Proceedings of the IASS 2024 Symposium
Redefining the Art of Structural Design
August 26-30, 2024, Zurich Switzerland
Philippe Block, Giulia Boller, Catherine DeWolf,
Jacqueline Pauli, Walter Kaufmann (eds.)

Generative Design Methodology Using Non-uniform Mesh Subdivision Based on Internal Force

Ziying SHI [a,*], Hang DAI [a]
[a,*] Department of Building Technology and Science, School of Architecture, Southeast University
Southeast University, Nanjing 210096, China
Shiziyingcn@gmail.com

Abstract
Mesh subdivide patterns plays a crucial role in morphology design and performance optimization of structure. Existing grid shells predominantly rely on three mesh subdivision patterns: triangular mesh, quadrangular mesh, and voronoi mesh, in which the relative shapes and scales of the mesh grids depend on the corresponding algorithms and surface geometry information. However, due to the non-uniform distribution of inner forces within the shell, the mesh subdivide patterns must respond. This paper addresses this challenge by presenting a novel parametric design approach for non-uniform grid shells, which integrates design boundaries, considerations of force and constraints, and textures order through featured stress lines and cloud diagrams to effectively control the density of mesh grids. Aiming to harness internal force data for morphological design, this paper introduces a novel design strategy that adjusts material distribution in response to internal force patterns, leading to intricate grid shell subdivisions and the creation of structural skin textures. By overlaying various internal force patterns, such as principal stress lines, isostress lines, and cloud diagrams, as subdivision criteria, diverse subdivisions are achieved, enabling control overthe arrangement of components and the density of material distribution. Several case studies are presented to validate the efficiency and feasibility of the proposed method. With the same amount of material utilized, comparisons were conducted between non-uniform subdivided grids outcomes and traditional subdivided grids. Finite element analysis (FEA) results validated that the shells with subdivision patterns featuring internal force attain higher structural efficiency. Additionally, the results validate that the heterogeneous patterns enable diversity in shell form designs, with potential implications for parametric architectural generation and structural optimization.

Keywords: Mesh subdivision patterns, Heterogeneous grid shells, Internal Force, Structural morphology, Parametric design, Texture generative design.

1. Introduction
In numerous engineering disciplines, mesh subdivision is recognized as a fundamental prerequisite for numerical calculations[1], playing an essential role in structural design and performance optimization. Extensive research has explored common subdivision forms and grid generation algorithms, such as triangular, quadrangular, and Voronoi meshing. Delaunay triangulation is widely used for generating and optimizing uniform grids on complex free-form surfaces, exhibiting a high degree of fluency[2]. Within the realm of quadrilateral mesh subdivision, the binary-tree method automates surface mesh generation for complex models, facilitating adaptive meshing and the direct creation of anisotropic meshes through selective subdivision[3]. Quadtree and Octree algorithms[4][5] excel in adaptive meshing and re-meshing, efficiently refining complex geometries to closely approximate surface boundaries and optimize computational resources. Furthermore, the Blossom-Quad algorithm effectively transforms triangular meshes into optimal quadrilaterals, enhancing mesh quality through

vertex adjustments and graph optimizations[6]. Building upon triangular and quadrangular subdivisions, polygonal meshes can also be formed and optimized based on geometric organization[7], such as the commonly observed Voronoi mesh. These studies underscore the necessity of research in the field of grid generation and lay the groundwork for non-uniform mesh subdivision. The discussions are centered on enhancing the uniformity of grid generation, the accuracy of finite element analysis model construction, and the reduction and smoothing of errors in curvature coverage on free surfaces. Non-uniform mesh generation is primarily aimed at accommodating areas with abrupt shape transitions and significant curvature variations, as well as obtaining more accurate finite element computational models[3].

Existing mesh subdivision methods underpin parametric design, allowing rapid adaptation to freeform surfaces. However, the shape and scale of mesh units depend largely on specific algorithms and surface geometry, neglecting the non-uniform internal force distribution within structural shells. Such limitations not only affect structural performance optimization but also constrain the aesthetic form's connection to natural forces. It is important to highlight the crucial role of non-uniform mesh subdivision in achieving accurate stress distribution in finite element analysis, especially in regions characterized by complex geometries and significant stress concentrations[3]. In structural form-finding and optimization, creating a strong connection between the form and forces is fundamental[8]. Modern computational tools and algorithms facilitate the visualization of shell internal forces (Figure1). A deep understanding of internal force distribution is essential for designs that marry mechanical principles with aesthetic appeal[8]. Thus, exploring non-uniform mesh subdivision methods that respond to internal force distributions is vital for enhancing structural efficiency and visual attractiveness, holding significant research and practical implications.

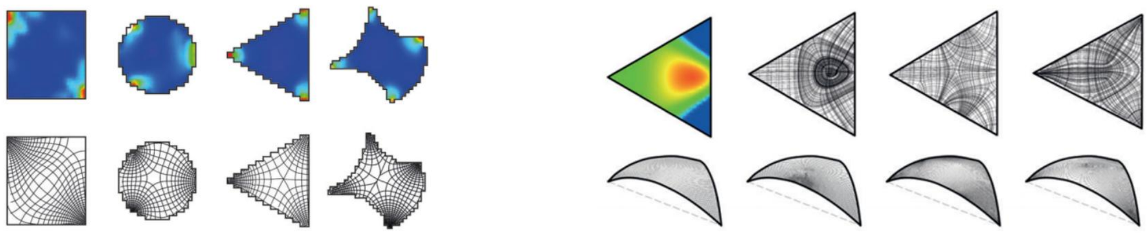

Figure 1: Patterns of shell form and force features. Left: Structural patterns derived from principal stress vectors in FE analysis. Right: Asymmetric shell with patterns derived from principal curvatures, gradients, principal moments, and principal stresses from left to right [8].

Building upon this, the study proposes a novel parametric texture generative design method that manipulates mesh density by leveraging internal force distributions within shells to create rich textures through grids and surface hollowing. By focusing on the non-uniform distribution of internal forces, this approach innovatively harnesses these forces to dictate the textural and structural composition of mesh grids. This design strategy enhances both the structural performance and the aesthetic appeal of architectural forms by exploiting detailed internal force data and morphological considerations to create dynamic, texture-rich surfaces. It appropriately manipulates mesh densities and patterns, using advanced computational tools and algorithms to overlay internal force patterns like principal stress lines and isostress clouds, which serve as the primary drivers for the subdivision process. This integration facilitates a deepened engagement with the materiality and structural logic, fostering a synergy between mechanical efficiency and visual richness. The potential of this methodology to revolutionize texture generative design is demonstrated through several case studies, illustrating how it surpasses traditional mesh subdivision techniques by achieving greater structural integrity and aesthetic diversity with equivalent material deployment. The findings not only underscore the method's practical and research implications but also highlight its potential to expand the boundaries of parametric design in architecture.

2. Background

2.1 Form generation Based on Internal Forces

The application of form-finding techniques in structural design, particularly those that utilize internal forces to shape form[8], represents a significant advancement in the field of structural engineering and architecture. Grounded in the principles of natural force, early pioneers like Michell[9] and Nervi [10]applied stress lines in structural design, thereby aligning structural forms with the natural flow of forces. This exploration of internal force-based form-finding techniques revolutionizes traditional design paradigms, contributing to both the theoretical discourse on structural morphology and the development of optimized, visually compelling designs.

With the advancement of computer-aided design tools, extensive research has expanded form generation methodologies to utilize internal forces effectively. Among the most notable methods are graphic statics[11] and topology optimization, which have closely linked form with force, significantly expanding the diversity of structural forms. However, the variability of structural forms is limited by the initial design constraints. Additional methodologies have also been explored to establish a connection between internal forces and structural form, leading to the optimization of structural efficiency and the creation of skin textures. Zomparelli and Naboni[13] have introduced a bio-inspired generative approach for designing isostatic ribbed slabs, leveraging anisotropic Reaction-Diffusion systems to harness principal stresses and moments, thereby facilitating the development of structurally efficient geometries featuring precisely controlled rib stiffener configurations. Cascone et al.[14] have introduced an innovative diagrid-like pattern for tall buildings that leverages principal stress trajectories, resulting in a structure that is not only highly efficient but also visually appealing due to its triangular units with variable angles, designed to enhance structural efficiency. Yu et al.[15] have expanded the application of Principal Stress Lines (PSLs), utilizing clustering algorithms for ordering and filtering stress lines, thereby demonstrating the diverse possibilities of arranging structural components based on stress line configurations. These studies transform traditional design paradigms by harmonizing structural forms with the natural flow of forces, thus enhancing both the discourse on structural morphology and the creation of optimized, visually engaging designs.

2.2 Structural Performance of Various Mesh Subdivisions

"Mesh subdivision" refers to the process of dividing a surface into smaller meshes, a technique widely applied to the gridshell of freeform roof surfaces and maintenance structures. To vividly demonstrate the significant impact of different subdivision patterns on stress distribution and overall structural behavior, a 2D rectangular beam with dimensions of 22m in length and 11m in height, supported asymmetrically at two points, is introduced. Controlling for the same material, cross-section, and boundary conditions, various mesh patterns are utilized for gridding. Under the consideration of self-weight load only, different mesh configurations significantly influence structural efficiency.

As shown in Figure 2 and Table 1, the structure experiences an equal self-weight of approximately 30kN. The outcomes from subdividing into triangular meshes (a, b, c), quadrilateral meshes (d, e), and voronoi polygon meshes (f) demonstrate significant differences in resistance to pressure and bending, due to the varied stiffness caused by different mesh subdivision patterns. For uniform triangular meshes, neither regular nor random subdivision patterns significantly impact the structural performance. However, distributing more material in areas of stress concentration can slightly reduce structural displacement, which is understandable. Interestingly, a notable difference exists in the displacement calculations between regular quadrilateral meshes (d) and quadrilateral meshes subdivided according to the trend of principal stress lines (f), as evidenced in Table 1.The variation in mesh subdivision alters structural behavior, thereby underscoring the importance of correlating mesh subdivision patterns with internal forces for structural form and optimization.

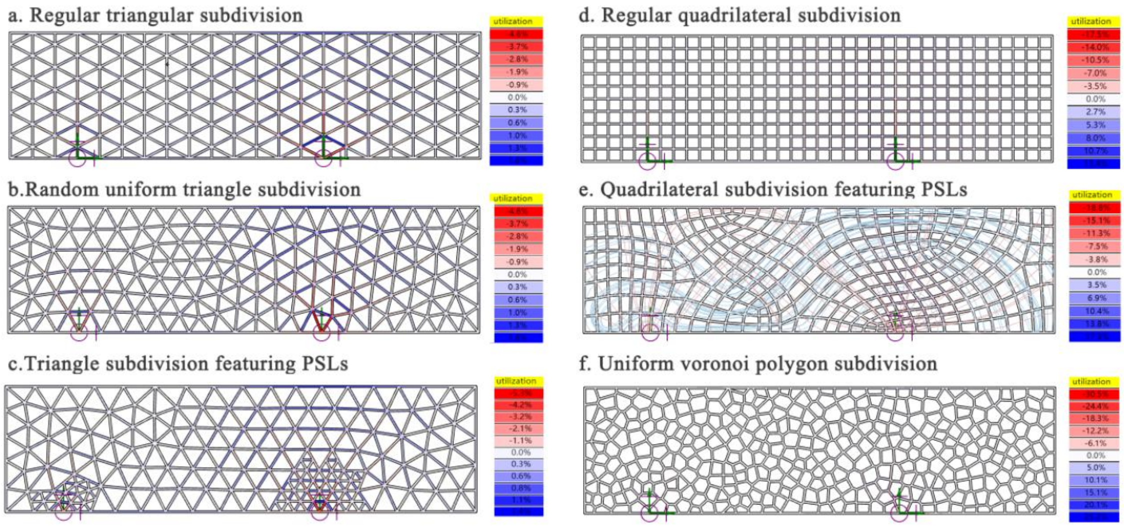

Figure 2: 2D mesh with different types of subdivision. (Gravity only)

Table 1: Comparative performance of 2D mesh with different types of subdivision

Meshing Model of 2D rectangular beam	Gravity (kN)	Maximum Displacement（mm）	Utilization (%)
a.Regular triangular subdivision	30.0	0.21	4.6
b.Random uniform triangle subdivision	30.4	0.24	4.6
c.Non-uniform triangle subdivision featuring Cloud Diagram	30.2	0.19	5.3
d.Regular quadrilateral subdivision	30.8	2.31	17.5
e.Quadrilateral subdivision featuring PSLs	30.0	1.02	18.8
f.Uniform voronoi polygon subdivision	30.0	0.85	30.5

Non-uniform mesh subdivision creates diverse structural surface patterns and textures; strategically placing materials on this basis can maximize structural and aesthetic benefits. In this study, the focus primarily lies on two issues: firstly, the extraction of internal force characteristics, which relates to the initial form of the structure and the order of construction components; secondly, using the information of internal forces as the basis for guiding the design of complex mesh patterns and spatial interfaces. In light of this, we employs parametric design tools (Grasshopper, Karamba, and Kangaroo) to propose a design strategy from the Inside-out theory. This approach aims to control material distribution by extracting features of internal forces, thereby enabling the creation of complex geometries and structurally efficient non-uniform meshes.

3. Methods

The key to mesh generation lies in the rational mapping between physical space and the mapping domain[1]. The process of generating meshes is illustrated in Figure 3. In parametric design, architectural concepts direct the initial design domain of the shell, including its relationship with the environment, the structural load conditions, material construction methods, and dimensions. Constraints and loads can be determined based on the structural function, thereby establishing a fundamental force field. Factors such as construction and spatial dimensions provide an original order for mesh generation. Finite Element Analysis (FEA) enables the acquisition of shell unit data. Internal forces can be visualized in two main forms: one is curve diagrams, such as isostress lines, principal stress lines; the other is cloud diagrams, which map internal force values to RGB colors, primarily including moment cloud diagrams and stress cloud diagrams. Architects can select appropriate internal force results based on the shell's primary resistance to internal forces, and then, combining construction and spatial requirements, determine the type of mesh subdivision and the initial geometric order.

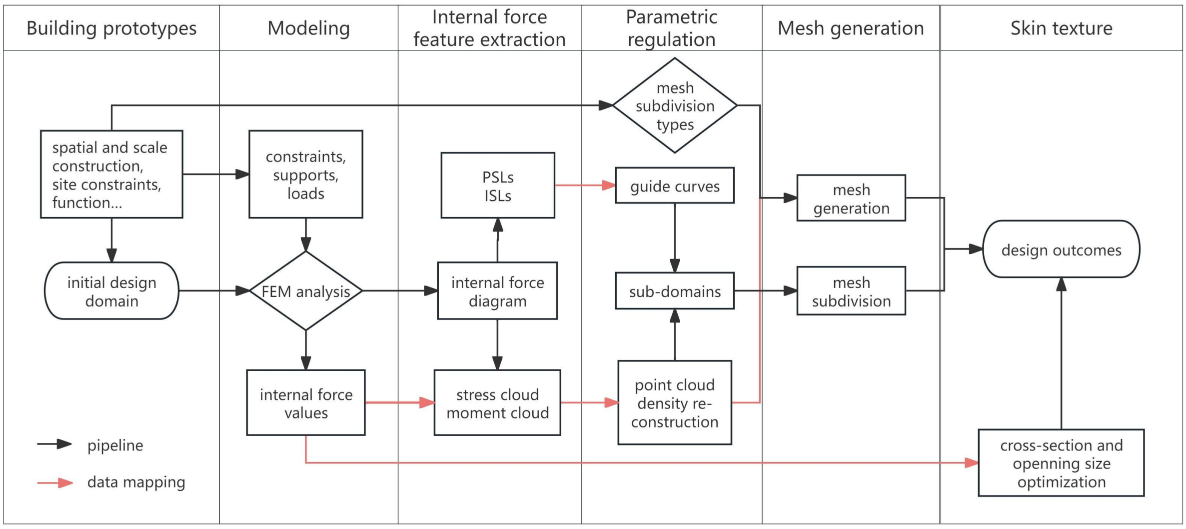

Figure 3: Process of mesh generation.

3.1 Mesh Generation Featuring Internal Force Curves

3.1.1.Principal Stress Lines（PSLs）

Principal Stress Lines reveal the areas and directions most susceptible to structural failure, as material typically fails along the path of maximum principal stress. Aligning materials with PSLs during design can significantly enhance the structure's efficiency and stability. In the context of mesh generation, using PSLs can result in varied grid patterns.

As shown in the middle of Figure 4, by employing the Quad-Remesh component within the Grasshopper software and using principal stress lines as guiding curves, varied quadrilateral grids are produced. To the right in Figure 4, a Quadtree algorithm is utilized to continuously reinforce the mesh along the direction of the PSLs. These two methods of mesh subdivision, referencing PSLs, provide a more complex pattern compared to traditional geometric triangular coverings or latitude-longitude grids.

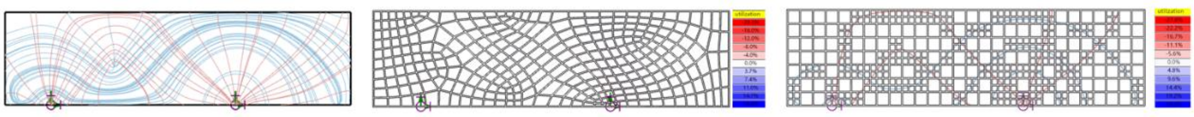

Figure 4:Mesh generation based on PLSs. (Conducting FEM analysis on the initial surface to extract PSLs, and then generating mesh with PSLs as feature curves using the Quad-Remesh algorithm and Quadtree algorithms.

3.1.2.Isostress Lines (ISLs)

Isostress Lines connect points within the structure that have the same stress value. Unlike the vector field represented by principal stress lines, they reflect the scalar field of internal forces. These lines reveal the concentration of stress and its gradient of change. Utilizing ISLs, the initial design domain can be subdivided into subdomains for grid reconstruction.

Figure 5 illustrates the ISLs of a 12m by 20m planar roof supported at three points under gravity load alone. By adjusting the spacing of the ISLs, simplified ISLs can be obtained. Following these curves, the surface is segmented, and the Quad-Remesh component is employed for mesh reconstruction, resulting in the grid pattern shown in the lower left of Figure 5. However, these mesh edges are discontinuous. By extracting the central points of the mesh faces and using polygons for reconstruction, a continuous mesh is formed, as shown in the lower right of Figure 5.

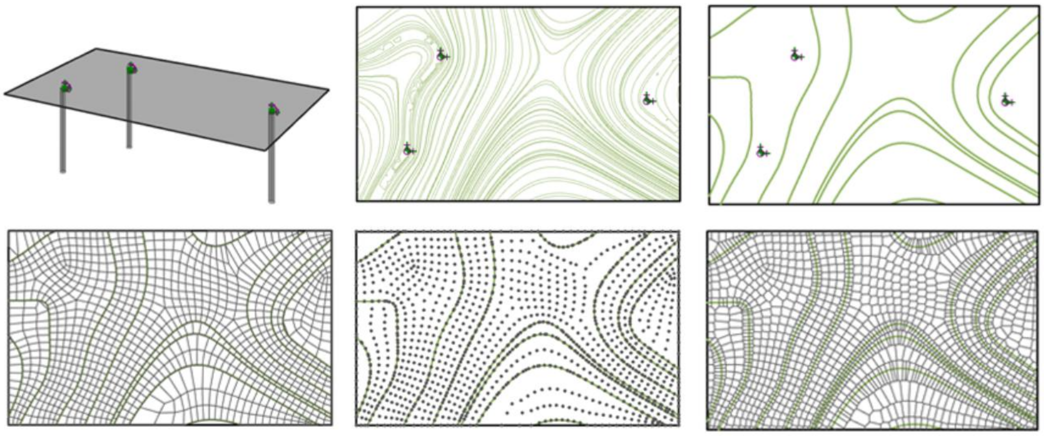

Figure 5: Remeshing based on ISLs for a triply supported planar roof.

3.2 Mesh Generation Featuring Cloud Diagram

Cloud diagrams, as a visualization result of finite element analysis, reflect the scalar field of internal forces. In the process of generating non-uniform meshes, the grid can be reconstructed based on cloud diagrams. Figure 6 illustrates the process of this method, initially extracting uniform discrete points on the shell surface, followed by subdividing the mesh into multiple levels of subdomains based on the stress variation areas indicated by the cloud diagrams. Within each subdomain, a mapping relationship is established between the point cloud density and the stress intervals.

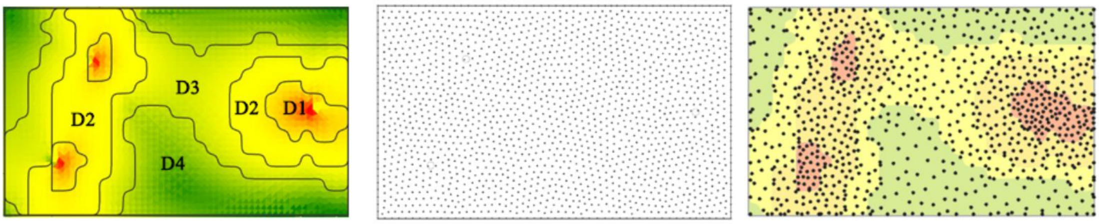

Figure 6: Subdomain division based on internal force cloud diagram for controlling point cloud density.

In this process, by mapping each point's stress value interval to the point's radius, points that are too close are eliminated, resulting in a point cloud with optimized density. The obtained point cloud is then used for further mesh subdivision, where the mesh is not necessarily in ideal triangular or polygonal forms. The from Kangaroo can be applied to exert linear elastic forces on mesh elements, achieving a smooth transition between areas of high and low density. As shown in Figure 7, the mesh in the middle undergoes bouncy optimization, leading to a more smooth transition in the mesh depicted on the right.

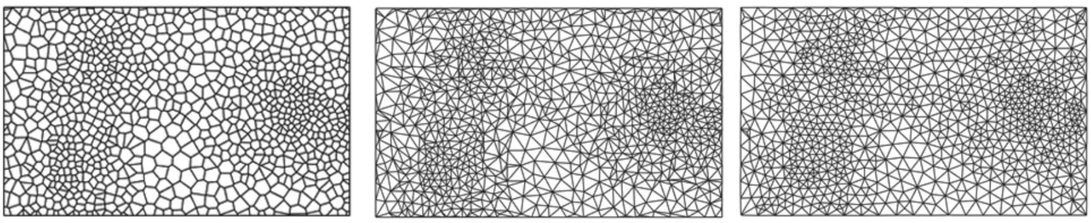

Figure 7: Results and optimization of mesh generation.

4. Case Study and evaluation

To validate the applicability and necessity of generating mesh morphologies based on internal forces, this paper demonstrates the impact of non-uniform meshes on the structural form and mechanical performance through representative case studies in shell design, utilizing the proposed method.

4.1 Free-form shell non-uniform subdivision

Free-form surfaces are increasingly utilized in architectural form design. Parametric tools such as Rhino Vault or Kangaroo can identify ideally compressed surfaces, for example, through the hanging method. However, little attention has been paid to whether the density variation of the initial mesh used for form-finding becomes more rational after deformation. Figure8 shows the result of generating a simple free-form shell surface with Kangaroo, where it can be observed that the triangular mesh sizes near the supports are significantly larger than those in the middle or at the edges, which appears counter intuitive to structural logic. When we reconstruct the mesh using Quad-Remesh, the result on the left side of Figure 9 is produced, whereas Quad-Remesh featuring principal compression stress lines yields a different mesh pattern on the right side of Figure 10. Near the supports, the mesh grids form an arch shape with the density altered.

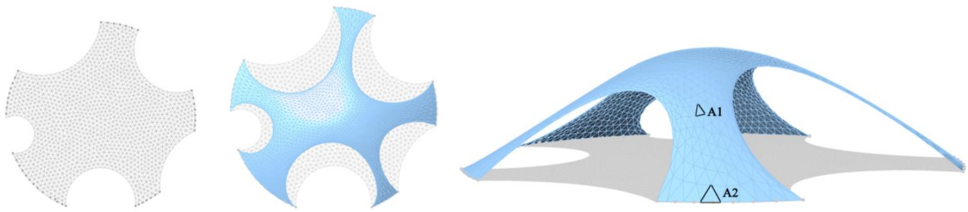

Figure 8: Results and optimization of mesh generation.

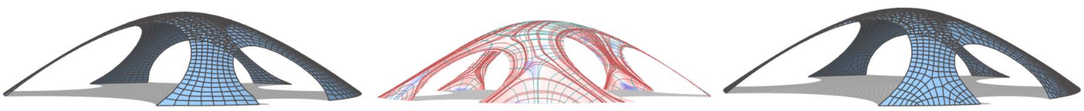

Figure 9: Mesh rebuilt featuring PSLs.

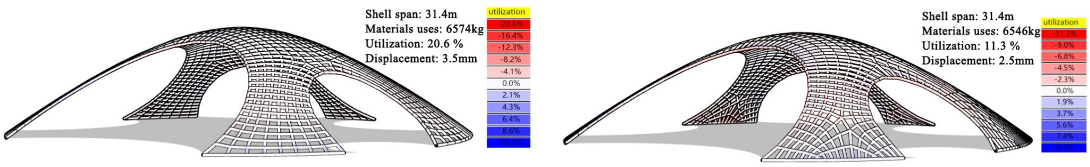

Figure 10: Comparison of structural performance of quadrilateral mesh results. (Gravity only)

The Stress Cloud Diagram (SCD) can also serve as a foundation for generating meshes on free-form shell structures. Figure 11 illustrates the results obtained by employing a fractal algorithm and performing secondary subdivision in areas of stress concentration to achieve grids b, as well as grids c generated through the point cloud density adjustment method described in section 3.2. Both methods are capable of producing non-uniform grids on the surface.

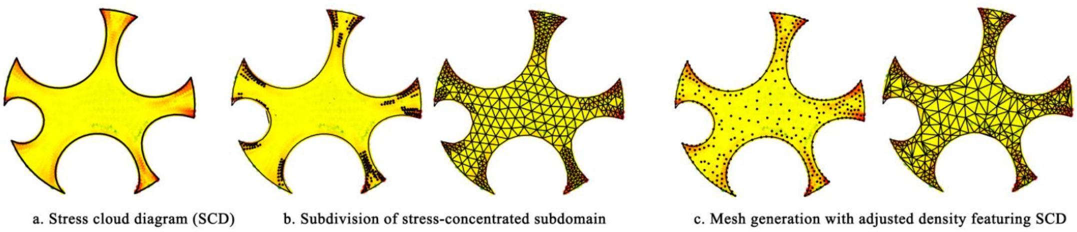

Figure 11: Mesh generation results from stress cloud diagram.

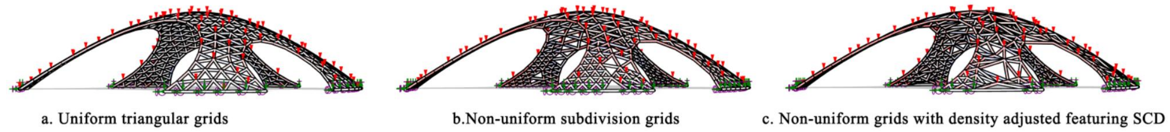

Figure 12: Comparison of structural performance between uniform and non-uniform triangular grids under vertical & gravitational loads.

Table 2: Comparative performance of uniform and non-uniform triangular grids

Meshing Model of 2D rectangular beam	Mass (kg)	Maximum Displacement （mm）	Utilization (%)
a. Uniform triangular grids	5380	11.2	62.9
b.Non-uniform subdivision grids	5263	6.8	23.7
c.Non-uniform grids with density adjusted featuring SCD	5323	7.1	24.3

Figure 2 and Table 1 have already demonstrated the structural advantages of triangular grids, yet shells with local density variations show no significant advantage over uniform grids under self-weight alone. To address this, Figure 12 depicts a hundred vertical downward point loads uniformly applied across the shell surface, as indicated by the red arrows, to approximate a uniformly distributed load of 2 kN/m², while minimizing variations in material usage. The structural performance outcomes, as shown in Table 2, indicate that both types of non-uniform grids significantly enhance structural efficiency and reduce maximum displacement. Furthermore, the non-uniform distribution of materials leads to a visual perception of discontinuity in the shell's structural form pattern, adding a sense of depth and layering to the space visually.

4.2 Internal force-driven surface texture design

To validate this approach for continuous shell interfaces, this paper employs the Bruges Pavilion[17] as a design scenario, briefly demonstrating the diversity brought by the mesh generative strategy in surface texture design. As shown in Figure 13, the shell coverage varies due to changes in supports (potentially determined by site conditions), resulting in different internal force patterns. Cloud diagrams reflecting stress, bending moments, and displacements indicate varied concentrations of the internal force field. In structural design, it is necessary to consider both the resistance to bending moments due to surface domain transitions and the convergence of compressive stresses near supports. Thus, when controlling point cloud density based on cloud diagrams, different internal force fields can be overlaid. The selection of mesh subdivision types leads to the final mesh grids.

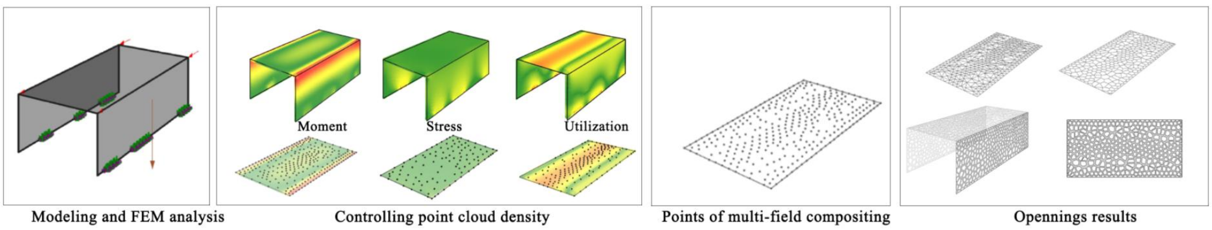

Figure 13: The process of surface texture design driven by internal forces

Figure 14 shows the uniform mesh subdivisions at the top (abc) and the non-uniform subdivisions adapted to internal forces at the bottom. Among them, Figure 14d is generated from an optimized point cloud, and on the basis of triangular grids, polygonal grids can be created. The calculation results in Table 3 verify that this method ensures structural efficiency across different types of mesh subdivisions. The generated patterns can be used in subsequent design phases to perform hollowing or optimize component cross-sections of the shell, forming structurally sound and aesthetically pleasing textures.

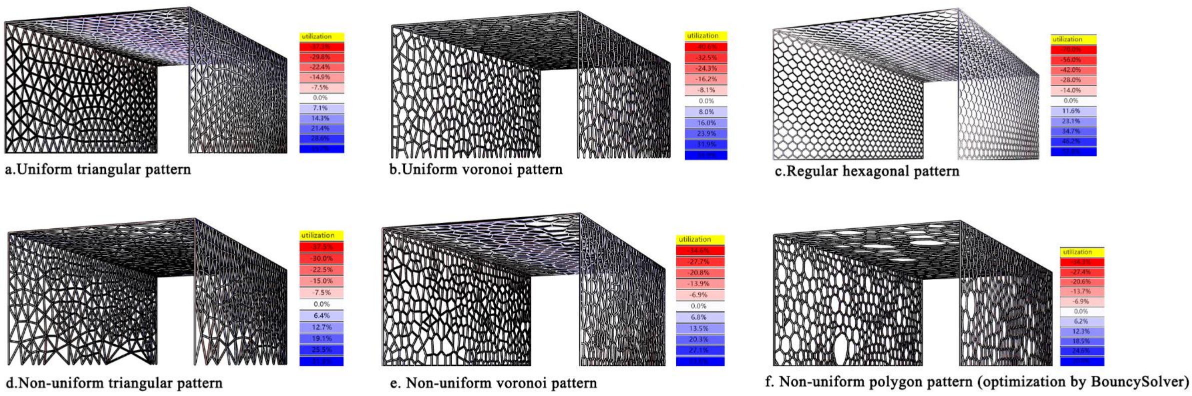

Figure 14: Verification of structural performance with different surface textures under gravitational loads.

Table 3: Comparative Performance of uniform and non-uniform grids

Results	Mass (kg)	Maximum Displacement （mm）	Utilization (%)
a.Uniform triangular pattern	2216	55.6	37.3
b.Uniform voronoi pattern	2182	68.6	40.6
c.Regular hexagonal pattern	2572	86.1	70.0
d.Non-uniform triangular pattern	2835	54.6	37.5
e. Non-uniform voronoi pattern	2216	55.6	34.6
f. Non-uniform polygon pattern	2058	55.3	34.3

5. Result and discussion

This paper unveils the structural differences among various types of mesh subdivisions and introduces novel methods for generating meshes, utilizing internal force data to guide the creation of non-uniform mesh subdivisions for shell structures. Leveraging a parametric platform, internal forces are visualized, and non-uniform meshes are produced through methods including mesh subdivision guided by internal force curves, subdivision of subdomains, and the reconstruction of point cloud densities based on stress cloud diagrams.

Our findings indicate that strategically applying stress cloud diagrams and stress lines as guiding elements for grid reconstruction can optimize structural configurations. Through case studies and structural validations, this method has proven to enhance structural efficiency and integrate structural form with formal aesthetics effectively. Non-uniform mesh subdivision is adaptable to various grid forms and can be applied to both planar and curved shells, and it is foundational for designing and optimizing surfaces with openings, integrating different internal force representations into the texture of spatial interfaces. Future research will expand the internal force-guided design strategy to more complex scenarios, exploring further how internal forces can influence external forms.

6. Conclusion

This study introduces an innovative parametric design method for non-uniform mesh subdivisions guided by internal forces, significantly enhancing structural efficiency compared to traditional approaches. Furthermore, we underscore the potential of the internal force-guided design strategy to foster a symbiotic relationship between structural integrity and architectural beauty. The adaptability of our methods across different shell forms—planar or free-form surfaces—highlights its versatility and applicability in contemporary architectural design. This endeavor not only contributes to the field of structural morphology but also opens new avenues for innovation in parametric architectural generation, promising a future where design is seamlessly aligned with the principles of natural force distribution and aesthetic expression.

References

[1] Baker T J. Mesh generation: Art or science?[J]. Progress in aerospace sciences, 2005, 41(1): 29-63.

[2] Wang Q, Ye J, Wu H, et al. A triangular grid generation and optimization framework for the design of free-form gridshells[J]. Computer-Aided Design, 2019, 113: 96-113.

[3] Ju C, Zhang J, Xiao R, et al. Automatic surface mesh generation by a binary-tree method[J]. Engineering Analysis with Boundary Elements, 2023, 152: 473-495.

[4] Reichel R, Klinkel S. A non-uniform rational B-splines enhanced finite element formulation based on the scaled boundary parameterization for the analysis of heterogeneous solids[J]. International Journal for Numerical Methods in Engineering, 2023, 124(9): 2068-2092.

[5] Jaillet F, Lobos C. Fast Quadtree/Octree adaptive meshing and re-meshing with linear mixed elements[J]. Engineering with Computers, 2022, 38(4): 3399-3416.

[6] Remacle J F, Lambrechts J, Seny B, et al. Blossom-Quad: A non-uniform quadrilateral mesh generator using a minimum-cost perfect-matching algorithm[J]. International journal for numerical methods in engineering, 2012, 89(9): 1102-1119.

[7] Weyer S, Fröhlich A, Riesch-Oppermann H, et al. Automatic finite element meshing of planar Voronoi tessellations[J]. Engineering fracture mechanics, 2002, 69(8): 945-958.

[8] Adriaenssens S, Block P, Veenendaal D, Williams C. Shell structures for architecture: form finding and optimization[M]. Routledge, 2014.

[9] Michell A G M. LVIII. The limits of economy of material in frame-structures[J]. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 1904, 8(47): 589-597.

[10] Halpern A B, Billington D P, Adriaenssens S. The ribbed floor slab systems of Pier Luigi Nervi[C]//Proceedings of IASS Annual Symposia. International Association for Shell and Spatial Structures (IASS), 2013, 2013(23): 1-7.

[11] Bertagna F, Schwartz J, D'Acunto P. Graphical methods as the key to holistic design: bringing together structural design and solar control strategies[C]//Proceedings of IASS Annual Symposia. International Association for Shell and Spatial Structures (IASS), 2022, 2022(8): 1-8.

[12] Kamyab H, Yeganeh M, Bolhassani M. Structural muqarnas: Reconstructing muqarnas using graphic statics[C]//Proceedings of IASS Annual Symposia. International Association for Shell and Spatial Structures (IASS), 2022, 2022(10): 1-11.

[13] Zomparelli A, Naboni R. Generative design of isostatic ribbed slabs using anisotropic Reaction-Diffusion[C]//Proceedings of IASS Annual Symposia. International Association for Shell and Spatial Structures (IASS), 2023, 2023(18): 1-12.

[14] Cascone F, Faiella D, Tomei V, et al. Stress lines inspired structural patterns for tall buildings[J]. Engineering Structures, 2021, 229: 111546.

[15] Yu Z, Dai H, Shi Z. Designing truss in architecture: Method and applications based on clustering algorithms and principal stress lines[J]. Computer-Aided Design, 2022, 151: 103330.

[16] Danhaive R, Mueller C. Structural Patterning of Surfaces[C]//Humanizing Digital Reality: Design Modelling Symposium Paris 2017. Springer Singapore, 2018: 559-572.

[17] Buntrock D. Toyo Ito and Masato Araya's experiments in the structural use of aluminium[J]. The Journal of Architecture, 2016, 21(1): 24-54.