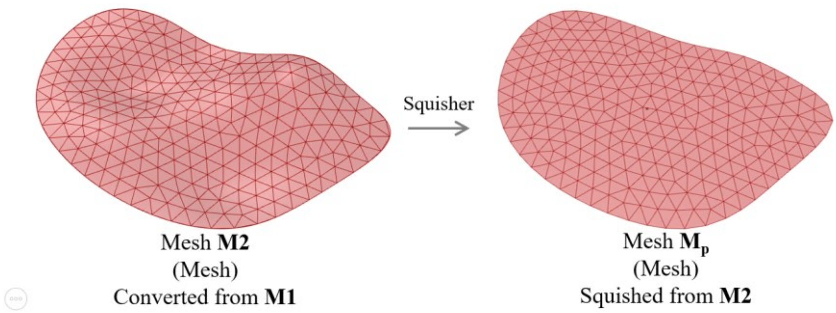


Figure 1 Mapping the Mesh to a Plane

*(3) Generating a Base Mesh on the Plane:*

Near the center of the flattened mesh **Mp**, a base point **Pt** is chosen for the generation of a foundational mesh, referred to as Base Mesh **B1**. This foundational mesh is constructed from equilateral triangles to form a planar grid, ensuring it completely covers the area of **Mp**.

The base mesh is generated using several parameters:

$d_x$ and $d_y$ represent the vector components from a generation point p0 to the reference point Pt, effectively positioning the base mesh in relation to **Pt**.

$l$ specifies the length of the sides of the mesh edges in the base mesh, determining the scale of the grid.

$\alpha$ denotes the clockwise rotation angle of the base mesh around the point **Pt**, which helps align the mesh with any directional features of the underlying surface.

$n_x$ and $n_y$ are the numbers of divisions in the x and y directions respectively, defining the extent and resolution of the base mesh across the plane.

This configuration allows for the base mesh to adapt to the underlying topography and geometry of **Mp**, facilitating subsequent steps in mesh processing or design applications. The parameters $d_x$, $d_y$, $l$, $\alpha$, $n_x$, and $n_y$ provide the flexibility to tailor the base mesh to various design needs and constraints, ensuring a robust and versatile foundation for further architectural modeling or structural analysis.

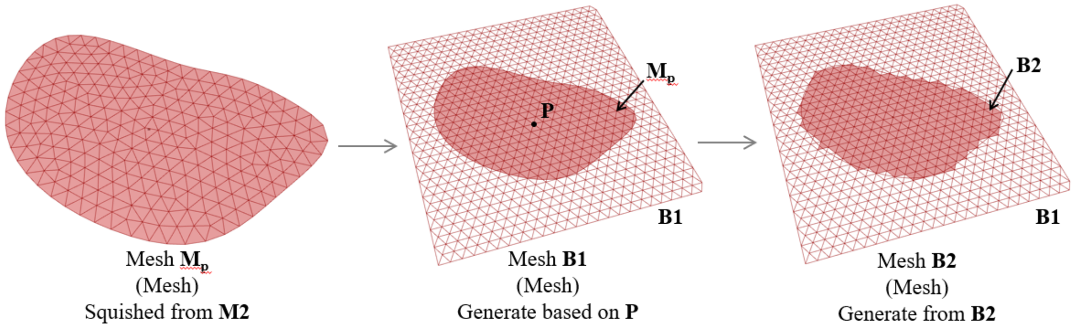


Figure 2 Generating base mesh **B1** and sub-mesh **B2**

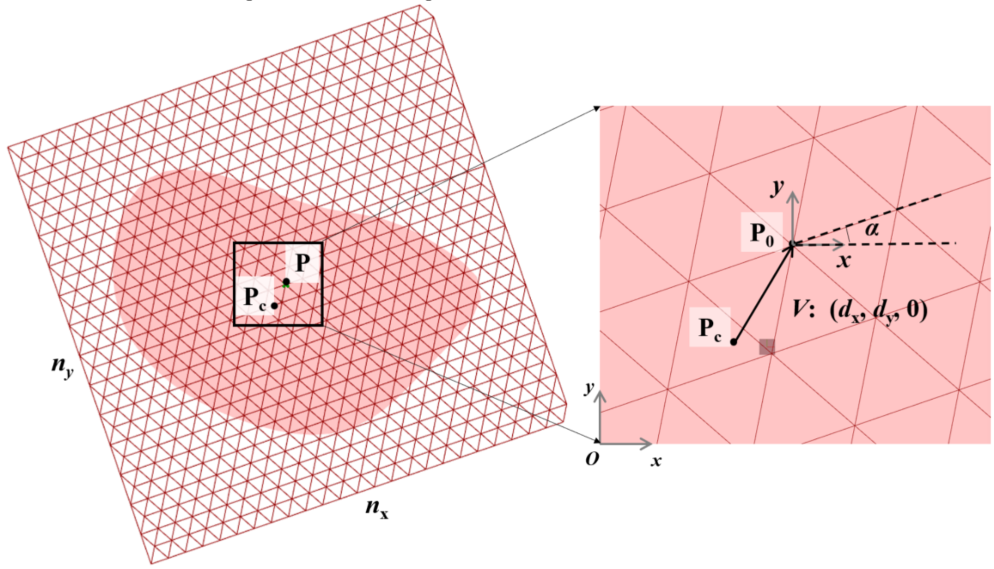


Figure 3 Schematic of Base Mesh Generation

*(4) Generating a Submesh Closely Resembling the Base Mesh*

Following the creation of the base mesh **B1**, a submesh that closely resembles the shape of mesh Mp is generated based on **B1**. This is achieved by retaining the mesh faces in **B1** that are present on mesh Mp, resulting in mesh **B2**. The edges of mesh **B2** exhibit a jagged appearance, necessitating the adjustment of the edge vertices of **B2** to align with mesh **Mp**, thus producing mesh **B3**. This process is illustrated in Figure 2.

*(5) Mapping Back to the Original Mesh*

The geometry of mesh **B3** is identical to that of mesh **Mp**; however, the internal vertices in **B3** are connected by six rods, resulting in enhanced fluidity. Subsequently, **B3** is mapped back onto the original mesh using the SquisherBack tool, generating mesh **R1**.

*(6) Mesh Smoothing*

Due to the potential irregular changes in member lengths caused by the mapping process, further smoothing is necessary to achieve the final mesh shape. Common smoothing methods include Laplacian smoothing and techniques based on inter-particle forces.

In this paper, a staged smoothing approach is adopted: Initially, the nodes at the edges of the mesh are smoothed. This preliminary step focuses on adjusting the boundary conditions of the mesh to ensure that the transitions between the edges and the internal area are gradual and natural. Following the edge smoothing, the internal nodes of the mesh undergo smoothing. This step is crucial as it directly impacts the overall structural integrity and aesthetic quality of the mesh.

The final result is Mesh R2, which exhibits a more uniform and aesthetically pleasing geometry. This staged smoothing process not only enhances the visual appearance of the mesh but also improves its mechanical properties by distributing stresses more evenly throughout the structure.

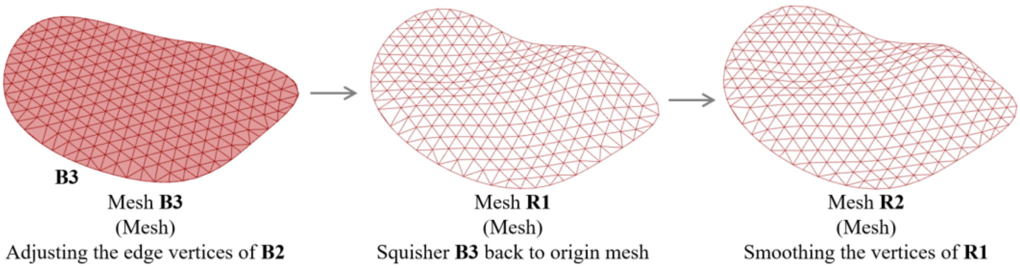


Figure 4 Squisher back and smoothing

## 2.2 Discussion about the method

This method leverages the principle of homeomorphism from topology to transform the challenge of structured mesh partitioning into a surface cutting task on a plane. With an existing surface shape, the process begins by unfolding the surface onto a plane. Given a set of base mesh parameters, a cropped mesh is generated through a predefined procedure and subsequently mapped back to the original geometric shape. A smoothing process ensures a division scheme without internal singularities. This method is computationally efficient, with each calculation requiring less than one second, making it suitable for further optimization computations.

However, the method has certain limitations:

*(1) Not suitable for surfaces with high curvature:*

The base mesh generation does not account for the curvature of the original geometric shape, which may result in suboptimal performance on surfaces with significant curvature. Given that the undulations of common roofing surfaces are relatively mild, this method is applicable to nearly all roof design scenarios.

*(2) The quality of a single generation depends on the base mesh:*

Since the final mesh retains the same topological relationship as the base mesh, the outcome of this method heavily depends on the base mesh's shape. If the base mesh has many inflection points at its edges, the final surface modeling may not be ideal. Therefore, optimization algorithms can be employed to perform extensive computations to achieve better mesh solutions.

# 3. Multi-objective optimization workflow

Building upon the mesh generation method introduced in Section 2, a comprehensive optimization workflow can be proposed, as illustrated in Figure 5. The input parameters for the optimization include the designed surface shape and some fundamental design parameters. The optimization variables comprise control parameters for both the surface shape and mesh partitioning. Following these inputs, the workflow automatically generates the mesh configuration, which is then subjected to multifaceted evaluations covering both geometric and mechanical properties. By optimizing the design variables, the process aims to yield the optimal configuration of surface shape and mesh partitioning. This systematic approach leverages the intrinsic characteristics of the design surface to enhance both the aesthetic and functional aspects of the final structure.

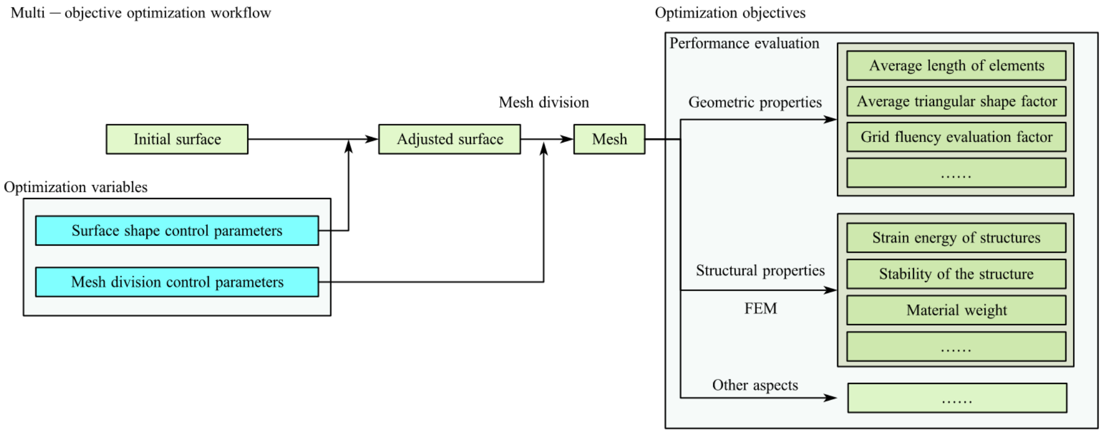


Figure 5 The workflow of multi-objective optimization

### 3.1 Optimization variables

In various optimization scenarios, the optimization variables may include parameters for controlling surface shapes as well as those for mesh partitioning.

*(1) Fixed Architectural Surface Shape:*

When the architectural surface shape is strictly fixed, only the mesh partitioning scheme can be optimized. Using the division method described in Section 2 of this paper, the mesh partitioning parameters include $d_x$, $d_y$, $l$, $\alpha$ .

*(2) Adjustable Surface Shape:*

When the architectural surface allows for fine-tuning, joint optimization of both surface modeling and mesh partitioning can be conducted. The optimization variables are expanded to include control parameters for the surface shape, which may involve adjustments to the control points of the initial surface. This scenario introduces more optimization variables, significantly expanding the search space and enabling the optimization process to identify superior designs for both surface shape and mesh partitioning.

### 3.2 Multi-objective optimization

In multi-objective optimization, selection of evaluation metrics can be tailored according to the specific needs of optimization. When deriving a mesh configuration from surface geometries and optimization variables, the mesh can be evaluated from various perspectives:

*(1) Architectural Form:*

Architectural form often aims for a smooth and uniform mesh design, which can be assessed using the following criteria:

Uniformity of Members: This evaluates the consistency in the length and thickness of members within the mesh. Uniform members contribute to visual harmony and economical use of materials.

Triangular Shape Factor: This involves analyzing the quality of triangular elements within the mesh and optimizing the proportion of equilateral triangles to enhance the stability and aesthetic appeal of the structure.

Node Fluidity: This examines the smoothness and coherence of connections at the nodes, facilitating an even distribution of forces and overall structural beauty.

*(2) Structural Performance:*

Structural performance focuses on the rationality of the force distribution under load conditions[8], reflected in the following aspects:

Strain Energy of the Structure: Measures the energy stored in the structure as it deforms under load. Optimization of strain energy can enhance the structure's resistance to deformation.

Structural Stability Performance: Assesses the structure's ability to maintain its shape and functionality under various loads, aiming to improve its safety factor.

Non-linear Stability Load Bearing Capacity: Analyzes the behavior of the structure under high loads, including its capacity within the non-linear range, which is crucial for safe design.

Material Usage under Given Load Conditions: Optimizes the design to reduce material consumption while meeting structural performance requirements, thus enhancing material efficiency and reducing costs.

*(3) Manufacturability:*

Manufacturability significantly affects the overall construction costs, with indicators including but not limited to:

Number of Node and Member Types: Evaluates the diversity of nodes and members in the design. Reducing the number of types can simplify the production process and lower costs.

Ease of Assembly: Considers the convenience of assembling structural elements, including connection methods and assembly time, to improve construction efficiency.

Manufacturing Constraints: Adjusts the design based on manufacturing and processing limitations to ensure the feasibility of the design, such as modifying complex geometric shapes to fit existing manufacturing technologies.

### 3.3 The tools of multi-objective optimization

In this study, the Octopus plugin was employed, which is a powerful multi-objective optimization tool designed for the Grasshopper environment. The Octopus extension enhances Grasshopper's capabilities, enabling it to perform complex multi-objective optimization tasks suitable for applications in architectural design and engineering fields. Octopus supports the exploration of optimal design solutions across multiple performance indices by integrating various evolutionary algorithms, including Genetic Algorithms (GA) and Particle Swarm Optimization (PSO). These algorithms are capable of exploring potential solutions within the design space and identifying sets of solutions that achieve the best trade-offs among multiple objectives, known as the Pareto frontier.

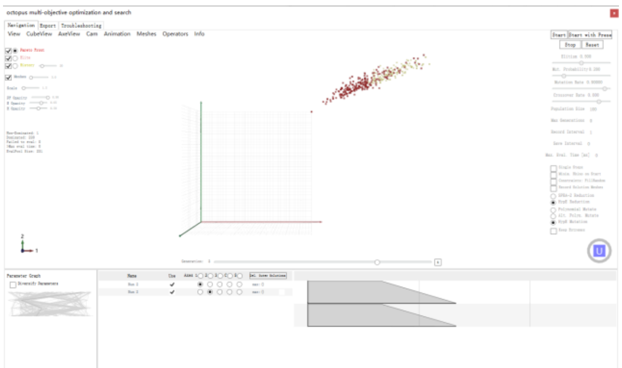

Figure 6 The user interface of the Octopus software

### 3.4 Optimization results

In multi-objective optimization, the outcome is typically represented by a set of solutions that achieves an optimal trade-off among the given objectives. This set is known as the Pareto frontier. The Pareto frontier consists of non-dominated solutions, meaning that no other solution is superior across all objectives simultaneously. The Pareto frontier provides all potential optimal trade-off solutions, allowing designers to select the most suitable one based on their specific requirements and preferences.

## 4. Multi-Objective Optimization Case Study and Results

Building on the optimization method introduced previously, this section presents a case study focused on a dual-objective optimization. The objectives considered are the uniformity of mesh members and the minimization of structural strain energy under constant loading.

### 4.1 Case Scenario

The architectural surface model is depicted in Figure 7. The members feature an I-beam cross-section, specifically H250x120x6x8, and are made of aluminum alloy 6061-T6.

Six hinged supports are strategically positioned around the perimeter of the surface, as illustrated in Figure 7.

The optimization variables include mesh division parameters $l$, α, $d_x$, $d_y$. The value of $l$ ranges from 1800 to 2000. The ranges for $d_x$ and $d_y$ are both $[-l, l]$, and the angle $\alpha$ ranges from 0 to 60 degrees.

Two optimization objectives are selected:

Objective 1 aims for the most uniform variation in the length of the mesh members, represented by the coefficient of variation r, which is to be minimized.

Objective 2 seeks to minimize the strain energy under a constant distributed load of 1.0 kN/m².

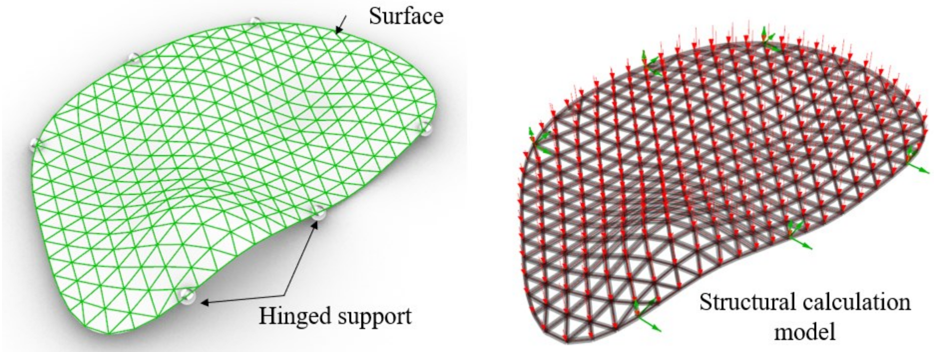


Figure 7 Surface and supports of optimization

### 4.2 Optimization Process

The workflow for optimization, based on the Grasshopper platform, is illustrated in Figure 8. This visual representation outlines the sequence and interaction of various components involved in the optimization process.

### 4.3 Final Results

Following the optimization calculations conducted with the Octopus tool, a Pareto front as shown in Figure 9 was obtained. The diagram identifies eight prominent points along the front, and five distinct mesh designs are showcased. These results demonstrate a balance between the uniformity of member lengths and the rationality of the structural forces. Among these, A1 exhibits the best structural performance, while A8 offers the smoothest grid continuity. Architects can choose a design from these options based on their preferences to finalize their architectural plan.

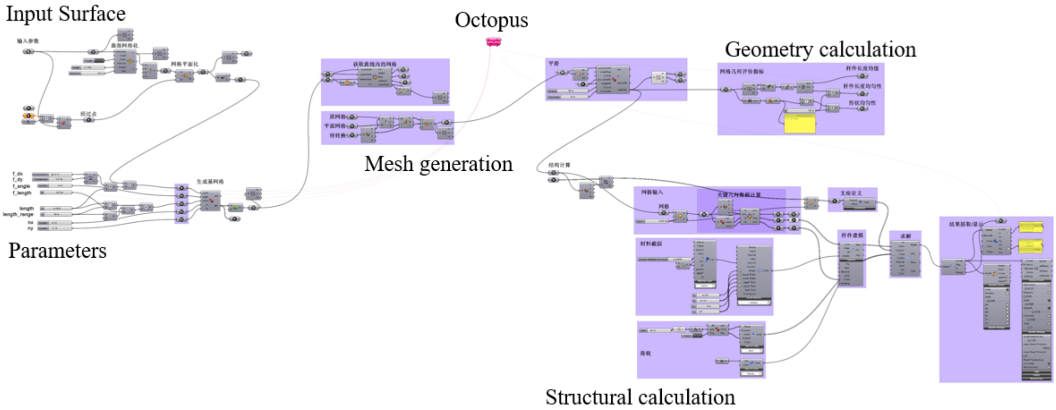

Figure 8 Workflow of Grasshopper script

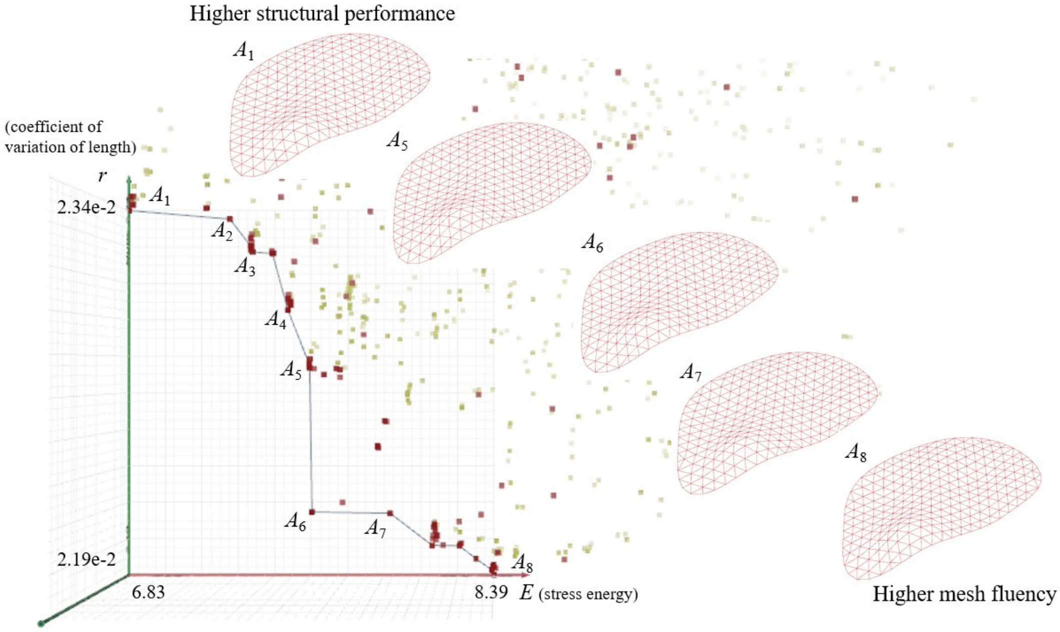

Figure 9 Pareto frontier of optimization

This case study demonstrates that with such a workflow, simply inputting the surface shape and some necessary parameters is sufficient to initiate the optimization computations. This approach yields multiple optimized results, providing designers with a variety of choices for their design decisions. This system's efficiency and efficacy significantly streamline the design process, allowing for rapid iteration and refinement to achieve optimal structural and aesthetic outcomes.

## 5. Discussion and conclusion

### 5.1 The main work completed in this article

The structured mesh generation method outlined in this paper utilizes principles of topology and surface unfolding to provide an efficient and accurate approach for creating meshes in architectural applications, particularly for roof structures. Once a surface shape is specified, this method can generate a smoother mesh configuration based on several basic parameters.

Building on this mesh generation technique, the paper proposes a multi-objective optimization workflow for surface grid shell structures, which allows for the refinement of the mesh partitioning of surfaces. The optimization variables include control parameters for the surface shape and mesh partitioning, with optimization goals that comprehensively consider the geometric characteristics of the mesh, structural force characteristics, and manufacturability.

Finally, through an optimization case study, this workflow is demonstrated. The case study focused on a surface aiming to minimize strain energy and achieve the most uniform member lengths as its dual objectives. This resulted in a series of Pareto front solutions, illustrating the practical utility of this method in engineering design. The outcomes showcase how this approach can effectively balance multiple objectives, providing valuable insights for designers and engineers seeking to optimize complex architectural forms.

### 5.2 Limitations and future research directions

The structured mesh generation method and the accompanying optimization workflow presented in this study provide a comprehensive and efficient approach to architectural design and structural analysis. The ability to convert complex 3D geometries into manageable 2D meshes allows for enhanced control over the design and optimization processes, making it a valuable tool in the field of architectural engineering.

Despite its demonstrated efficiency, the method's effectiveness is somewhat limited by the geometric complexity it can handle, particularly in terms of curvature. Future research could focus on enhancing the method's adaptability to more complex surfaces and exploring more advanced algorithms for base mesh generation that can more effectively accommodate varying curvatures and complexities.

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