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Form-finding of cable-strut structures with sliding cables

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Abstract

Cable-strut structures with sliding cables have been used in various fields. Form-finding (i.e., determining the initial shape and force distribution) is a key issue in the design of cable-strut structures. There have been a number of form-finding methods for normal cable-strut structures, but the form-finding of them with sliding cables has been seldom studied. The main difference in the form-finding of cable-strut structures with sliding cables is that the forces of the segments that belong to the same sliding cable should be identical. In the application of existing cable-strut structures with sliding cables, they are mainly generated by replacing the adjacent cables with the same force by continuous sliding cables based on structural symmetry. In this paper, a general energy-based form-finding method of cable-strut structures with sliding cables is proposed. The potential energy function of sliding cables is derived, and the equilibrium equations of the system are obtained based on the principle of stationary energy. The LM algorithm is used to solve the equilibrium equations. Various numerical examples are adopted to demonstrate the effectiveness and practicality of the proposed method. The results show that the proposed method has a good convergence property and can find different configurations of cable-strut structures with sliding cables.

Keywords: cable-strut structures, sliding cable, form-finding.

1. Introduction

Cable-strut structures, including cable trusses, cable domes, and tensegrity, are composed of cables in tension and struts in tension. Cable-strut structures have been widely used in the field of structural engineering due to their lightweight and large span. Sliding cables are sometimes used in cable-strut structures. In suspended dome structures, the application of sliding cables can reduce the peak structural internal force and improve the mechanical properties under asymmetric loading [1]; in active-controlled structures with sliding cables (clustered tensegrity), the number of active cables can be reduced[2, 3]; In bridges and high-rise buildings, the frictional dissipation of energy by sliding cables can be utilized to improve the dynamic performance of the structure [4-6].

Determining the initial shape and force distribution is a key step in the design of cable-strut structures. Existing methods can be divided into two main categories. The first one is called force-fiding, which is to determine the force distribution based on given structural shapes. The second one is called form-finding, which finds the shape and force simultaneously. The basis of force-finding is the equilibrium matrix analysis theory of pin-jointed systems[7-9]. A series of orthogonal base vectors satisfying the equilibrium equations can be obtained by singular value decomposition (SVD) of the equilibrium matrix, and they are defined as self-stress modes. The force-finding always falls into the optimization of the combination coefficients of self-stress modes, and different objective functions have been utilized, such as the global stiffness[10], the variation of internal forces[11], and the structural total mass[12-14]. The number of unknown parameters (element forces and nodal positions) exceeds the number of equations (equations equations), and thus additional conditions should be given in the form-finding procedures.

Different methods have been developed when different additional conditions are provided, such as the force density method [15, 16], the dynamic relaxation method[17], the nonlinear finite element method[18, 19], and heuristic algorithms[20, 21].

Despite the developments of form-finding of cable-strut structures, the form-finding of cable-strut structures with sliding cables has not been considered. The existing applications of the sliding cables are based on the structural symmetry or designing experience[22-25]. The main designing process is replacing the adjacent cables with the same forces as sliding cables without influencing the structural equilibrium. Yu[26] solved for the combination coefficients of self-stress modes that make the forces of specified adjacent cables identical, to achieve the condition of applying sliding cables, but whether the solution exists or not depends on the initial given structural shape. Due to the lack of form-finding methods for cable-strut structures with sliding cables, the structural forms and their application scenarios are very limited. In this paper, a novel energy-based form-finding method of cable structures with sliding cables is proposed, the main idea is to find the structural shape based on given cable forces. By assigning special internal force distributions, the conditions for the use of sliding cables can be satisfied.

This paper is organized as follows. Section 2 gives the basic assumptions, the mathematical model, and the solving method. In Section 3, several numerical examples are adopted to show the ability of the method to find structural forms with sliding cables. In section 4, a typical physical model is built to demonstrate the feasibility of the methods. Finally, Section 5 concludes this paper.

2 Method

2.1 Basic assumptions

The following assumptions are adopted in this paper:

(1) The elements are linear elastic and bear only axial forces.

(2) No element failures nor strut buckling occurs.

(3) There are no frictions between the cables and nodes.

Note that assumption (3) does not affect the correctness of the form-finding result when considering the friction. If a cable-strut structure with sliding cables is in equilibrium without friction, the structural equilibrium must be satisfied when considering friction.

2.2 Potential energy function

Suppose the cables give forces as ideal elements with 0 rest length and 0 stiffness (the force does not change with the variation of cable lengths). Then the internal forces in the cables remain at the initially given values regardless of the final structural equilibrium state, and thus the cable forces in the form-finding can be controlled. The potential energy of the ideal element is the product of the given cable force and the final element length. When using the nodal coordinates as unknown parameters of the form-finding, the potential energy of ideal cables that connect nodes *i* and *j* can be written as

$$\pi_{ij}^{c}(\mathbf{x}) = T_{ij}\left\|\mathbf{x}_i - \mathbf{x}_j\right\|_2 \tag{1}$$

where $\mathbf{x}$ is the vector unknown parameters (i.e., nodal coordinates). T_{ij} is the given cable forces in the form-finding. $\mathbf{x}_i$ and $\mathbf{x}_j$ are vectors of nodes *i* and *j*, respectively. $\|*\|_2$ represents the 2-order norm of $*$.

The strut and non-sliding cables are assumed as normal linear elastic elements with a relatively large stiffness. So that the final lengths of those elements can be very similar to the given rest length to achieve better control of the structural shape. Similarly, the potential energy of the element connecting nodes nodes *i* and *j* can be written as

$$\pi_{ij}^{s}(\mathbf{x}) = \frac{1}{2}k_{ij}(\|\mathbf{x}_i - \mathbf{x}_j\|_2 - l_{ij}^{0})^2 \tag{2}$$

where k_{ij} and l_{ij}^{0} are the axial stiffness and rest length of the element, respectively.

To ensure the structure satisfies the given conditions under external loads, the potential loads should also be considered(e.g. self-weight load). If the external load is acting on the kth degree of freedom (DOF), the potential energy can be written as

$$\pi_{k}^{l}(\mathbf{x}) = -f_k x_k \tag{3}$$

where f_k is the magnitude of the external load, x_k is the nodal coordinate corresponding to the kth DOF.

Note that, in the form-finding process, the final lengths of the cables are unknown, so the self-weight of the structure cannot be pre-determined. In this paper, the self-weight load is determined by an iteration process. Firstly, the equilibrium state of the structure without self-weight load is determined; then, the corresponding self-weight of the equilibrium state is used in the next iteration; following this scheme, the self-weight is iteratively updated until its change becomes negligible.

Considering the total potential energy of the structure, the following indexes are defined:

$$\begin{gathered} \delta_{ij}^{1} = \begin{cases} 1, \text{if node } i \text{ and } j \text{ are connected by a cable} \\ 0, \text{if node } i \text{ and } j \text{ are not connected by a cable} \end{cases} \\ \delta_{ij}^{2} = \begin{cases} 1, \text{if node } i \text{ and } j \text{ are connected by a strut} \\ 0, \text{if node } i \text{ and } j \text{ are not connected by a strut} \end{cases} \\ \delta_{k} = \begin{cases} 1, \text{there is external load on the } k\text{-th DOF} \\ 0, \text{there is no external load on the } k\text{-th DOF} \end{cases} \end{gathered} \tag{4}$$

Then, the total potential enegry of the cable-strut structures can be written as

$$\Pi(\mathbf{x}) = \sum_{i=1}^{n}\sum_{j=i+1}^{n} \delta_{ij}^{c}\pi_{ij}^{c}(\mathbf{x}) + \sum_{i=1}^{n}\sum_{j=i+1}^{n} \frac{1}{2}\delta_{ij}^{s}\pi_{ij}^{s}(\mathbf{x}) + \sum_{k=1}^{3n} \delta_{k}^{l}\pi_{k}^{l}(\mathbf{x}) \tag{5}$$

Substituting Eqs. (1)-(4) to Eq. (5), the total potential energy in terms of nodal coodinates is obtained and it can be written as

$$\Pi(\mathbf{x}) = \sum_{i=1}^{n}\sum_{j=i+1}^{n} \delta_{ij}^{c}T_{ij}\|\mathbf{x}_i - \mathbf{x}_j\|_2 + \sum_{i=1}^{n}\sum_{j=i+1}^{n} \frac{1}{2}\delta_{ij}^{s}k_{ij}(\|\mathbf{x}_i - \mathbf{x}_j\|_2 - l_{ij}^{0})^2 - \sum_{k=1}^{3n} \delta_{k}^{l}f_k x_k \tag{6}$$

2.3 Form-finding based on Levenberg-Marquardt algorithm

According to the principle of potential stationary energy, the node coordinates corresponding to the equilibrium state of the structure are the stationary values of the potential energy function, and the corresponding first-order derivative vector of the overall potential energy function is **0**, which can be written as:

$$\frac{\partial \Pi_2(\mathbf{x})}{\partial \mathbf{x}} = \mathbf{0} \tag{7}$$

The most commonly used Newton iteration-based method needs the inverse of the stiffness matrix (i.e., the Hessian matrix of the total potential energy), which cannot be used when the Hessian matrix is singular or near singular. To avoid solving the inverse of the stiffness matrix, some vector-based methods, such as the dynamic relaxation method (DRM) and the vector form intrinsic finite element method (VFIFE), are developed [17, 27]. The main idea of the methods is that the nodes move along the direction of unbalanced forces until reaching the balanced position. The methods are essential to finding the local minimum of the potential energy, and the applicability of these methods is based on the fact that a stable equilibrium state is a local minimum of the potential energy. However, in the form-finding process, virtual constitutive relations are used for the cables, hence the potential energy is different from that of the real structure, and the stable equilibrium state may not be a local minimum of the virtual potential energy [28]. Miki and Kawaguchi have shown that the form-finding based on minimizing the potential energy may not converge when all the cable forces are given as the initial conditions [29].

The Levenberg-Marquardt (LM) algorithm, which converts the solving of non-linear equations to a least-squared problem, is an efficient method to solve non-linear equations with global convergence [30]. By adaptive modification of the Hessian matrix, non-linear equations can be solved even though the Hessian matrix is singular or the solution is not a local minimum of the potential energy. It has been proved to be valid for the form-finding of tensegrity systems without sliding cables [31].

Considering a general nonlinear equation $f(\mathbf{x}) = \mathbf{0}$, then the objective of the LM is as follows

$$\mathbf{x} = \arg.\min \|f(\mathbf{x})\|_2 \tag{8}$$

where $f(\mathbf{x})$ is the residual error in terms of $\mathbf{x}$.

Let $\mathbf{J}_m$ and $\mathbf{R}_m$ denote respectively the hessian matrix and the residual errors in the m^{th} iteration; λ_m denotes a parameter to modify the Hessian matrix. The main steps of the LM are as follows (more details of the LM can be referred to in [30]).

(1) Input the initial guess of the solution $\mathbf{x}_0$, the computational parameters $\lambda_0 > 0$ and $\alpha > 1$; and set $m = 0$.

(2) Compute the Hessian matrix $\mathbf{J}_m$ and the initial residual errors $\mathbf{R}_m$; solve the variations of $\mathbf{x}$ in the m^{th} iteration (Eq. (11)).

$$d\mathbf{x}_{m+1} = (\mathbf{J}_m^{\mathrm{T}}\mathbf{J}_m + \lambda_m\mathbf{I})^{-1}\mathbf{R}_m \tag{9}$$

(3) Update $\mathbf{x}$ as follows

$$\mathbf{x}_{m+1} = \mathbf{x}_m + d\mathbf{x}_m \tag{10}$$

(4) Check the convergence criterion $\|f(\mathbf{x}_{m+1})\|_2 < \varepsilon$. If the criterion is satisfied, terminate the iteration, and $\mathbf{x}_{m+1}$ is the final solution; otherwise, go to step (5).

(5) Judge the condition $\|f(\mathbf{x}_{m+1})\|_2 < \|f(\mathbf{x}_m)\|_2$, if it is satisfied, set $\lambda_{m+1} = \lambda_m/\alpha$, and go to step (6); otherwise, set $\lambda_{m+1} = \lambda_m\alpha$, and go to step (2).

(6) Set $m = m + 1$, and go to step (2).

For illustration, the computational framework of the form-finding method is shown in Fig. 1.

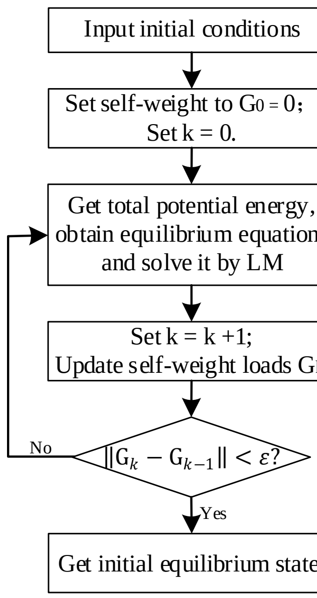


Fig. 1 Computation freamworks

3 Numerical examples

Example 1: Prism tensegrity structure

The topology, the node numbering, and the element numbering of the prism tensegrity structure are given in Fig. 2. There are 9 cables and 3 struts in the structure. There are no constraints and no external loads on the nodes. The cable forces and strut lengths for the form-finding are given in Table. 1. The structural equilibrium state under the condition is solved based on the proposed method, and the final nodal positions are listed in Table 2. Under the special cable force distribution in Table 1, all the cable forces are the same, and two different configurations of prism tensegrity structures with sliding cables can be obtained. As shown in Fig. 3, in the first one (Fig. 3(a)), the three cables on the bottom side are connected as an integral sliding cable; in the second one (Fig. 3(b)), the four cables in the lateral side are connected as a sliding cable.

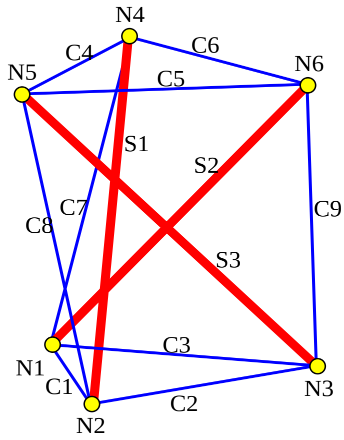


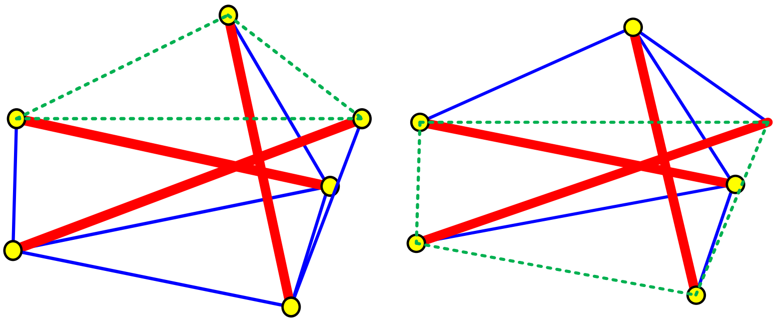
Fig. 2 Prism tensegrity structure considered in example 1

Table 1. Given condition of cable forces and strut lengths of prism tensegrity

Element No.	Force of cables (kN)									Length of struts(m)		
	C1	C2	C3	C4	C5	C6	C7	C8	C9	S1	S2	S3
Given parameters	1	1	1	1	1	1	1	1	1	4	4	4

Table 2. Nodal coordinates of equribrium state of prism tensegrity (m)

Node No.	Nodal coordinates
N1	(1.228, −0.399, 0.556)
N2	(−1.142, −1.034, 0.593)
N3	(0.593, −2.770, 0.566)
N4	(1.576, −1.034, −0.662)
N5	(−0.794, −0.399, −0.625)
N6	(−0.159, −2.77, −0.635)

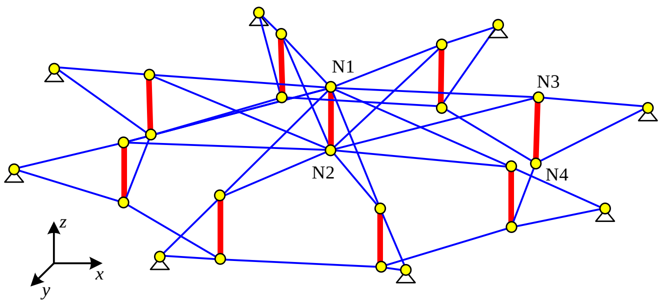


(a) sliding bottom cable (b) sliding lateral cable

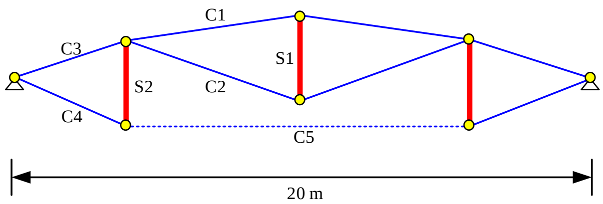
Fig. 3 Prism tensegrity structure with sliding cables

Example 2: Geiger-type cable dome

A Geiger cable dome as shown in Fig. 4 is considered in this example. The span of the structure is 20 m. There are 40 cables and 9 struts in the structure. Considering the symmetry of the system, the cables and struts are divided into 5 groups (C1-C5) and 2 groups (S1, S2), respectively. The 8 nodes of the outer ring are fixed. No external loads are considered. The cable forces and strut lengths are given in Table 3. Some nodal coordinates in the equilibrium state are shown in Table 4, the positions of other nodes can be obtained based on the structural symmetry. Similarly, two different configurations of Geiger-type cable dome with sliding cables can be obtained based on the special cable force distribution in Table 4. As shown in Fig. 4, the first one (Fig. 5(a)) uses a sliding hoop cable, and the second one (Fig. 5(b)) uses sliding ridge cables that overlap at the second layer of the ridge line.



(a) perspective view: structural configuration and partial node numbering



(b) side view: element grouping

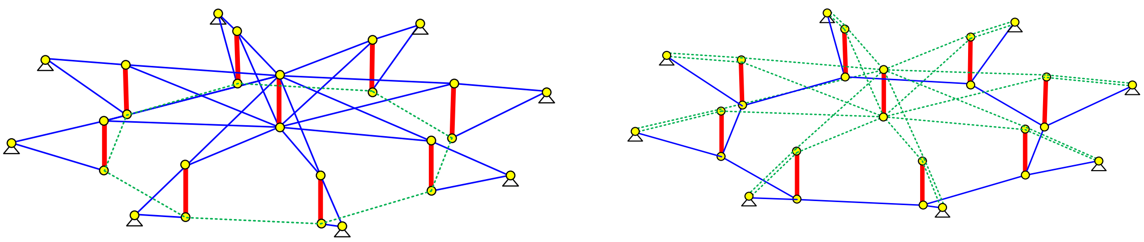
Fig. 4 Geiger-type cable dome

Table 3. Given condition of cable forces and strut lengths of cable dome

	Force of cables (kN)					Length of struts(m)	
Group No.	C1	C2	C3	C4	C5	S1	S2
Given conditions	1	1	2	1	1	4	4

Table 4. Nodal coordinates of equilibrium state of cable dome (m)

Node No.	Nodal coordinates
N1	(0, 0, 3.137)
N2	(0, 0, −0.853)
N3	(6.54, 0, 1.142)
N4	(6.452, 0, −2.854)

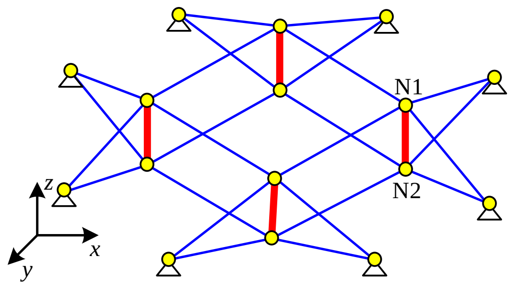


(a) sliding hoop cable (b) sliding ridge cables

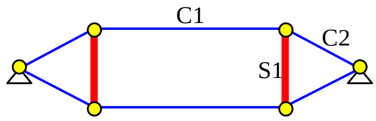
Fig. 5 Geiger-type cable dome with sliding cables

Example 3: Orthogonal cable truss

As shown in Fig. 6, a simple orthogonal cable truss with 2×2 planar cable trusses is considered. The span of the structure is 10 m. There are 24 cables and 4 struts in the structure. Considering the symmetry of the system, the cables and struts are divided into 2 groups (C1-C2) and 1 group (S1), respectively. The 8 nodes of the outer ring are fixed. No external loads are considered. Some nodal coordinates in the equilibrium state are shown in Table 6. Two configurations of the cable truss with sliding cables can be obtained based on the special cable force distribution in Table 5. As shown in Fig. 7, the first one (Fig. 7(a)) uses a sliding hoop cable, and the second one (Fig. 8(b)) uses sliding string cables.



(a) perspective view: structural configuration and partial node numbering



(b) side view: element grouping

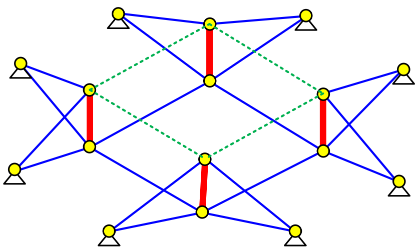
Fig. 6 Orthogonal cable truss

Table 5 Given condition of cable forces and strut lengths of cable truss

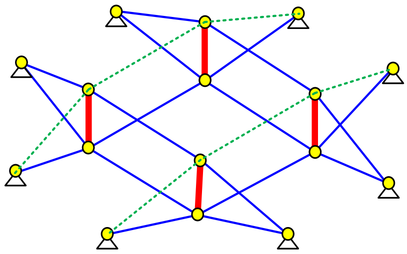
Group No.	Force of cables (kN)		Length of struts(m)
	C1	C2	S1
1	1	1	4

Table 6. Nodal coordinates of equilibrium state of cable truss (m)

Node No.	Nodal coordinates
N1	(5.551, 5.551, 1.998)
N2	(5.551, 5.551, −1.998)



(a) sliding hoop cable

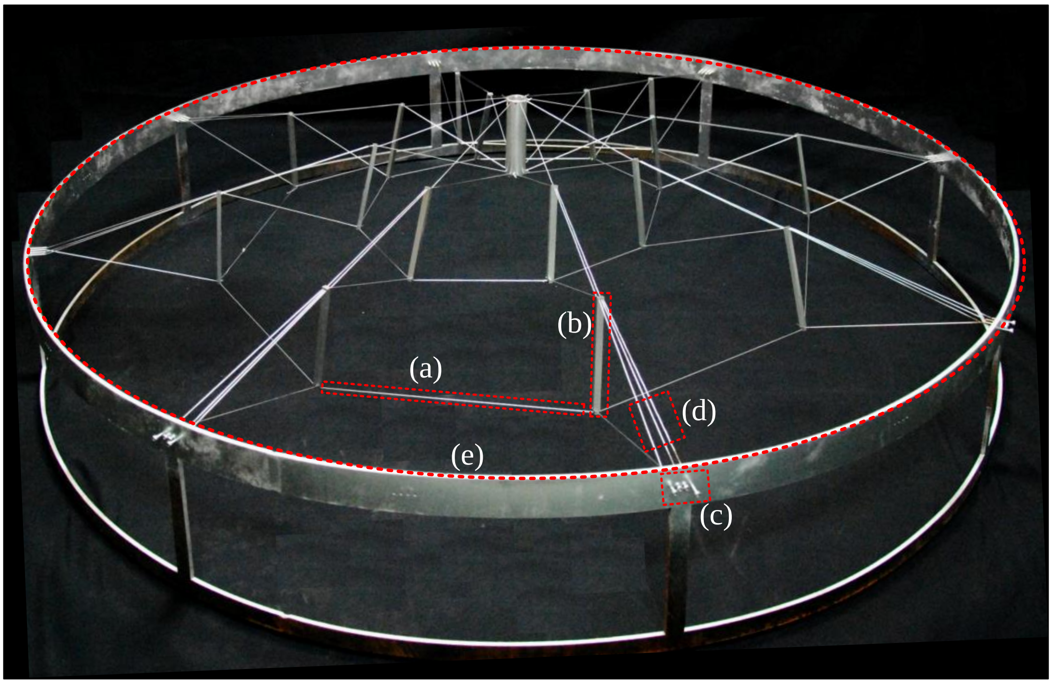


(b) sliding string cables

Fig. 7 Orthogonal cable truss with sliding cables

4 Physical model

As is shown in Fig. 8, to further demonstrate the feasibility of the form-finding method, a simple physical model of a Geiger-type cable dome with sliding ridge cables (Fig.5(b)) is built. The model is composed of eight planer trusses and two layers of loop cables. The span of the physical model is 1.2 m, the cables are made of Nylon threads, and the struts are made of aluminum alloy. The cables are tensioned by adjusting the screws in the supported nodes.



(a) Cable made of Nylon thread (b) Strut made of aluminum alloy (c) Tensioning screws (d) Overlap of cables (e) Steel supported system

Fig. 8 Physical model of Geiger-type cable dome with sliding ridge cables.

5 Conclusions

The distribution of cable force in a cable-strut structure with sliding cables needs to satisfy certain constraints. This paper proposes a form-finding method by assigning special cable forces of specific cables to make it satisfy the condition of using sliding cables. The potential energy function of the structure under the given conditions of cable forces and strut lengths is derived, and the form-finding is transformed into solving the stationary value of the energy function. The Levenberg-Marquardt is used to solve the stationary value. The feasibility of the method is demonstrated by numerical examples. Various configurations of cable-strut structures with sliding cables are obtained based on the assigning special cable force distributions of a prism tensegrity, a Geiger-type cable dome, and an Orthogonal cable truss. A physical model is built to further demonstrate the feasibility of obtained structural configurations. After replacing adjacent cables with the same force with sliding cables, the resultant force at each node remains unchanged and the cables do not slide, and thus the structure remains in equilibrium.

Note that only the equilibrium condition are considered the proposed method. The replacement of sliding cables changes the constraint conditions of the structure, and thus the geometrical stability of the structure may be influenced and requires further judgment based on the positive definiteness of the tangent stiffness matrix.

References

[1] X. Cui, Y. Guo, and K. Ye, "Application of Gliding Hoop Cable Joint in Suspen-dome Structures," *Journal of Tongji University*, vol. 32, no. 10, pp. 1300-1303, 2004

[2] N. Veuve, A. C. Sychterz, and I. F. C. Smith, "Adaptive control of a deployable tensegrity structure," *Engineering Structures*, vol. 152, pp. 14-23, 2017.

[3] S. Ma, M. H. Chen, and R. E. Skelton, "Dynamics and control of clustered tensegrity systems," *Engineering Structures*, vol. 264, p. 114391, Aug 2022.

[4] J. H. He, L. Xu, W. C. Yuan, X. Z. Dang, and Ieee, "Transverse Seismic Design of Asymmetric Single Tower Cable-stayed Bridge under Strong Earthquake Excitations," in *International Conference on Smart City and Systems Engineering (ICSCSE)*, Shanghai, 2018, pp. 161-166, 2018.

[5] S. Bagheri, S. S. Shishvan, M. Barghian, and B. Baniahmad, "A new energy dissipative cable bracing system," *Advances in Structural Engineering*, vol. 22, no. 14, pp. 3134-3146, Oct 2019.

[6] M. Kurata, R. T. Leon, and R. DesRoches, "Rapid Seismic Rehabilitation Strategy: Concept and Testing of Cable Bracing with Couples Resisting Damper," *Journal of Structural Engineering*, vol. 138, no. 3, pp. 354-362, Mar 2012.

[7] S. Pellegrino and C. R. Calladine, "Matrix analysis of statically and kinematically indeterminate frameworks," (in English), *International Journal of Solids and Structures*, Article vol. 22, no. 4, pp. 409-428, 1986.

[8] S. Pellegrino, "Structural computations with the singular-value decomposition of the equilibrium matrix," *International Journal of Solids and Structures*, vol. 30, no. 21, pp. 3025-3035, 1993 1993.

[9] S. Pellegrino, "Analysis of prestressed mechanisms," *International Journal of Solids and Structures*, vol. 26, no. 12, pp. 1329-1350, 1990.

[10] S. Lee and J. Lee, "Optimum self-stress design of cable-strut structures using frequency constraints," *International Journal of Mechanical Sciences*, vol. 89, pp. 462-469, Dec 2014.

[11] X. Xu and Y. Z. Luo, "Force Finding of Tensegrity Systems Using Simulated Annealing Algorithm," *Journal of Structural Engineering*, vol. 136, no. 8, pp. 1027-1031, Aug 2010.

[12] Y. F. Wang, X. Xu, and Y. Z. Luo, "Minimal mass design of active tensegrity structures," (in English), *Engineering Structures*, Article vol. 234, p. 111965, May 2021.

[13] M. Quagliaroli, P. G. Malerba, A. Albertin, and N. Pollini, "The role of prestress and its optimization in cable domes design," *Computers & Structures*, vol. 161, pp. 17-30, Dec 2015.

[14] N. Pollini, "Gradient-based prestress and size optimization for the design of cable domes," *International Journal of Solids and Structures*, vol. 222, p. 111028, Jul 2021, Art. no. 111028.

[15] H. J. Schek, "The force density method for form finding and computation of general networks," *Comput. Meth. Appl. Mech. Eng*, no. 3, pp. 115-134, 1974.
[16] J. Y. Zhang and M. Ohsaki, "Adaptive force density method for form-finding problem of tensegrity structures," *International Journal of Solids and Structures,* vol. 43, no. 18-19, pp. 5658-5673, 2006.
[17] R. Motro, "Forms and forces in tensegrity systems," presented at the International Conference on Space Structures, Amsterdam, 1984.
[18] B. Tabarrok and Z. Qin, "Nonlinear analysis of tension structures," *Computers & Structures,* vol. 45, no. 5-6, pp. 973-984, Dec 1992.
[19] L. Y. Zhang, Y. Li, Y. P. Cao, and X. Q. Feng, "Stiffness matrix based form-finding method of tensegrity structures," *Engineering Structures,* vol. 58, pp. 36-48, Jan 2014.
[20] X. Xu and Y. Luo, "Form-finding of nonregular tensegrities using a genetic algorithm," *Mechanics Research Communications,* vol. 37, no. 1, pp. 85-91, 2010.
[21] Y. Li, X.-Q. Feng, Y.-P. Cao, and H. Gao, "A Monte Carlo form-finding method for large scale regular and irregular tensegrity structures," *International Journal of Solids and Structures,* vol. 47, no. 14-15, pp. 1888-1898, 2010.
[22] Z. You and S. Pellegrino, "Cable-stiffened pantographic deployable structures .1. Triangular mast," *Aiaa Journal*, vol. 34, no. 4, pp. 813-820, Apr 1996.
[23] Z. You and S. Pellegrino, "Cable-stiffened pantographic deployable structures .2. Mesh reflector," *Aiaa Journal,* vol. 35, no. 8, pp. 1348-1355, Aug 1997.
[24] H. J. Peng, L. Fei, Z. Y. Kan, and P. L. Liu, "Symplectic Instantaneous Optimal Control of Deployable Structures Driven by Sliding Cable Actuators," *Journal of Guidance Control and Dynamics*, vol. 43, no. 6, pp. 1114-1128, Jun 2020.
[25] K. W. Moored and H. Bart-Smith, "Investigation of clustered actuation in tensegrity structures," *International Journal of Solids and Structures,* vol. 46, no. 17, pp. 3272-3281, 2009.
[26] Yu Feng, "Computational theory of cable slip analysis and its application to cable-rod girder membrane structures," PhD thesis, Zhejiang University, 2015.
[27] Y. Zhang, D. S. Chen, and H. L. Qian, "Computational method for the deformation mechanism of non-prestressed cable net structures based on the vector form intrinsic finite element method," *Engineering Structures,* vol. 231, Mar 2021, Art. no. 111788.
[28] Y. Xue, Y. Luo, and X. Xu, "Form-finding of cable-strut structures with given cable forces and strut lengths," *Mechanics Research Communications,* vol. 106, p. 103530, 2020/06/01/ 2020.
[29] M. Miki and K. i. Kawaguchi, "Extended force density method for form finding of tension structures," vol. 51, no. 4, pp. 291-303, 2010.
[30] J. Pujol, "The solution of nonlinear inverse problems and the Levenberg-Marquardt method," *Geophysics,* vol. 72, no. 4, pp. W1-W16, Jul-Aug 2007.
[31] X.-F. Yuan, S. Ma, and S.-H. Jiang, "Form-finding of tensegrity structures based on the Levenberg–Marquardt method," *Computers & Structures*, vol. 192, pp. 171-180, 2017.