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## Modularizing Schwarz D-Surface structures with hypar based timber shell components

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### Abstract

The building construction sector is an important contributor to global CO<sub>2</sub> emissions and a major producer of waste. Modular timber structures can enable a higher degree of circularity and reuse in the building construction sector, and strengthen the role of timber buildings as carbon sinks. This research builds upon and continues recently initiated investigations on reconfigurable construction systems that employ hypar based timber components as basic modules, and their potential for integrating panel shaped wood production waste. This paper presents new insights on the design space of such systems, as well as new ways of designing and prefabricating the modular units. These findings are implemented in a design proposal for a pavilion that is also intended as a contribution to the IASS 2024 Design Competition. The results of initial FEA studies are presented, which provide first insights on the structural performance and behavior of the pavilion, and the suitability of the proposed construction logic and method. The focus here is on the global behavior of the structure assuming a rigid connection between the components. We are opening the research for future studies on semi-rigid behavior, making use of the FEA extension of COMPAS – An Open-Source Computational Platform for Research in Architecture and Engineering. Finally, we identify possible applications and provide an outline of future work.

**Keywords:** Timber and bio-based structures, modular timber structures, circular construction, Schwarz D-Surface, hyperbolic paraboloids, reconfigurable structures, upcycling wood production waste, conceptual design, digital modelling and fabrication.

### 1. Introduction and background

Implementing a *Circular Economy* (CE) in the building construction sector is a means of reducing both CO<sub>2</sub> emissions and waste. Recent studies suggest that modular timber buildings have the potential of increasing the degree of circularity in the construction sector, and to contribute to a circular built environment [1]. Modular systems can involve volumetric, planar, or linear base units. While being a promising contribution to circular construction, building with modules also bears the risk of producing buildings that suffer from monotonous repetition, a feature that contributed to the vast public rejection of such buildings when they emerged in the 1960s. While being based on a similar rational logic, modular systems with non-orthogonal units have the potential of breaking up respectively avoiding this monotony. Using timber as a construction material can turn buildings into carbon sinks [2,3], even more so if waste wood is included. An increasing number of studies investigates the potential of upcycling wood production waste [4-6] and reclaimed wood [7].

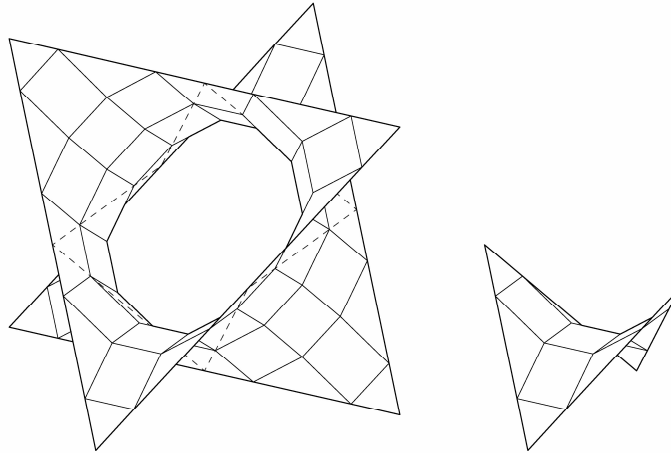


Figure 1: Six identical hypars, with a hypar surface that can be inscribed in a cube, can be combined into a star-shaped higher-order unit. This example also shows how neighboring triangular mesh faces, on the boundary of adjacent hypars, can be merged into quads [8].

This paper builds upon recently initiated research [8] on reconfigurable construction systems that employ hypar based timber components as basic modules, demonstrating their potential for integrating panel shaped wood production waste. The integration of panel shaped wood waste is partially enabled by a particular geometric property of hypars, which allows for the generation of flat-quad meshes and thus for building doubly-curved structures from flat wooden panels. This system goes beyond planar or box-shaped units, and breaks up the monotony inherent to conventional modular buildings. As part of these studies, it has been shown how six hypar components can be combined into star-shaped higher order units, see Figure 1. Six of such units were used as modules in the construction of a proof-of-concept demonstrator, the Hypar Up pavilion, the winning entry to the IASS 2023 Design Competition. The realization of this pavilion was made possible by a collaboration between the design team, based at Aarhus University, and a fabrication team, based at the University of Melbourne.

## 2. Design space explorations: from Hypar Up to Schwarz D

In this paper, we introduce another configuration in which four star-shaped higher order units are combined in a partially overlapping way, as shown in Figure 2. Some scholars refer to this geometry as Schwarz D-Surface or full Schwarz D-Surface [9]. In addition to introducing a new configuration, the paper also presents a new approach to panelization and modularization, and a pavilion design based on these insights.

In [8], it has been shown that hypar surfaces can serve as basis for segmented timber shells made of planar wooden elements. Moreover, it has been shown that six of such hypar based components can be combined into star-shaped higher order units. In the case of the Hypar Up pavilion, six of these higher order units are combined, using translational and/or rotational symmetry. The underlying geometry of the Schwarz D pavilion, on the other hand, features rotational and reflective symmetry.

The Schwarz D pavilion uses the same hypar geometry as basic unit as the Hypar Up pavilion. However, there are differences in the size of each unit and the number of units used in the two configurations. Some of the key differences are summarized below:

- Hypar Up Pavilion
  - 36 hypars, embedded in six higher order units
  - 360 panel elements in total
  - A single hypar fits in a cube with an edge length of 45 cm
  - A bounding box of 180 x 90 x 270 cm

- Schwarz D Pavilion
  - 18 hypars, prefabricated as 18 octagonal modules
  - 190 panel elements in total
  - A single hypar fits in a cube with an edge length of 100 cm
  - A bounding box of 300 x 300 x 300 cm

What is also interesting about the Schwarz D-Surface configuration is that it can be placed on the ground in three different ways: 1) As shown in Figure 2 (left), 2) as a tower with one star-shaped unit as support, 3) with a three-point support. In the latter case, one would take off the outer triangular panels to increase the support area.

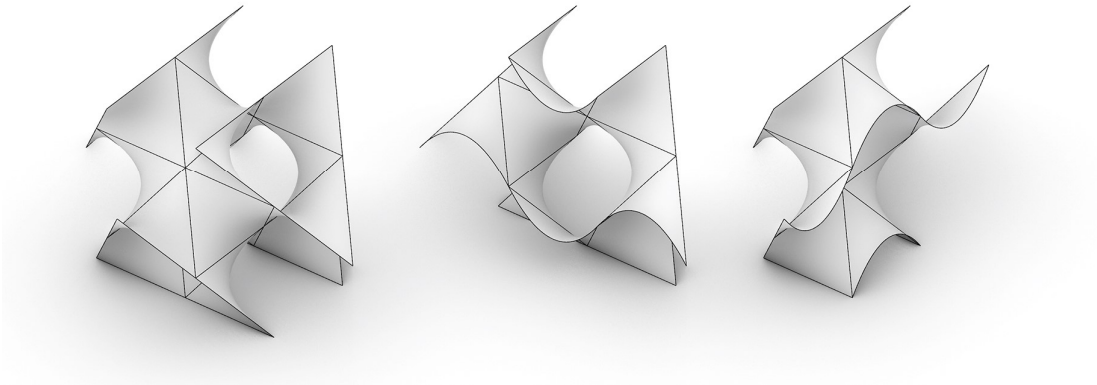


Figure 2: The here proposed configuration (left) is comprised of four partially overlapping star-shaped units. Two cross-section drawings (middle, right) provide insights on the interior space of the structure.

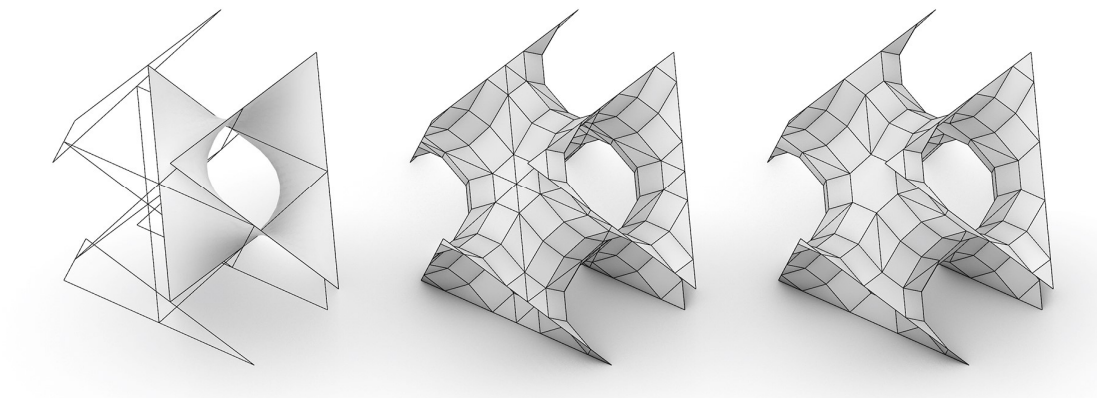


Figure 3: Schematic representation of the pavilion geometry with smooth hypars, highlighting one star-shaped higher order unit (left); segmented hypar modules, where the boundary of each module corresponds to that of the underlying hypar (middle); new design for the prefabricated modules (right).

### **2.1. Modularization strategy: octagonal modules and hexagonal connector panels**

In addition to introducing a new overall geometry, a new approach towards the panelization and modularization is proposed as well. To achieve the Schwarz D-Surface configuration, breaking up the star-shaped higher order units is required. One way of designing the hypar-based modular units is illustrated in Figure 3 (middle). This leads to continuous edges along the modules. The preferred proposal suggests octagonal modules that are combined with hexagonal and trapezoid connectors and triangular corner pieces, as shown in Figure 3 (right). This approach could provide a structural advantage: The hexagonal and trapezoid connector plates interrupt the continuous straight edge lines between the modules, further reducing the risk of enabling a hinge mechanism between the modules.

The disadvantage is that instead of having only hypar-based modules, additional elements are included in the kit-of parts, which makes the assembly slightly more complex.

The octagonal modules are based on segmented hypar modules in which the corner triangles are removed. In locations where several hypars meet, the triangular pieces are merged into one hexagonal piece, which connects the hypars. In total there are four hexagonal plates, each of which connects to six octagonal modules.

## **2.2. Joint design, assembly and prefabrication**

In order to make the fabrication of the pavilion more efficient, we propose the use of a scaffolding system, as shown in Figure 4. The proposed scaffolding system consists of simple planar elements that are connected via interlocking joints. Straps can be attached during the curing period of the glue, to increase the pressure on the joints. The prefabricated octagonal modules can be stacked vertically for space-efficient transportation. The inner panel edges follow the half-space logic described in [8], and can be produced with one straight cut, for example with a circular saw. The global boundary edges are ruled surfaces, the production of which requires a 5-axis CNC router.

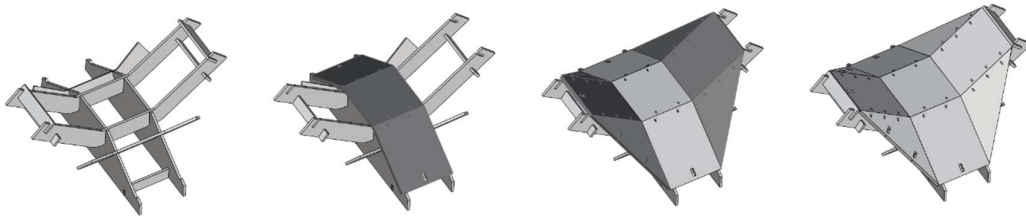


Figure 4: Proposed fabrication process of the octagonal modules with a scaffolding system, from left to right: Empty scaffolding structure, placement and connection of three panels in the center, placement of outer elements, all panels are connected into one module. A 5-axis CNC router is used for the production of the individual panels.

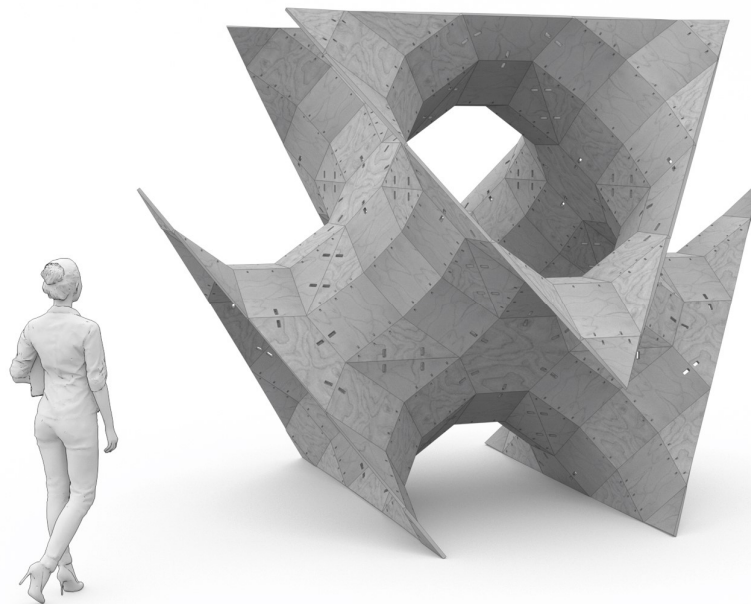


Figure 5: Visualization of the demonstrator, which also serves as a pavilion. The proposed configuration consists of 190 plywood panels with 21 mm thickness. The structure fits in a cubic bounding box with an edge length of 3 meters.

The initially proposed connection method is using screws, glue and cylindrical timber dowels between the panels of the prefabricated octagonal modules. Reversible connections are established between the modules, using worktop connectors, such as the Häfele Worktop Connecting Bolt, or similar.

The final design of the joints depends on the overall structural behavior of the pavilion [10,11]. By means of Finite Element Analysis (FEA), we investigated if rotationally stiff connections are required between the modules. In the very first FEA studies, the connections are considered to be made with glue and wooden dowels, and are assumed to be rotationally stiff.

### 3. Structural Analysis and Design

#### 3.1. Numerical Finite Element model

The structural model is built within the ABAQUS environment, utilizing shell elements with a uniform thickness of 21 mm (Figure 6a). This selection of element type is often preferred for thin-walled structures as this modeling technique captures the behavior of the structure efficiently while reducing the computational costs. The connections between the plates are modeled rigidly so no nonlinearity is rising from the plate connections based on the assumption that the combination of fastener and glue integrate these plates with sufficient robustness to transmit the expected loads. Consequently, flexibility is not anticipated for the stage that the structure is just bearing gravitational and partial live loads. For the material properties, since the structure is expected to operate within its elastic range, no significant plastic deformation is foreseen. Therefore, the material is considered to be isotropic and homogenous with a modulus of elasticity of 3723 MPa and a Poisson's ratio of 0.5. The model is discretized with a 10 mm mesh size and 3-node triangular shell elements are used during the analysis (Figure 6b). The total volume of the structure is 0.494 m<sup>3</sup> and the total mass of the structure is 247.1 Kg.

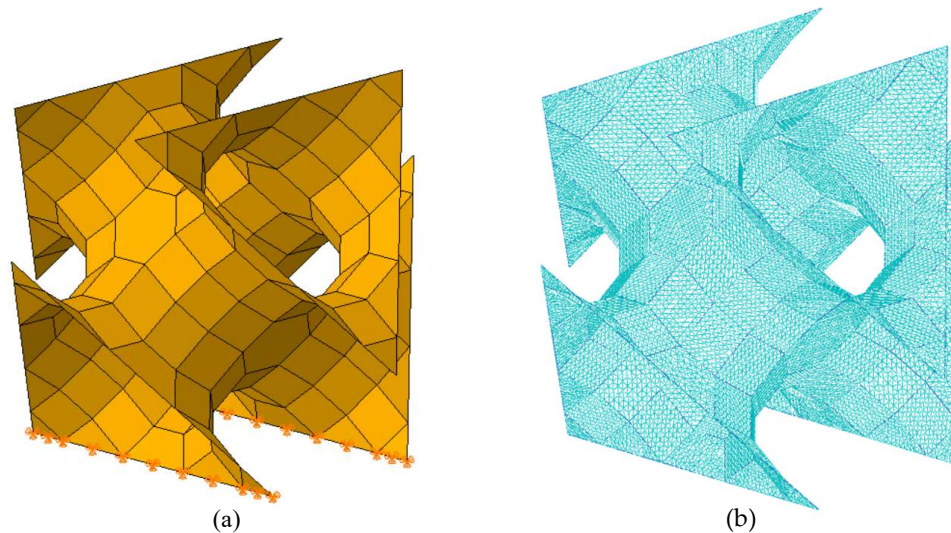


Figure 6: (a) Numerical model; (b) Meshed model.

#### 3.2. Force-flow and stress distribution mechanisms

The results of the analysis for self-gravitational loading of the structure are shown in Figure 6. Regarding stress values, Von Mises stress values (Figure 7a) indicates a maximum stress value of 0.0823 MPa which is significantly lower than the yield strength of the material and safely in elastic zone. For the principal stresses (Figure 7b), maximum tensile principal stress value of 0.0547 MPa and minimum compressive stress value of 0.0837 MPa are observed. These results are collectively suggesting that under gravitational loads, the structure experiences relatively low stress values which are well within the elastic range of material behavior. These results confirm the integrity and safety of design, and the capability of the structure to withstand gravitational loads without any significant deformations.



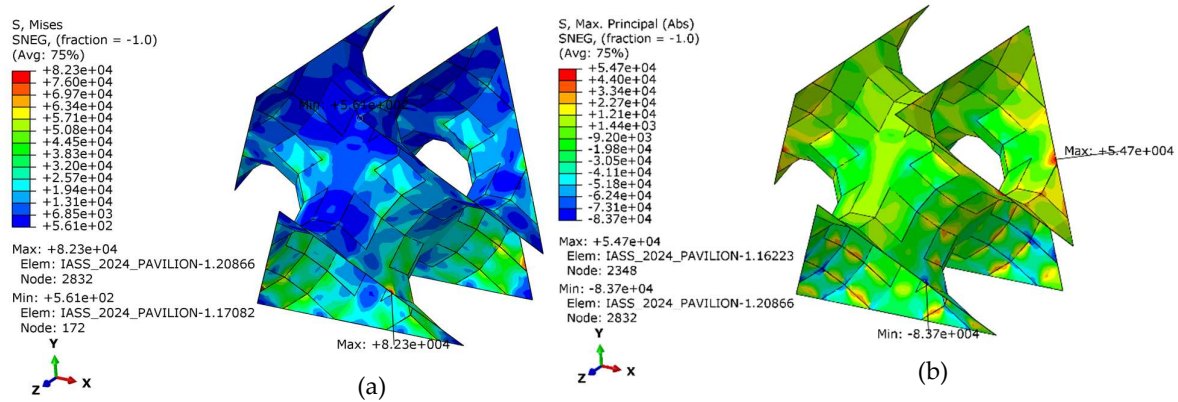


Figure 7: (a) Von Mises stress distribution, (b) Maximum and Minimum Principal Stress distribution.

### 3.3. Deformations

Considering the deformations of the structure under gravitational loads, the maximum displacement (Figure 8a) is 0.113 mm which happens at the edges of the end plates of the structure. This phenomenon is attributed to the cantilever-like behavior exhibited by these plates, which inherently have less stiffness compared to other parts of the structure. For the rotational deformation (Figure 8b), maximum rotation is 0.183 radians which is also related to the endplates with cantilever-like structure. The observed deformations are relatively insignificant compared to allowable deflections and deformations for serviceability criteria, and this confirms the robustness of structure for its self-weight load bearing.

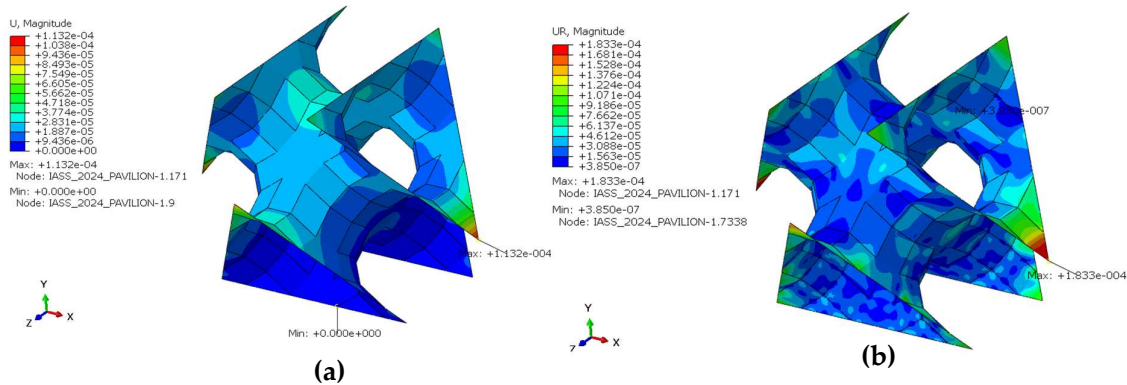


Figure 8: Deformations for dead load, (a) Absolute Displacement Magnitude, (b) Absolute Rotation Magnitude.

### 3.4. CAD-CAE integration and semi-rigidity of connections

The numerical simulation mentioned earlier is also integrated using COMPAS. This is an open-source platform based on Python, developed at the Swiss NCCR Digital Fabrication. Its purpose is to increase code reusability and simplify the development of collaborative design workflows. The COMPAS\_FEA extension is designed to create a seamless interface between CAD and FEA. This Python package initially creates a structure object linked to the 3D model. This object contains geometric information, elements, sections, and material properties. It then lets users specify loads and boundary conditions for structural analysis. The structure object is assembled using modules from the core COMPAS library and the FEA extension. This approach eliminates most repetitive scripting tasks and offers streamlined data post-processing support. Once the structure object is constructed, COMPAS\_FEA creates the native input file for the FE software, either ABAQUS or OpenSees. The model is then sent to the original solver for analysis. Lastly, data from the analysis results are extracted and returned to the collaborative design interface.

With full edgewise connectors established, and considering the FE Von Mises stress distribution, it's assumed that the connections perform rigidly in this investigation. However, the scalability success of such structures from a structural integrity perspective heavily depends on the connection performance at larger scales. Future research will incorporate the semi-rigidity of timber connections into the FE model. For this purpose, COMPAS integration is crucial. The connection links will be integrated into the simulation by defining spring elements (with 6 degrees of freedom) between the shell elements. The algorithm will also ensure that the nodes of each connection link correspond to vertices of the plate mesh.

#### **4. Discussion and conclusions**

Building on previous research, this paper illustrates the vast design space of a modular timber construction system based on segmented hypar shells. Using the same base unit as previous studies, it introduces a new configuration and a novel modularization strategy, and presents a method for producing the prefabricated components. The work also presents a design proposal for a pavilion, which incorporates these findings. FEA studies are employed to gain a better understanding of the structural soundness of the global design and the proposed connection method.

Regarding the installation of the pavilion, the boundary conditions will need to be verified, as it is not possible to install permanent foundations in the exhibition area. In order to activate the area between the two line supports as an arch, it might become necessary to establish a connection between these two supports. Even more so, when assuming the possibility of people entering the pavilion, which would go beyond the above-described scenario with dead load only. Future studies will complement the FEA results with experimental testing.

This work is based on a bottom-up and research by design approach. Nevertheless, the goal is to implement the proposed system on a larger scale. Possible applications identified in [8] include facades and roof structures. Additional ways of doing so are considered. One way of achieving a building scale application would be to scale up the configuration used for the pavilion, and to add layers for weather proofing and insulation, glazing. Cladding, for example made from sheet metal, could be added to create a watertight envelope. The sheet metal cladding could follow the same logic as the segmented timber hypars. In this scenario, one could also add floor slabs inside the interior space. Larger structures could also be achieved by combining, multiple Schwarz D-Surface units, for example to form space trusses.

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