Proceedings of the IASS 2024 Symposium
Redefining the Art of Structural Design
August 26-30, 2024, Zurich Switzerland
Philippe Block, Giulia Boller, Catherine DeWolf,
Jacqueline Pauli, Walter Kaufmann (eds.)

Uncertain analysis of deployable structures with interval parameters based on finite particle method and Chebyshev polynomial method

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Abstract

Deployable structures have broad application prospects in engineering due to their advantages. In engineering, deployable structures may contain uncertain parameters which may lead to the deviation of the dynamic response of deployable structures from design. In this study, the uncertain analysis of deployable structures is conducted based on finite particle method (FPM) and Chebyshev polynomial method, which have low computational cost. Firstly, the modeling method of deployable structures and the corresponding elements of FPM are briefly described, respectively. Subsequently, the Chebyshev polynomial method is introduced and a non-intrusive uncertainty analysis method is proposed by combining FPM and the Chebyshev polynomial method. Finally, the uncertain dynamic analysis of a typical deployable structure is conducted.

Keywords: deployable structure; dynamic response; uncertainty; finite particle method (FPM); Chebyshev polynomial method.

1. Introduction

Deployable structures have many advantages compared to conventional structures. Hence deployable structures have broad application prospects in civil engineering [1], aerospace engineering [2], rescue engineering [3], medical engineering [3], etc. Many researchers focus on the deployment process and dynamic responses of deployable structures. Zheng et al. [4] proposed an enhanced simplified modeling method of Bennett linkage considering link cross-sectional sizes and contacts to analyze its dynamic response during deployment process.

Most previous studies on the dynamic analysis of deployable structure have focused on the deterministic parameters of deployable structure. However, the deployment structure may inevitably contain uncertainty parameters in engineering. The main challenge of studying the dynamics of deployment structure with uncertain parameters is the description of uncertain parameters. There are two main types of methods to describe the uncertain parameters: the probabilistic methods [5] which are usually used to analyze the random parameters with known probability density functions and the non-probabilistic methods [6] which are usually used to analyze the uncertain parameters with bounds. In engineering, it is well known that the complete probabilistic information of the parameters is often difficult to obtain, and the probabilistic methods cannot solve the above problems.

The interval method is a commonly used non-probabilistic method, and is widely applied in analyzing uncertainty issues involving interval parameters. Revol et al. [7] proved that the results obtained by using the interval method can satisfy the containment property. Chebyshev polynomial method [6, 8] is a well-developed interval method in uncertainty analysis, which can determine the lower and upper bounds by solving an interval polynomial function [6, 8]. It is noting that Chebyshev polynomial method can build a surrogate model between the uncertain interval parameters and the structural responses of deployable structures based on a small number of interpolation points, which is suitable for analyzing the uncertainty of deployable structure in engineering. Therefore, Chebyshev polynomial method is used to quantify the uncertainty of the structural responses in this study.

The deployment process of deployable structures involves the coupling of rigid-body motion and structural deformation. Common dynamic analysis methods, including finite element method (FEM) and multi-body dynamics (MBD), have difficulties when analyzing the dynamic responses of deployable structures. FEM has difficulties in analyzing the motion of deployable structure. MBD [9, 10] might have difficulties in obtaining structural internal forces. The finite particle method (FPM) is an alternative approach developed based on vector-form mechanics and has been successfully applied to the analysis of structures involving rigid-body motion, including kinematically indeterminate structures [11], deployable structures [4] and mechanism with clearance joints [12-14]. FPM can separate structural deformation from rigid-body motion by virtual reverse motion. Moreover, FPM exhibits excellent scalability and can be integrable with other analysis approaches easily. Therefore, it is possible to effectively conduct the uncertainty analysis of the entire process of deployable structure by combining Chebyshev polynomial method and FPM.

In this study, the uncertain analysis of deployable structures is conducted based on finite particle method (FPM) and Chebyshev polynomial method, which have low computational cost. Firstly, the modeling method of deployable structures and the corresponding elements of FPM are briefly described, respectively. Subsequently, the Chebyshev polynomial method is introduced and a non-intrusive uncertainty analysis method is proposed by combining FPM and the Chebyshev polynomial method. Finally, the uncertain dynamic analysis of a typical deployable structure is conducted.

2. Analysis method of deployable structures based on FPM

In FPM, deployable structures are discreted by particles and elements. Particles have mass and are subjected to forces, while elements characterize the interaction relationships between particles. The equations of motion of a particle follow Newton's second law. A particle has three translational DOFs and three rotational DOFs, and the equations of motion are expressed as:

$$m\ddot{\mathbf{d}} = \mathbf{F}^{\mathrm{ext}} - \sum \mathbf{f} - \mathbf{F}^{\mathrm{dmp}} \tag{1}$$

$$\mathbf{I}\ddot{\boldsymbol{\theta}} = \mathbf{M}^{\mathrm{ext}} - \sum \mathbf{m} - \mathbf{M}^{\mathrm{dmp}} \tag{2}$$

where m is the mass of the particle, $\mathbf{d}$ is the particle translational displacement vector, $\mathbf{F}^{\mathrm{ext}}$ is the particle external force vector, $\sum \mathbf{f}$ is the particle internal force passed from beam elements, $\mathbf{F}^{\mathrm{dmp}} =$ is the damping force, $\boldsymbol{\theta}$ is the particle rotational displacement vector, $\mathbf{M}^{\mathrm{ext}}$ is the particle external moment vector, $\sum \mathbf{m}$ is the particle internal moment vector passed form beam elements, $\mathbf{M}^{\mathrm{dmp}}$ is the damping moment, $\mathbf{I}$ is the inertia matrix of the particle, and $\dot{\mathbf{d}}$ and $\dot{\boldsymbol{\theta}}$ are the particle translational and rotational velocities, respectively.

Deployable structures typically comprise links and revolute hinges, as illustrated in Figure 1. Beam elements and revolute hinge elements in FPM are adopted to model the links and revolute hinges of deployable structures, respectively. The internal forces of beam elements are calculated based on the Euler-Bernoulli beam theory and fictitious reverse motion. The motion of the particles in revolute hinge elements are modeled by coupling the translational degrees of freedom and the degrees of freedom perpendicular to the rotational axis. Interested readers are referred to the literature [4] for more details of the derivation.

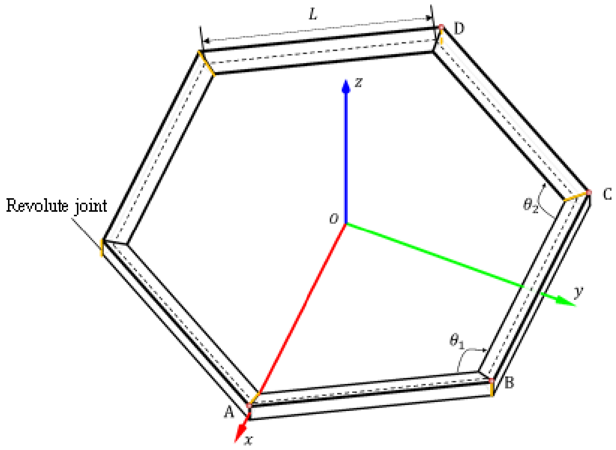

Figure 1: Physical model of deployable structure

Additionally, it is essential to consider physical phenomenon during the deployment process of deployable structures, such as size effects of links and contact constraints between the end cross-sections. Virtual beam elements and face-to-face contact elements of FPM are developed to model the size effects and contact constraints mentioned above, respectively. Torsion spring elements have been developed to provide the expanding driving force of deployable structures. The details of the derivations of the above analysis elements can be referred to the literature [4].

3. Approximation method of dynamic response based on Chebyshev polynomial method

Chebyshev polynomial method is a commonly used for approximating continuous polynomials. It has been demonstrated that the approximation accuracy of Chebyshev polynomials is better than most other types of truncated series. Additionally, the Chebyshev polynomial method does not require the high-order derivatives of interval functions. The upper and lower bounds of the responses can be determined by Chebyshev surrogate models, which is one of the most computationally efficient ways to evaluate the mentioned bounds and effectively controls wrapping effect from the interval arithmetic operations. The details of the Chebyshev polynomial method are provided in the literature [8].

A *k*-dimensional continuous function $f(\mathbf{x})$ can be approximated by the multi-dimensional Chebyshev polynomials. For a multi-dimensional problem, the polynomial of degree n of Chebyshev polynomials is expressed as:

$$C_{i_1 i_2 \dots i_k}(\mathbf{x}) = C_{i_1 i_2 \dots i_k}(x_1, x_2, \dots, x_k) = \cos(i_1\theta_1)\cos(i_2\theta_2)\dots\cos(i_k\theta_k) \tag{3}$$

where the subscript $i_i = 0,1,2,\dots,n$ represents the degree of the Chebyshev polynomial, $\theta_i = \arccos\left(\frac{2x_i-(\overline{X}_i+\underline{X}_i)}{\overline{X}_i-\underline{X}_i}\right) \in [0,\pi]$, and $\mathbf{x} = [x_1, x_2, \dots, x_k]^{\mathrm{T}}$ is a *k*-dimensional vector. Then the *k*-dimensional continuous function $f(\mathbf{x})$ can be approximated as

$$f(\mathbf{x}) \approx \sum_{i_1=0}^{n} \dots \sum_{i_k=0}^{n} \left(\frac{1}{2}\right)^l f_{i_1,\dots,i_k} C_{i_1,\dots,i_k}(\mathbf{x}) \tag{4}$$

where the constant coefficient $f_{i_1,\dots,i_k}$ in the multi-dimensional problem can be calculated as [8]

$$f_{i_1,\dots,i_k} \approx \left(\frac{2}{n+1}\right)^k \sum_{i_1=1}^{n} \dots \sum_{i_k=1}^{n} f\left(x_1^{j_1}, \dots, x_k^{j_k}\right) C_{i_1\dots i_k}\left(x_1^{j_1}, \dots, x_k^{j_k}\right) \tag{5}$$

where $x_i^{j_i} = \frac{\overline{X}_i+\underline{X}_i}{2} + \frac{\overline{X}_i-\underline{X}_i}{2}\cos\theta_i^{j_i}$ and $\theta_i^{j_i} = \frac{2j_i-1}{n+1}\frac{\pi}{2}$, $j_i = 1,2,\dots,k$ and $i = 1,2,\dots,n+1$.

When analyzing the deployment process of a deployable structure, the system is assumed to contain k uncertain parameters of which the bounds have got. The above uncertain system can be transformed into $(n+1)^k$ systems with deterministic parameters based on the Chebyshev polynomial method, which effectively reduced computational costs. Then the above systems can be established computational models and analyzed the dynamic responses of them by using FPM. Furthermore, a surrogate model between the interval parameters and structural responses is established through the Chebyshev polynomial method. Finally, the upper and lower bounds of the target response during the entire deployment process of the deployable structure are obtained through iterative calculations and interval arithmetic operations.

4. Uncertain analysis of deployable structure.

In this section, a typical deployable structure with an interval parameter is analyzed. It is noted that discussion focuses on the system with only one uncertain parameter is discussed in this paper.

Bennett linkage is a widely used deployable structure, whose deployment process is shown in Figure 2 The uncertain responses of the mechanism is analyzed by the non-intrusive algorithm based on Chebyshev polynomial method and FPM. The schematic of Bennett linkage is shown in Figure 3. Bennett linkage is composed of four links and four revolute joints. The parameters of Bennett linkage are given in Table 1. The side length of square cross-section of links s assumed to contain 1% of its nominal value, which is expressed as

$$\hat{a} = a(1 + 0.01\xi) \tag{6}$$

Bennett linkage is modeled by beam elements, revolute hinge elements, virtual beam element, face-to-face contact elements, and torsion elements. The details of the computational model can be referred to the literature [4]. The surrogate model between $\hat{a}$ and the structual responses is established by 3rd-order Chebyshev polynomials. The uncertain displacement response and uncertain velocity response of Bennett linkage are analyzed, respectively.

Table 1: Geometrical and material parameters of Bennett linkage

Parameter	Value
Length of link L (m)	0.45
Side length of square cross-section of link a (m)	0.035
ω (°)	54.7
μ_B (°)	22.5
λ (°)	45.0
Young's modulus of link E (Pa)	1×10^9
Poisson's ratio of link ν	0.3
Density of link ρ (kg/m^3)	800
Contact stiffness per unit length of face-to-face contact element K^C (N/rad)	1×10^4
Torsional stiffness per unit length of torsion spring element K^F (N/rad)	5

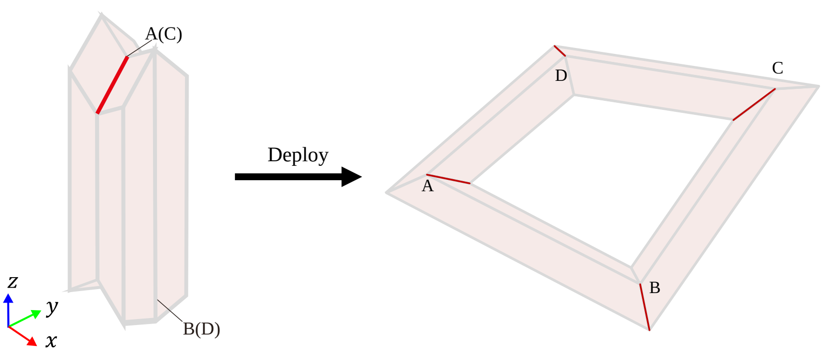

Figure 2: Deployment process of Bennett linkage

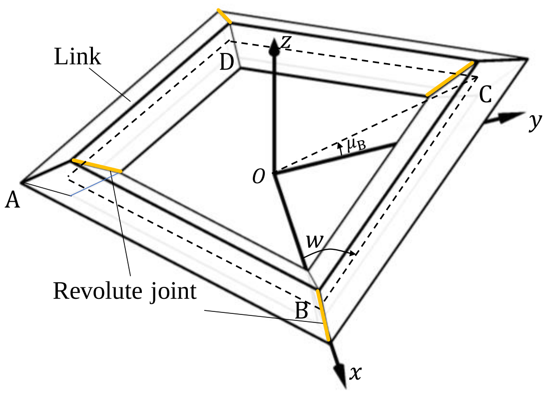

Figure 3: Schematic of Bennett linkage

Figure 4(a) depicts the time histories of the distance between point B and D. It can be observed that the distance responses of the mechanism with an interval parameter $\hat{a}$ are close to that of the mechanism with deterministic parameters during the deployment process for a period of 1 s. The upper and lower bounds of the distance BD are similar to each other during the first-time deployment process, indicating a minor influence of $\hat{a}$ on the uncertainty of distance response. The difference between the bounds does not increase significantly during the second-time deployment process, with a maximum difference of 0.063 m.

Figure 4(b) illustrates the time histories of the velocity of the *z*-directional velocity of point B. It can be observed that before the first-time collision between the end cross-sections, the upper and lower boundary curves essentially coincide with each other; after the first-time collision, the oscillation of the curve intensifies, and the difference between the velocity response of the mechanism with an interval parameter $\hat{a}$ and that with deterministic parameters gradually increase, which indicates that the collision may intensify the uncertainty of response. During the second-time deployment process, the difference between the bounds increases, with a maximum difference of 4.38 m/s.

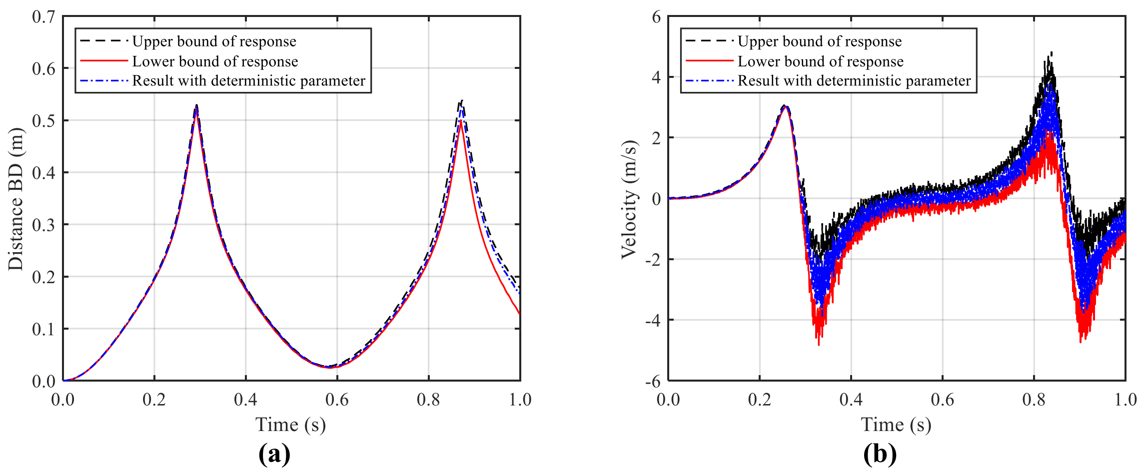

Figure 4: Time histories of dynamic response of Bennett linkage: (a) distance BD; (b) z-directional velocity of point B

5. Conclusion

In this study, the uncertain analysis of deployable structures is conducted based on FPM and Chebyshev polynomial method, which have low computational cost. The uncertain dynamic responses of a typical deployable structure are analyzed considering an interval parameter by the proposed method. The results indicate that the uncertainty of the side length of square cross-section of link has a weaker impact on the structural response uncertainty due to the value of it being relatively small. Moreover, the collision during the deployment process may enhance the response uncertainty, and the strong interactions of deployable structures during the deployment process should be avoided.

Acknowledgements

This work was supported by the National Key Research and Development Program of China (Grant No. 2023YFC3805600). This research was also supported by the National Natural Science Foundation of China (Grant Nos. 52378203 and 52238001).

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