Proceedings of the IASS 2024 Symposium
Redefining the Art of Structural Design
August 26-30, 2024, Zurich, Switzerland
Philippe Block, Giulia Boller, Catherine DeWolf,
Jacqueline Pauli, Walter Kaufmann (eds.)

Reliability analysis of tensile membrane structures using a metamodeling approach

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Abstract

Tensile membrane structures (TMS) offer an aesthetically pleasing and functional solution for covering large spaces like stadiums, public areas, and amphitheaters. However, the inherent uncertainties associated with TMS make the efficient design of these structures challenging. Uncertainties arise due to the complex material behavior, estimation of wind and snow loads, challenges in form finding, application of prestress and construction. The prevailing design methodology for TMS involves a stress factor approach, where the permissible stress of the material fabric determines the structural design. This approach differs from the limit state methodologies that prioritize achieving a target reliability. The first-order reliability method (FORM) is commonly used to find the reliability of any structure for its simplicity and efficiency. However, assumptions are made on the shape of the failure surface or limit state surface which can lead to inaccuracies in safety assessment. Furthermore, while Monte Carlo simulation (MCS) methods can yield reliable results, they demand extensive evaluations of the limit state function, resulting in high computational costs (sometimes unmanageable for large and complex structures). In this study, the reliability analysis of diverse TMS shapes is comprehensively examined, exploring the use of a metamodeling approach while ensuring both efficiency and accuracy of the solution. The studied TMS include both frame and cable-supported structures across various limit states, and are compared with both the FORM and MCS results.

Keywords: tensile membrane structures, reliability analysis, surrogate model, metamodel, kriging

1. Introduction

Tensile membrane structures (TMS) are innovative and lightweight architectural solutions that offer designers the freedom for creative and dramatic artistic expressions. TMS have found their diverse applications in various structures, including light canopies, airport terminals, sports stadiums, and more. The fundamental elements of a TMS include a highly flexible fabric held under tension to provide structural stiffness, one-dimensional flexible elements like cables to form ridges and boundaries, and rigid support members to sustain compression and bending forces. The prestress in fabric membranes is achieved through the application of forces at the boundaries, ensuring the membrane's stability and performance (Lewis [1]).

Tensile membrane structures (TMS) are becoming more popular in developing economies such as India, because they are cost-effective and attractive. However, they are complex to design and construct, which increases the risk of failure. The adoption of TMS has not been as extensive as expected, when compared to some other popular materials like concrete or steel. One of the main reasons behind this may be that the design philosophy of TMS has not been standardized in most of the countries. Besides, wherever such standards exist, they follow highly conservative and non-economic practices. In Europe, CEN/TC/250 WG5 has been working on standardization of TMS and has recently published a technical specification [2]. Similar limit states based design specifications following the LRFD format need to be developed for many countries. Considering that this will indispensably require a large number of reliability analyses, the current study's focus rests on developing an efficient reliability analysis strategy for various limit states of TMS.

A preliminary reliability study of TMS was carried out by Gosling *et al.* [3] relying on the first-order reliability method (FORM). Reliability-based optimization methods were discussed by Dutta *et al.* [4]. More recently, reliability analysis for code calibration has been studied by De Smedt *et al.* [5]. Teichgräber *et al.* [6] and Fusseder *et al.* [7] explored the ambiguity in nonlinear structural models for reliability-based code calibration. Additionally, Thomas and Schoefs [8] and Thomas *et al.* [9] conducted reliability analyses of pneumatic tensile structures. Most of these studies used FORM for reliability analysis of TMS. FORM gives an approximate estimate of the structure's probability of failure. While probabilistic simulations, such as Monte Carlo simulation, can give accurate estimates, they are computationally expensive because of the thousands (or even millions) of structural analyses involved. For the reliability analysis of TMS, we need methods that are computationally affordable while acceptably accurate as well.

The current study focuses on developing an efficient reliability analysis methodology for TMS by adopting metamodeling or surrogate modeling techniques. The rest of this paper is organized as follows. Sec. 2. briefly describes the current approach to form-finding, analysis, and design of TMS. The basics of reliability analysis and metamodeling strategies that can be used for reliability analysis is elaborated in Sec. 3. and the case studies are illustrated in Sec. 4.. Finally, Sec. 5. concludes the key findings of the study.

2. TMS Design

Design of membrane structures involves a three-step process consisting of form-finding, load analysis and patterning. TMS possess unique shapes that cannot be described by simple mathematical functions. Before analysing the structure subjected to external loads such as wind or snow (*i.e.*, *load analysis*), the initial stable shape of the structure under pretension forces must be found. This process is known as *form-finding*, where the initial shape is determined to achieve equilibrium with the prestressing forces and boundary conditions. Due to their flexibility, TMS exhibit visible load-shape interactions. As loads are accommodated, the TMS membrane experiences changes in surface tension and significant displacements, leading to geometrically nonlinear behavior. Even under working loads within the elastic limit, these large deflections necessitate accounting for overall geometry changes in the analysis, including the identification of wrinkling directions, if present. For TMS construction, the three-dimensional shape of the tensioned surface must be translated into two-dimensional cutting patterns through a process known as *patterning*. This step requires precision to cut the fabric into strips with minimal material wastage and distortions.

2.1. Form-finding using the updated weight method

Form-finding methods for TMS include physical models, force density method, dynamic relaxation, geometric stiffness methods, and optimization-based methods. An example of an optimization-based method is the *updated weight method* (UWM), which has been observed to be robust. In this approach, the energy functional for a cable-supported TMS is expressed as the combination of two parts: (i) the energy associated with membrane prestress and (ii) the energy stemming from cable prestress. In UWM, this energy functional is minimized, with the objective function defined as a weighted expression of the structural configuration:

$$f_{UW} = \sum_{i=1}^{N_a} \sum_{j=1}^{3} W_i^j (L_i^j)^2 + \sum_{i=1}^{N_c} w_i l_i^2 \tag{1}$$

For each ith constant-strain triangular (CST) element discretizing the membrane, L_i^j and W_i^j denote the pseudo-cable side length and the side weight, respectively, of the jth side ($j = 1, 2, 3$). The boundary cables' weight and length are denoted by w_i and l_i, respectively. The summation encompasses all N_a membrane CST elements and N_c boundary cable elements. Details of UWM implementation for diverse TMS types can be found in the works of Marbaniang *et al.* [10].

2.2. Static load analysis using the modified minimum potential

TMS undergo noticeable deformations when subjected to wind pressure or snow loads, involving a complex relationship between their shape and the applied forces. This aspect needs to be taken into account when performing reliability analysis with limit states defined by these loads (Dutta *et al.* [4]). The load analysis of a form-found TMS can be conducted by minimizing a modified energy functional with respect to the material fiber basis direction, as outlined in detail by Marbaniang *et al.* [11]. In their method, adopted in the present, the membrane is again modeled using CST elements, while boundary cable elements are modelled using a two-noded line element. The form-found shape using UWM is used as the prestressed reference configuration for the load analysis. The modified potential energy is defined with respect to this configuration, using a total Lagrangian approach. The external loading is applied to the membrane surface as distributed vertical loads (dead or snow loads) or pressure normal to the surface (wind loads). The equilibrium initial prestress is also incorporated into the potential energy and the wrinkling state check as described in Marbaniang *et al.* [11]. The state of stress for an element can be either *taut* (biaxial tension), *wrinkled* (uniaxial tension) or *slack*, determined by the following criteria:

$$\begin{aligned} \text{Taut:} \quad & S_{\min} > 0 \\ \text{Wrinkled:} \quad & S_{\min} \leq 0 \quad \text{and} \quad E_{\max} > 0 \\ \text{Slack:} \quad & E_{\max} \leq 0 \end{aligned} \tag{2}$$

where, $S_{\min}$ and $E_{\max}$, respectively, are the minimum principal (second Piola-Kirchoff) stress and the maximum principal (Green-Lagrange) strain with respect to the reference configuration.

2.3. Design process

In TMS design, ensuring structural integrity involves maintaining structural responses within acceptable limits. These structural responses include stress levels, deflections, and wrinkling/slacking/ponding. Achieving structural integrity in a TMS design involves selecting the appropriate membrane materials, determining the required prestressing forces, and establishing suitable boundary conditions. The dominant design methodologies for TMS today rely on an approach known as the *permissible stress* or *stress factor approach*. The induced stresses due to the external loads is made sure to be within the permissible limits. In this approach, the fabric's strength is reduced by certain factor to establish a permissible stress threshold. This permissible stress value is then compared to the maximum fabric stress values obtained from a load analysis using unfactored (characteristic) loading conditions. The chosen stress factor implicitly accounts for various uncertainties, such as variations in material property, loading conditions, long-term material wear and tear, and construction tolerances. Engineering communities worldwide have adopted a variety of alternative stress factors, derived through diverse methodologies (Gosling *et al.* [3]).

3. Structural reliability analysis

Reliability-based design in structural engineering involves accounting for inherent uncertainties in design parameters, "scientifically". By incorporating probabilistic models and partial safety factors, it aims to control the probability of failure to meet desired safety levels. This approach allows for optimized resource allocation and risk-based decision-making, although it requires more data and more complex calculations to arrive at the design safety factors than the deterministic design methods.

Each possible failure mode of a structure corresponds to specific limit states, which serve as critical conditions beyond which the structure is no longer capable of performing its intended function or has experienced failure. The mathematical expression linking the structural resistance to the applied loads is termed as the *performance function*, or more commonly, the *limit state function*. It forms a hyper-surface in the probability space that distinguishes safe and unsafe designs. As both load and resistance parameters are inherently random variables, the performance function itself is also a random variable. In mathematical terms, the limit state function is expressed as $g(R, S)$, with R and S representing resistance and load variables, respectively. The probability associated with the limit state function taking a negative value is referred to as the probability of structural failure. Often represented as $P_f = \mathbb{P}(g(x) < 0)$, this probability serves as a crucial measure for evaluating the reliability and safety of engineered structures.

The assessment of structural safety and performance is also quantified through the *reliability index* (β). The Eurocode [12] defines reliability as the 'ability of a structure or a structural member to fulfil the specified requirements, including the design working life, for which it has been designed.' The reliability index for a specific limit state can be related to the corresponding probability of failure using the standard normal operator Φ:

$$P_f \approx \Phi(-\beta) \tag{3}$$

In most practical cases, the limit state will be nonlinear and hence a first-order approximation is sought at the 'design point' which is found by a constrained minimization of the distance from origin. These methods come in the class of *first-order reliability methods* (FORM) and works very well for limit states that are mildly nonlinear and composed of Gaussian or close-to-Gaussian random variables. However, for heavy-tailed non-standard distributions or correlated random variables, FORM can lead to inaccurate reliability estimates.

To overcome these issues, numerical probabilistic simulation methods, such as the Monte Carlo simulation (MCS), have remained popular in practical reliability analyses. MCS is a robust and versatile tool for probabilistic simulation, and is resilient to factors like nonlinearity and the order of the limit state function, correlation among variables, and the distribution types of random variables (RVs). However, its strength comes at a cost – it is computationally expensive. For instances where the probability of failure (P_f) is in the range of 10^{-6} to 10^{-4}, millions of simulations are often necessary, each involving a deterministic structural analysis. Particularly for intricate finite element (FE) models of structures, a "crude" approach to MCS may not be realistically feasible due to its cost and complexity. In case of TMS reliability analysis, both form-finding and load analysis are computationally expensive, making the adoption of MCS a very costly proposition in practical applications.

3.1. Metamodels

Metamodeling, also known as *surrogate modeling*, is a technique used for simplifying the large number of performance function evaluations in simulation based reliability analysis. It involves building a simplified model, called the metamodel or surrogate model, which approximates the behavior of a more complex and computationally expensive model or system. Instead of using the original model which might be computationally expensive and time-consuming, probabilistic simulations such as MCS are performed on the matamodel, reducing the computational cost and time required significantly.

To create a metamodel, an experimental design is chosen using sampling techniques, *e.g.*, Latin Hypercube sampling, and are evaluated using the original structural model. This input-output data is trained using a "machine learning" model, such as regression, polynomial chaos expansion, kriging and its combinations, support vector regression, or neural networks. In this study, kriging [13] is used for obtaining the metamodel. A kriging metamodel $\mathcal{M}^{\mathcal{K}}(x)$ can be described by [14]

$$\mathcal{M}^{\mathcal{K}}(x) = \beta^{\mathrm{T}} f(x) + \sigma^2 Z(x, \omega) \tag{4}$$

where, $\beta^{\mathrm{T}} f(x) = \sum_{j=0}^{P} \beta_j f_j(x)$ is the mean value of the Gaussian process (*i.e.*, its 'trend'), consisting of P arbitrary functions such as polynomials ($f_j;\ j = 1, \ldots, P$) and the corresponding coefficients ($\beta_j;\ j = 1, \ldots, P$). σ^2 is the variance of the Gaussian process and $Z(x, \omega)$ is a zero-mean, unit-variance, stationary Gaussian process. ω is the underlying probability space with hyperparameters θ and is defined in terms of a correlation function $R = R(x^i, x^j; \theta)$ that describes the correlation between two sample points x^i and x^j in the input space.

The Gaussian assumption leads that the vector formed by the prediction at a new design point x, *i.e.*, $\hat{Y}(x)$ and the true model responses $\mathcal{Y}$, has a joint Gaussian distribution defined by

$$\left\{ \begin{array}{c} \hat{Y}(x) \\ Y \end{array} \right\} \sim \mathcal{N}\left(\left\{ \begin{array}{c} f^{\mathrm{T}}(x)\beta \\ F\beta \end{array} \right\}, \sigma^2 = \left[\begin{array}{cc} 1 & r^{\mathrm{T}}(x) \\ r(x) & R \end{array} \right] \right) \tag{5}$$

where,

- F is the observation (design) matrix of the Kriging metamodel trend,
  $F_{ij} = f_j(x^{(i)}), \qquad i = 1, \ldots, N; \qquad j = 0, \ldots, P$
- $r(x)$ is the vector of cross-correlations between the prediction point x and each one of the observations, $r_i = R(x, x^{(i)}; \theta), \qquad i = 1, \ldots, N$
- R is the correlation matrix with elements: $R_{ij} = R(x^{(i)}, x^{(j)}; \theta), \qquad i, j = 1, \ldots, N$

The prediction follows a normal distribution $\hat{Y}(x) \sim \mathcal{N}(\hat{\mu}_{Y(x)}, \hat{\sigma}^2_{Y(x)})$ in which the mean and the variance are conditional on the observed data $\mathcal{X}$ and $\mathcal{Y}$:

$$\mu_{\hat{Y}(x)} = f(x)^{\mathrm{T}} \hat{\beta} + r(x)^{\mathrm{T}} R^{-1} (\mathcal{Y} - F\hat{\beta}) \tag{6}$$

$$\sigma^2_{\hat{Y}(x)} = \sigma^2 \left(1 - r^{\mathrm{T}}(x) R^{-1} r(x) + u^{\mathrm{T}}(x) (F^{\mathrm{T}} R^{-1} F)^{-1} u(x) \right) \tag{7}$$

where $\hat{\beta} = (F^{\mathrm{T}} R^{-1} F)^{-1} F^{\mathrm{T}} R^{-1} \mathcal{Y}$ is the generalized least-squares estimate of β and $u(x) = F^{\mathrm{T}} R^{-1} r(x) - f(x)$.

Kriging stands out as one of the most extensively utilized metamodels in reliability analysis. Notably, it offers an estimation of uncertainty associated with the metamodel prediction, a crucial aspect in reliability analysis. Despite the complexity involved in constructing this metamodel, readily available software packages like the UQLab Kriging module ([14]) simplify the process, making it accessible for practical applications.

4. Case studies

This section illustrates the reliability analysis of two TMS using kriging metamodels. The two TMS are adopted from the work of Gosling *et al.* [15]. These are relatively simpler forms of TMS where FORM has been found to work acceptably and no significant advantage of metamodeling can be expected over the simple FORM-based reliability analysis. However, this will allow a comparison between the two methods in terms of their accuracy, where the MCS is assumed to give the most accurate reliability estimates.

Two load cases are considered for these case studies: (i) snow load acting vertically downward under gravity and (ii) uniform wind uplift that acts normal to the surface of the membrane. Prestressing forces are applied before applying wind or snow loads. In these case studies, two critical limit states are examined. The stress criterion, denoted as LSF1, mandates that the maximum principal stress ($\sigma_{\max}$) within the membrane must not surpass the permissible fabric strength (σ_{per}). This permissible strength is taken as the tensile strength of the fabric divided by a stress reduction factor. On the other hand, LSF2 or deflection limits, require that the maximum nodal displacement ($\delta_{\max}$) remains within the predefined allowable displacement threshold (δ_{all}). This threshold is arbitrary and can be set to prevent clashes with the support structure and avoid issues like ponding. One should, however, note that membrane structures are known to deform significantly under applied wind or snow loads, and in some cases, a higher limit of allowable displacement may be considered.

For the reliability analysis of TMS, deterministic and probabilistic parameters are primarily sourced from a round-robin exercise (De Smedt *et al.* [16]) and presented in Table 1. Poisson's ratio is chosen as 0.4 and a reduction factor of 5 (De Smedt *et al.* [5]) applied to the ultimate fabric strength. Material strength and their distribution type are obtained from PD CEN/TS 19102:2023 [2], whereas their mean and standard deviation are calculated based on the JRC report [17].

To establish a benchmark for the reliability index, Monte Carlo simulation is performed using 100,000 Latin hypercube samples (LHS). Reliability analyses employing FORM and the kriging metamodel are conducted using UQLab, a widely-used Matlab toolbox for uncertainty quantification ([18]).

Table 1: Statistical characteristics of design variables (adopted from De Smedt *et al.* [16])

Variable	Distribution	Mean (μ)	Standard deviation (σ)	Unit
Prestress in warp direction (P_{warp})	Normal	4	0.75	kN/m
Prestress in fill direction (P_{fill})	Normal	4	0.75	kN/m
Stiffness in warp direction (E_{warp})	Normal	600	40	kN/m
Stiffness in fill direction (E_{fill})	Normal	600	40	kN/m
Shear modulus (G)	Normal	30	3	kN/m
Snow load (Q_{s})	Gumbel	0.66	0.198	kN/m^2
Wind load uplift (Q_{w})	Gumbel	0.7	0.245	kN/m^2
Tensile strength (σ) for Type I PES/PVC	Lognormal	55	3.04	kN/m
Tensile strength (σ) for Type II PES/PVC	Lognormal	80	6.08	kN/m

4.1. Case study 1: Hypar TMS

A hyperbolic paraboloid ('hypar') is a simple anticlastic shape commonly emplyed in TMS The chosen hypar is characterized by alternating high and low points. This is a square hypar measuring 6 m on each side with a 2 m height difference between the low and high points (Fig. 1) (Gosling *et al.* [15]). This is a cable-supported TMS with a cable diameter of 12 mm with an elastic modulus of 205 kN/mm^2,

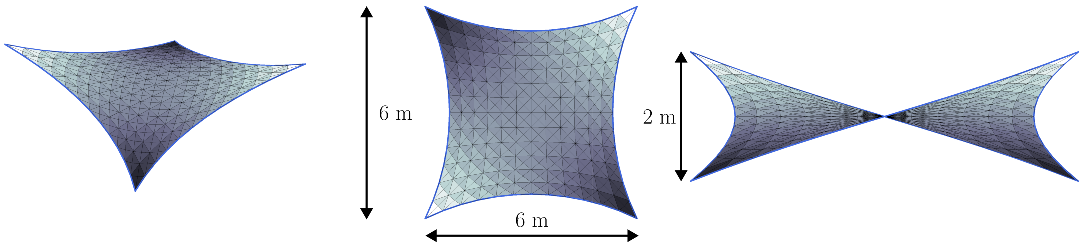

Figure 1: Cable supported 'hypar' TMS (adopted from Gosling *et al.* [15])

and a cable force of 30 kN. The analysis encompasses two limit states, considering material strength corresponding to Type I PVC coated polyester (PES/PVC) fabrics. Its maximum allowable deflection (δ_{all}) is considered as 100 mm.

The reliability indices (β) obtained through simulation, FORM and kriging metamodel, for both snow and wind load cases, are tabulated in Table 2. This table also shows the relative error in the FORM-based and kriging-based estimates, with respect to the MCS based reliability values. It can be clearly observed that reliability indices found using the kriging metamodel approach is more accurate than that found using FORM.

Table 2: Reliability indices for hypar TMS and error in the FORM- and metamodel-based estimates

Load case	MCS	FORM		Metamodel	
	β_{MCS}	β_{FORM}	Relative error (%)	$\beta_{\text{Metamodel}}$	Relative error (%)
LSF1: Stress criterion ($\sigma_{\max} \leq \sigma_{\text{per}}$)					
Snow	2.48	2.68	8.06	2.55	2.98
Wind	1.97	2.08	5.95	1.99	1.05
LSF2: Deflection control ($\delta_{\max} \leq \delta_{\text{all}}$)					
Snow	1.54	1.58	2.62	1.55	0.76
Wind	1.38	1.43	3.20	1.39	0.66

4.2. Case study 2: Conic TMS

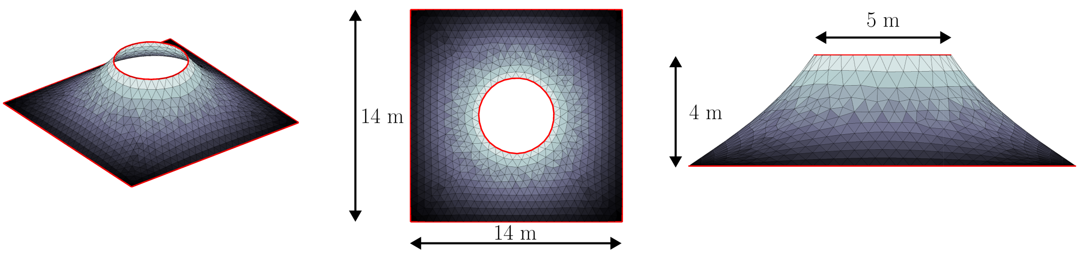

Figure 2: Frame supported 'conic' TMS (adopted from Gosling *et al.* [15])

The second example is that of a frame-supported conic adopted from Gosling *et al.* [15]. The structure is a high-point structure having a square base of 14 m × 14 m with a fixed circular head ring of diameter 5 m at a height of 4 m above the base (Fig. 2). The membrane material is assumed as Type II PES/PVC fabric and all other parameters remain the same as in Case Study 1. The maximum allowable deflection is assumed to be 200 mm.

The results of the reliability analysis for MCS, FORM and metamodel is given in Table 3 for both snow and wind loads. Similar to the hypar TMS of Case Study 1, it is seen here that the relative error for the metamodel approach is less than the FORM-based estimation, for both limit states LSF 1 and LSF 2. FORM-based estimates show a significantly high maximum error of (approximately) 45% while the maximum error for the metamodel is at (approximately) 17%. This clearly highlights the effectiveness of using the suggested metamodel approach over FORM. Furthermore, it underscores the simplicity and straightforwardness of its application, making it a highly advantageous choice in TMS reliability analysis.

Table 3: Reliability indices for conic TMS and error in the FORM- and metamodel-based estimates

Load case	MCS	FORM		Metamodel	
	β_{MCS}	β_{FORM}	Relative error (%)	$\beta_{\mathrm{Metamodel}}$	Relative error (%)
LSF1: Stress criterion ($\sigma_{\max} \leq \sigma_{\mathrm{per}}$)					
Snow	1.05	1.52	44.97	1.23	17.21
Wind	1.83	2.01	10.21	1.89	3.52
LSF2: Deflection control ($\delta_{\max} \leq \delta_{\mathrm{all}}$)					
Snow	2.13	2.21	3.99	2.09	1.65
Wind	2.15	2.11	2.13	2.14	0.58

5. Conclusion

Reliability analysis plays a crucial role in understanding and mitigating the complexities of TMS failures. Despite the high frequency of TMS failures, there is a scarcity of comprehensive studies addressing this issue. One of the primary reasons for this may be the computational complexities involved in conducting a thorough reliability analysis for these highly nonlinear structures.

The application of a metamodeling approach emerges as a promising solution. This work shows that the adopted kriging metamodeling technique yields more accurate results as opposed to approximate methods such as FORM. On the other hand the computational savings are significant as compared to MCS or LHS. This study underscores the efficacy and simplicity of the metamodeling approach in TMS reliability analysis.

Future works in this direction should focus on more complex TMS problems and other metamodeling approaches.

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