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Bending-active molds for pre-fabricated concrete shells

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Abstract

This paper introduces a method for prefabricating curved concrete shells using fabric formwork. This formwork is supported by geodesic grids composed of bamboo lamellae or other materials that are elastic and strong, to maintain the shape of the formwork during fabrication. This approach offers a practical alternative to traditional, resource-intensive formworks for curved structures, addressing their inherent inefficiencies. Our research focuses on fabricating large, thin, curved concrete panels by dividing the curved shell into manageable patches, setting it apart from techniques that rely on small, planar elements. Each patch is designed for off-site fabrication followed by on-site assembly. These patches are formed by applying concrete onto fabric formworks, tailored and stitched to match the specific curvature of the shell. The fabric formwork is supported by robust frames and a grid that traces the geodesic curves of the surface, suspended beneath the frame. The geodesic grids supporting these patches are optimized based on the shell's curvature and the elastic properties of the bamboo lamellae. By precisely positioning and connecting the lamellae at the vertices of the geodesic grid, we ensure that the fabric formwork is constrained by the grid, thereby achieving the intended shape.

Keywords: Bending-active mold, gridshell structures, geodesic grids, fabric formwork, pre-fabricated concrete shells.

1. Introduction

Shells are efficient structures ideal for large construction due to their ability to cover wide areas with minimal materials (Rombouts et al. [1]). However, traditional rigid formwork for concrete shells is costly and time-consuming, often requiring customized components (Li et al. [2]). In response, research has shifted towards more sustainable flexible formwork systems, such as fabric formwork, which are less resource-intensive and reusable (West [3]). This approach uses a flexible membrane to shape the concrete, potentially supported by rigid materials, wires, or bending-active elements to maintain shape and prevent deformation (Rombouts et al. [1]). The following sections will explore the application of fabric formwork and bending-active gridshells in architectural practices.

1.1. Fabric Formwork

Fabric formwork is renowned for its lightweight and easy-to-transport nature, allowing both on-site and off-site fabrication (West [3]). It offers a cost-effective and environmentally friendly alternative to traditional formwork due to the reusability and recyclability of many fabric types, reducing construction costs and greenhouse gas emissions. This method facilitates the creation of organic shapes with natural tension geometries that are difficult to achieve with rigid molds. Fabric formworks are also tear-resistant and non-adherent to concrete, making the demolding process easier (Wagiri et al. [4]). In the 1970s, architect Miguel Fisac used fabric formwork for the Juan Zurita residence, creating precast panels with rope and plastic sheeting (Figure 1). Recent advancements have been made by researchers like Professor West from the University of Manitoba, who developed methods for creating concrete elements suitable for both precast and in-situ construction (West [3]). Experiments by West's students at the CAST laboratory (Schmitz [5]) and by students at the National Taiwan University of Science and Technology (NTUST) have further demonstrated the adaptability of fabric formwork (Wagiri et al. [4]), particularly for creating precast wall panels (Figure 1). A primary challenge with fabric formwork is managing the deformation under the weight of wet concrete. To address this, bending-active structures, such as geodesic grids, can be integrated. These act as molds and structural supports to produce lightweight, material-efficient concrete panels with complex geometries.

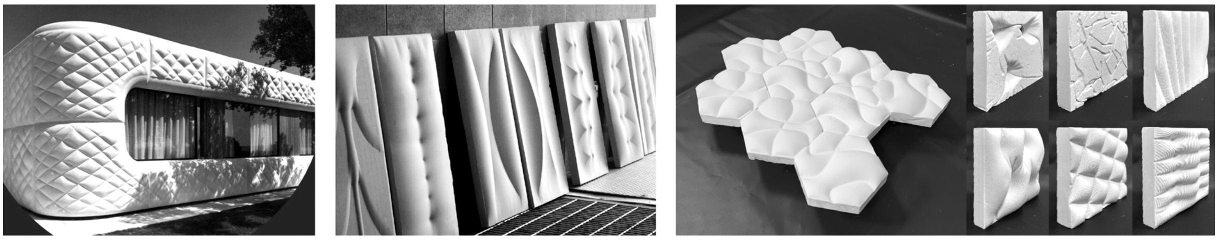

Figure 1: Juan Zurita residence by Miguel Fisac [5], precast concrete panels by CAST laboratory students [5], and fabric-formed gypsum panels by NTUST students [4]

1.2. Bending-Active Gridshells

Bending-active gridshells are structures that transform flat or straight elements into complex, curved shells, making them ideal for flat-packed transport and on-site shaping. This approach simplifies construction by eliminating the need for custom joints and enhancing material reusability. rei Otto's 1970s work on the Multihalle Mannheim project, using timber laths, demonstrated the potential of gridshells for creating large, self-supporting structures (Liddell [6]). To ensure stability and desired shape, gridshells can be reinforced with diagonal members, tensioned cables, or membranes. Bamboo is particularly favored for these structures due to its strength, rapid growth, and environmental benefits (Ma et al. [7]), though other materials with similar elastic properties can also be used to achieve a range of curvatures. Combining bending-active elements with form-active membranes not only broadens the architectural possibilities but also enhances structural stability. Figure 2 shows examples of shells that incorporate bending-active grids or frames with textiles.

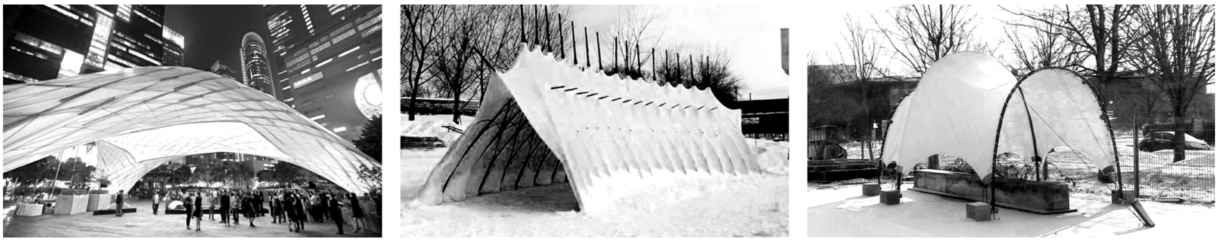

Figure 2: ZCB bamboo pavilion [8], Cocoon fabric-formed ice shell [9], FAB shell [10]

2. Methodology

This section presents a design framework for creating prefabricated concrete shells using bending-active molds (Figure 3). The goal is to fabricate thin, curved concrete panels off-site and then assemble them on-site into a curved shell structure. The process starts with designing the shell surface, then dividing it into smaller, manageable patches. Each patch is reinforced with geodesic grids made from flexible materials like bamboo lamellae, precisely positioned and interconnected at the grid vertices. These grids are optimized based on the shell's curvature and the bamboo's elastic properties to ensure a close fit to the intended shape. Following this, each patch is covered with a fabric membrane. To minimize potential wrinkles and folds, the fabric membrane is cut into segments and then tailored, ensuring it closely conforms to the panel's curvature before being securely fastened along the panel's perimeter. This fabric formwork is held in place by robust frames that follow the geodesic curves underneath, matching the design precisely. When the concrete is cast, the fabric and grid are stretched to their limits, ensuring the final shape accurately represents the planned design.

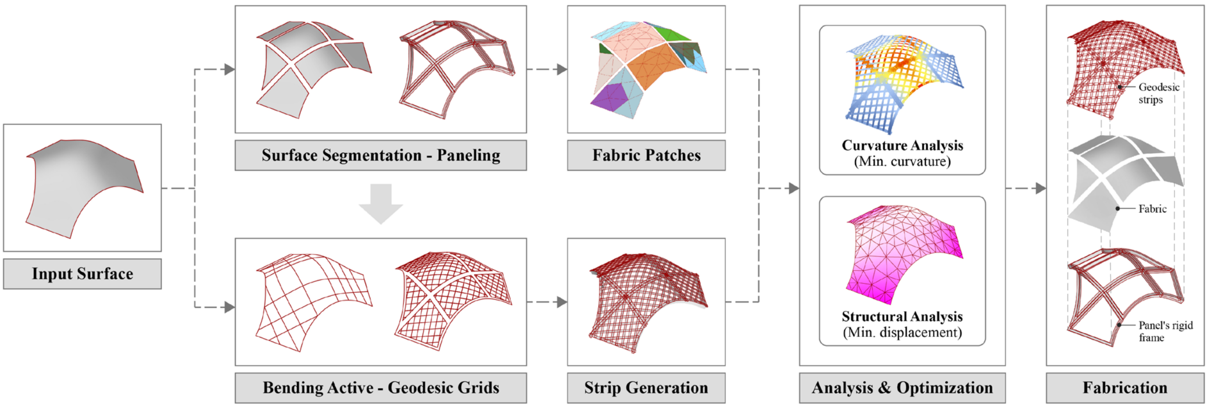

Figure 3: Research framework

2.1. Shell Designs

A shell structure can be either a continuous or a segmented surface. Continuous surfaces involve fewer joints, leading to smoother load paths and potentially enhancing global stiffness and stability. However, manufacturing and transporting large continuous elements can be challenging, requiring high precision in fabrication and assembly. In response, a segmented surface approach can be adopted, dividing the shell along geodesic or principal curves to maintain a smooth appearance. This method shifts fabrication to offsite prefabrication of panels or segments, offering flexibility in design and construction, easier manufacturing, transport, and onsite assembly. Additionally, it allows for the optimization of each segment and facilitates localized repairs, as damaged segments can be easily replaced or repaired. Despite these advantages, segmented shells introduce challenges such as potential weakening at joints and connections, more complex structural analysis due to discontinuities, and less uniform stress distribution. To enhance the performance of segmented shells, various joint types can be employed, including mechanical fasteners (e.g., bolts, screws, rivets), welded joints for metal structures, interlocking geometries, and tensioned cables or rods. The choice between continuous and segmented shells depends on factors like the structure's scale, manufacturability, transport feasibility, and the complexity of the shell's geometry.

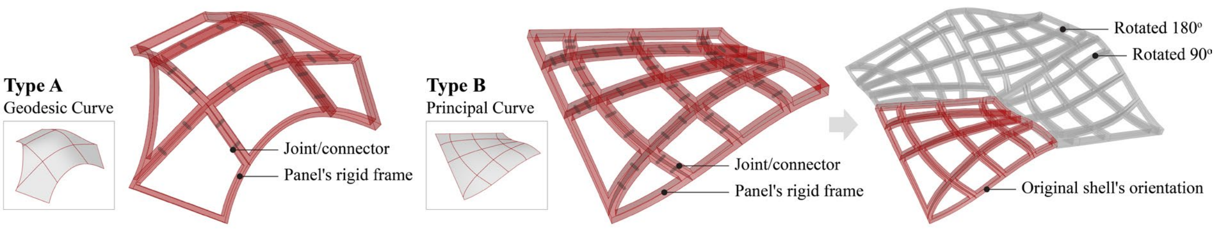

Figure 4: Segmentation strategies using geodesic curves for shell Type A and principal curves for shell Type B

In this study, we explore two distinct designs, Type A and Type B, as shown in Figure 4. Type A features a double-curved surface derived from form-finding simulations, resulting in an asymmetrical shape. In contrast, Type B is characterized by an osteomorphic surface (Dyskin et al. [11]), distinguished by a pattern of convex and concave curves shaped by a cosine wave. A notable feature of Type B's concave-convex surface is its stackability; by rotating it 90, 180, and 270 degrees, it can seamlessly fit together to create a continuous surface. For segmenting the shell surfaces into smaller, manageable patches, Type A utilizes geodesic curves, which trace the shortest path between points on a surface. The networks of geodesic curves vary in density and intersection angles, and while their start and end points can be chosen, their paths cannot be predetermined (Schling et al. [12]). This makes it challenging to divide the panel size symmetrically and equally based on geodesic curves. Conversely, Type B's segmentation is based on principal curves. Pottmann et al. [13] introduced the principal curve method to semi-discretize surfaces into developable strips, where the edges of these strips align with the surface's lines of maximum curvature, and rulings are oriented along lines of minimal curvature.

Many studies suggest segmenting surfaces into principal patches, which typically result in flat quads with limited control and are excessive for strip-based materials like bending-active structures. Instead, principal curves are more effective for constructing freeform surfaces (Chai et al. [14]). To generate principal curves on the surface of a shell, start by selecting an initial point and calculating the principal curvatures and directions at that point. From the initial point, move in the direction of one principal curvature by a specified step size to determine the next point. Continue this iterative process, calculating each new point in the same way, and connect these points sequentially to form the principal curve. If the principal curve forms a closed loop, calculate the distance between the new point and the starting point at each step. If this distance is less than the step size, close the curve directly to ensure a smooth loop. Balancing automated generation with manual intervention is essential when applying principal curves.

2.2. Grid Networks

Our study integrates networks of geodesic curves on shell segmentations. Although geodesic grids can be individually optimized for each surface segment, we aim to maintain the appearance of continuous grids across adjacent panels, which will create imprints on the formwork. Geodesic curves are selected for their ability to connect two points on a surface by taking the shortest path without lateral curving, also known as zero geodesic torsion (Schling et al. [12]). This is beneficial for materials like plywood or bamboo, which bend flexibly in one direction while remaining stiff in the other. Strips generated from geodesic curves can be projected onto a flat plane without distortion, stretching, or tearing. We identified two types of developable strips: one behaves like freely twisting strips, creating an arc-like line when unrolled; the other follows a geodesic curve, lying tangent to the surface (Figure 5). At intersections of the grids, they share a common normal vector, allowing for efficient rotational joints.

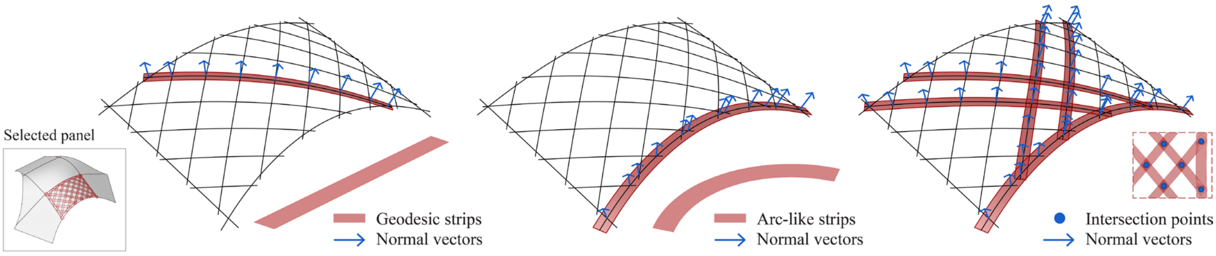

Figure 5: Developable strips based on geodesic and arbitrary curves (illustrated on Type A's panel)

To find the most effective geodesic grid patterns, we conducted a multi-objective optimization (Wagiri et al. [15]). This process involved adjusting the starting and ending points of the geodesic grids to evaluate their distribution across the surface. The optimization prioritized configurations with lower curvature and minimal displacement, which served as the fitness objectives. Figure 6 presents the curvature analysis using a color gradient. Blue areas represent regions of lower curvature where the lamellae are nearly flat, indicating less stress and a lower risk of breakage. In contrast, red areas indicate regions of higher curvature, which require more bending and therefore pose a higher risk of lamella failure. The curvature and grid distribution are significantly influenced by the shell's segmentation.

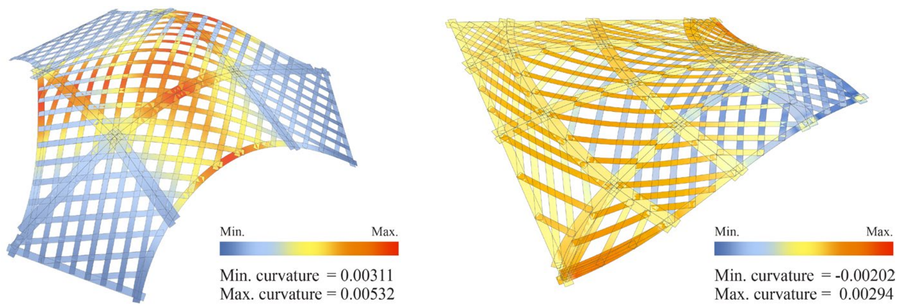

Figure 6: Type A has higher curvature concentrated in the middle panels, while Type B maintains a moderate curvature across its entire surface, indicated by the widespread yellow-to-orange coloration

The grids in our study are mechanically bi-stable, designed to transform flat materials into curved shapes through snap-through buckling (Huang et al. [16]). This process involves the system transitioning between two stable configurations, as shown in Figure 7. Similar to the auxetic structures studied by Huang et al. [16], when an external force is applied, the components start to bend. Upon reaching a critical threshold, the system rapidly switches to an alternate stable state. The post-buckling curve in Figure 7 illustrates the valid equilibrium states of the system after it surpasses the critical point on the load-deflection curve. These equilibrium states indicate the stable configurations the system can maintain through the snap-through buckling process, facilitating further fabrication methods.

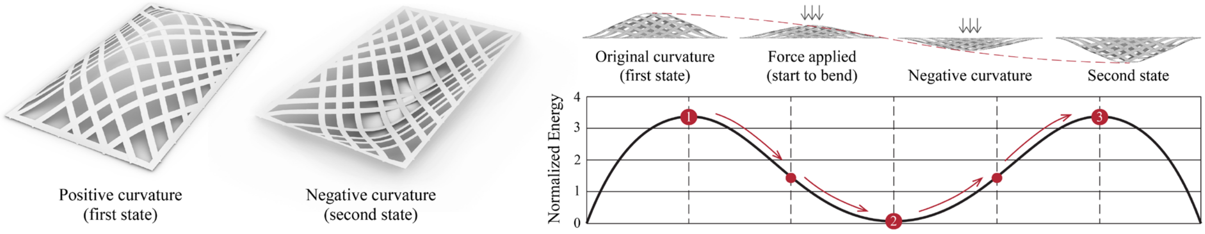

Figure 7: Geodesic strips provide a bi-stable structure

2.3. Fabric Patches

Selecting the right fabric membrane is essential, especially for covering complex curved surfaces. When the fabric is not elastic, it is recommended to divide it into patches (Figure 8). This method helps avoid wrinkles or unwanted folds as the concrete hardens. On the other hand, if the fabric is elastic, it can simply be stretched to conform to the shell's intended curvature. We utilized the Ivy plugin to segment and unroll the patches. The simulation results demonstrate that surfaces with higher curvature demand a greater number of patches, which can complicate the assembly process. Thus, it is essential to have detailed drawings and clear notations to ensure that each segment is correctly tailored.

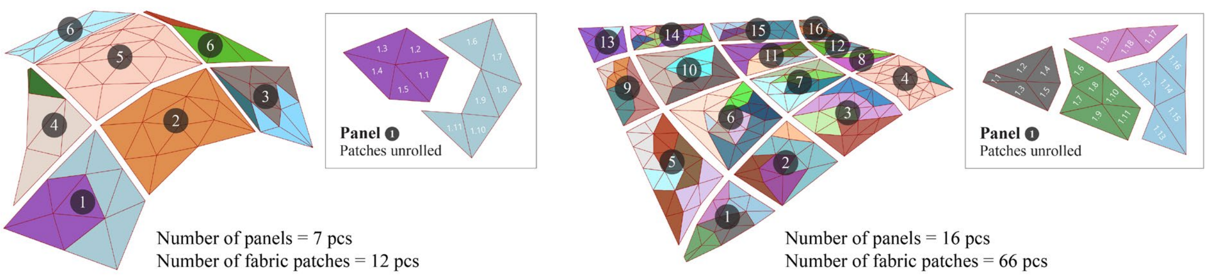

Figure 8: Fabric patches for curved surfaces (unrolling one panel as an example)

2.4. Simulation

We use Kangaroo to simulate the effect of wet concrete's weight on the fabric during fabrication. This simulation demonstrates the emergence of concrete patterns on the shell surface, influenced by the underlying geodesic grids (Figure 9). By modeling the concrete's weight distribution, Kangaroo predicts the fabric will deform, providing insights into the material's behavior. Although these simulations might not offer exact predictions due to varying fabric properties, they help us understand deformation patterns and make informed decisions to enhance the aesthetics and functionality of the structures. Additionally, these simulations facilitate design refinement, such as adjusting grid distribution to prevent overly large bulges caused by excessive grid spans.

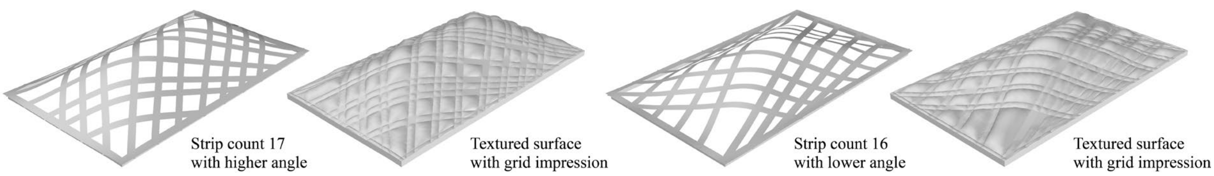

Figure 9: Inflated surface simulation of precast panels with two different grid distributions

2.5. Physical Testing

In our ongoing experiments, we used scaled gypsum models for preliminary studies and constructed geodesic grids with 0.3 mm thick bamboo sheets and 0.5 mm thick wax-coated paperboard. Panel frames, which require rigidity and strength, would typically use materials like timber or metal. However, for this case study, we used 3D printing to create the frames. The prototyping process is divided into three main stages: drawing preparation, prefabrication, and assembly. During the drawing preparation stage, we focus on creating production drawings, which include marking joint locations at the curve intersections and unrolling the bending lamellae onto the XY plane for two-dimensional drafting. The goal is to maximize the utilization of the bamboo or paper sheet for cutting and add notations to identify the order and orientation of the lamellae.

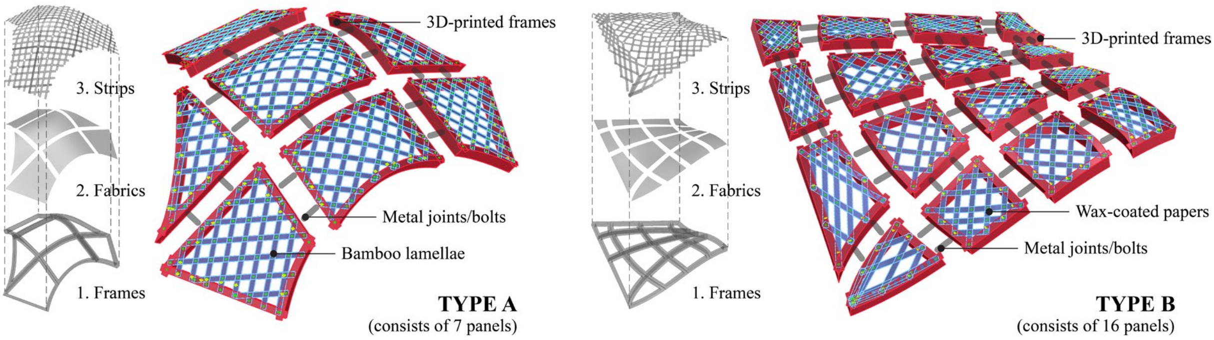

Figure 10: Assembly details of Type A (using bamboo lamellae) and Type B (using wax-coated papers)

In the prefabrication stage, the main objective is to construct precast panels. This involves marking, cutting, aligning, and pre-drilling the lamellae. The lamellae are then fastened with screws at their intersections and attached to the rigid frame using M5 bolts, ensuring the panels remain strong and rigid along one axis while allowing flexibility along the other for bending and twisting without causing damage. Ensuring the correct sequence, positioning, and orientation of the geodesic strips is crucial, so they are carefully labeled and marked according to the fabrication data. The grids are then covered with an elastic fabric tailored for double-curved surfaces, eliminating the need for fabric segmentation and ensuring a seamless covering over the panels. Figure 10 presents the assembled digital models for Types A and B. Bamboo was chosen for Type A and wax-coated paper for Type B. During assembly, it is recommended to assemble the panels sequentially, either row by row or column by column, rather than on a panel-by-panel basis. This approach streamlines the installation process, especially for shells with many panels in a single row or column, and is particularly useful when installing panels at sharp angles, as seen in Type B.

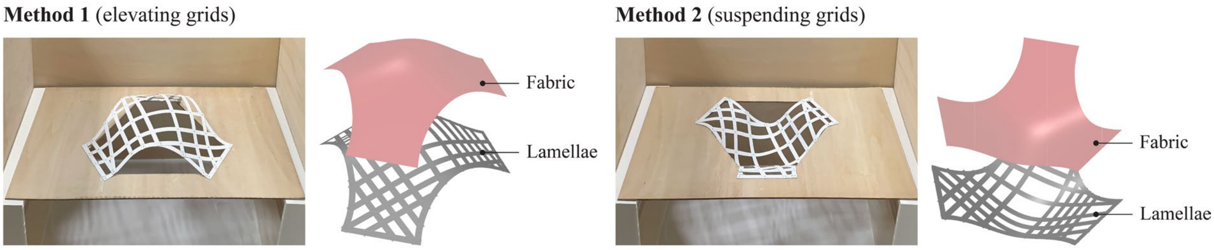

Figure 11: Two fabrication methods for precast panels. This study employed the second method, utilizing suspended grids, for the per-fabrication of the shell panels

Figure 11 illustrates two methods for casting panels. The first method uses planar lamellae, elevated to achieve the desired curvature. In this method, the lamellae must be strong enough to withstand the weight of the wet concrete or gypsum. The second method involves suspending geodesic grids and draping fabric over them, allowing gravity to shape the concrete as it is poured. These methods result in different finishes: the first method produces a smooth exterior, while the second creates a pattern reflecting the underlying grids. When choosing the most appropriate fabrication method, it is important to consider factors such as the size of the structure and the logistics of assembly, including whether it will occur offsite or onsite, as these affect transportation requirements.

A common challenge with pouring gypsum is that the center of the suspended panel tends to accumulate more material, becoming thicker than the edges. To manage this, it is essential to control the pouring sequence to limit thickness and prevent formwork deformation. This is typically achieved by applying the gypsum in layers, progressively increasing in stiffness and weight, starting at the shell's corners or edges and moving towards the center.

3. Results

In our preliminary tests, we used simple shapes to cast curved panels (Figure 12). These experiments revealed aesthetic possibilities by displaying the imprints of the formwork and cushioning on the shell's surface. However, we observed discrepancies from the digital models due to factors like uneven fabric stretch during gypsum casting, variations in lamellae dimensions, inaccuracies in joint locations, and differences in fabric properties. Despite these challenges, we considered these deviations an integral part of the system, demonstrating the method's success in terms of overall behavior and outcome.

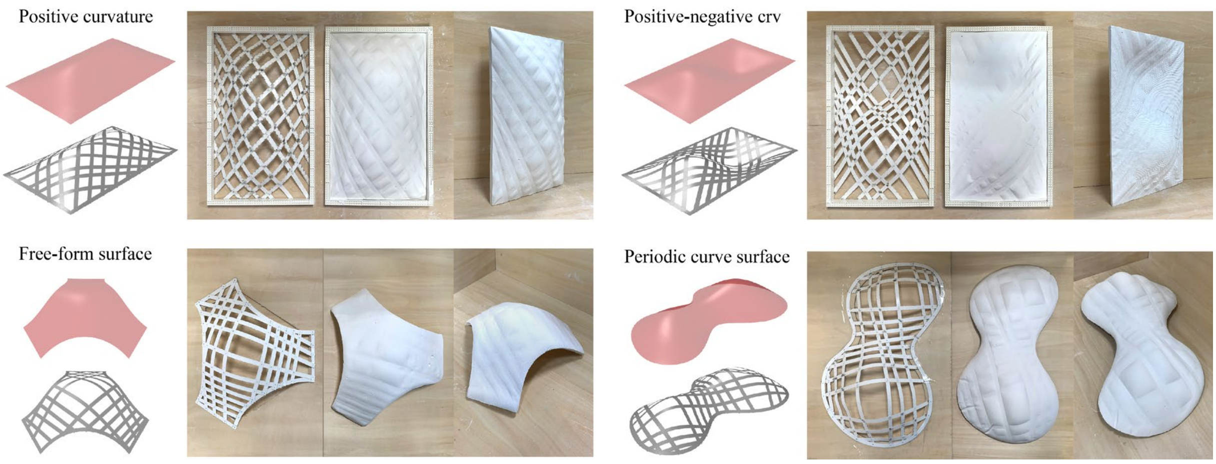

Figure 12: Preliminary tests using gypsum and wax-coated papers

Deciding on the surface finish was an important aspect of our project. For panels with uneven grid distributions leading to random bulge patterns and sizes, we considered two approaches. One option was to apply an additional layer of concrete or gypsum over the grids to achieve a smooth surface, hiding the grids but making the panels thicker and requiring additional structural supports for the increased weight. Alternatively, we could leave the grids and fabrics as stay-in-place formwork, preserving the textured appearance. For panels with a uniform grid distribution that produced a visually appealing finish, we recommended removing the molds to maintain this aesthetic (Figure 13). This approach allowed us to showcase the imprint patterns formed by the grids, enhancing the structure's visual appeal.

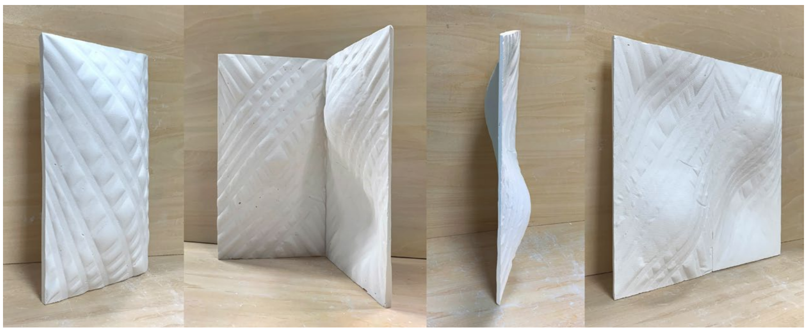

Figure 13: Results of preliminary tests reveal the influence of underlying grid structures on patterns and textures of the finished surface, once fabrics and grids have been removed

Figure 14: Fabrication results for Type A before and after mold removal

Figure 15: Fabrication results for Type B before and after mold removal

Another issue is the thickness of the panels. Variations in panel thickness occur due to the difficulty in controlling the exact thickness while pouring the concrete or gypsum. Even with careful marking, slight differences are inevitable. The robust and rigid frames help mitigate the visual impact of these differences by providing a consistent structural outline, ensuring that the overall appearance remains aesthetically pleasing and structurally sound. Figures 14 and 15 show the final results for Types A and B before and after mold removal. To highlight the distinctive bulges and patterns created by the fabric formwork, we removed the screws, grids, and fabrics, which left holes in the panel frames. For future prototypes, we plan to use thinner strips and frames to reduce material use and minimize gaps between panels, creating a more unified appearance.

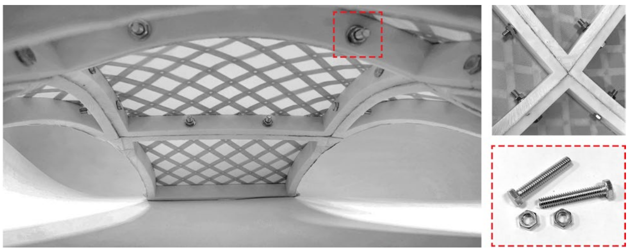

Figure 16: Connection between panels using M5 bolts and steel nuts

The panels are connected using M5 bolts and steel nuts (Figure 16). Careful analysis of the quantity and placement of these bolts is crucial for the overall structural stability. During concrete or gypsum pouring, specific precautions must be taken to keep the bolt holes clear and unobstructed. This ensures the proper insertion of bolts, guaranteeing a secure and stable connection between the panels.

4. Discussion

While this research successfully demonstrated the fabrication method for precast panels, dissecting the shell remains challenging. Shell structures should be tessellated according to the internal force pattern rather than using a superimposed rectangular grid, like Type A shell. The local curvature of the panels depends on the tessellation resolution, making this approach suitable for large panels, unlike techniques that use small, non-curved elements. Tessellating a shell surface using methods like the Force Density Method of Thrust Network Analysis (Block [17]) or principal curves (Chai et al. [14]) ideally aligns with the structure's internal forces. Principal curves represent major and minor stress trajectories, showing how the structure distributes and handles loads, which are useful for panelization (Pottmann et al. [13], Chai et al. [14]). By aligning the tessellation pattern with these curves, panels naturally follow the internal force distribution, significantly minimizing secondary stresses due to misalignment. This ensures that panels experience primarily axial forces (tension or compression), which are easier for materials to handle, reducing stress concentrations and promoting a uniformly stressed structure. Under uniform loading conditions, principal stress lines may form a grid-like pattern, ensuring efficient internal stress handling. In theoretical scenarios such as minimal surfaces or symmetric loading, internal forces align with principal curves, representing the directions of principal stress, where the stress is purely normal without shear components. This alignment makes the structure more robust and resilient. However, practical applications require a detailed analysis of the surface, material, and load conditions to determine the actual internal force distribution, as internal forces may not always perfectly follow the path of principal curves. Additionally, integrating the automated generation of principal curves with manual adjustments is necessary for achieving a more uniform force distribution.

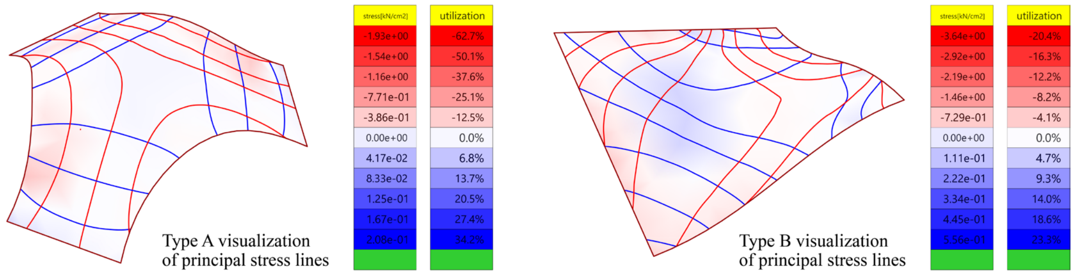

Figure 17: Principal stress lines with highlighted areas of high tension (blue) and high compression (red)

Figure 17 presents the principal stress lines, showing that Type A experienced a maximum tensile stress of 2.08 MPa and a compressive stress of 19.3 MPa, while Type B faced higher stresses, with 5.56 MPa in tension and 36.4 MPa in compression. This suggests potential compression issues, possibly causing wrinkles in the membranes that require careful stretching before pouring concrete. The study also calculated the structures' utilization, showing 34.2% in tension and 62.7% in compression for Type A, and 23.3% in tension and 20.4% in compression for Type B. These findings highlight the need for efficient material use. Although Type B shell is underutilized with a 20 mm concrete thickness, further reducing thickness might cause cracks during formwork removal. It is important to note that these findings, based on a conceptual model, might not fully align with real-world material behaviors.

5. Conclusion

This study presents an innovative approach to constructing prefabricated concrete shells, proposing a sustainable and cost-effective alternative to traditional concrete formworks. By integrating bending-active molds with fabric membranes, we have developed a flexible formwork system that is lightweight, reusable, and designed for both off-site production and on-site assembly. The effectiveness of this method has been validated through both digital simulations and physical models. However, our study reveals that the unique and specific shape of the shell, along with the variation in the panels, poses a challenge to the reuse of formwork and could potentially lead to increased material consumption. To address this challenge, we suggest designing shell structures with repetitive segments or modular panels to enhance the formwork's reusability and minimize material usage, thus promoting sustainability in construction. We acknowledge the limitations of our study, particularly our reliance on scale models, which may not capture the complexities of larger constructions. To address this, we intend to build full-scale prototypes by September 2024, employing materials like Glass-fiber Reinforced Concrete (GRC) and Ultra-High-Performance Concrete (UHPC). This development will mark a significant progression in our research, bringing us closer to achieving more efficient construction practices.

Acknowledgments

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