Proceedings of the IASS 2024 Symposium
Redefining the Art of Structural Design
August 26-30, 2024, Zurich Switzerland
Philippe Block, Giulia Boller, Catherine DeWolf,
Jacqueline Pauli, Walter Kaufmann (eds.)

Parametric design and digital fabrication of disassemblable elastic timber gridshells with principal curvature lines network and carpentry joints

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Abstract
This work explores the design and digital fabrication of elastic timber gridshells integrating principal meshes. These networks guarantee, among others, planar faces discretization, node offset properties and orthogonal meetings of the components what simplifies the joining solution for what demountable right-angled wood-wood carpentry joints have been used. A design-to-production workflow has been addressed employing a bottom-up and construction-aware design strategy to deal with geometric complexity in early design stages using computational design methods resulting on a much simpler manufacturing process by means of accessible carpentry machinery such a 3-axis CNC router. The use of active bending lets the structure to be erected from initially slender straight laths and to be covered by developable strips what makes it possible to machine all the elements on structural timber boards with just one tool, varying only the thickness of the support according to that required by each component. The methodology has been successfully validated through the construction of a small 7.5m span temporary prototype.

Keywords: Active bending, CNC machining, computational design, construction-aware design, design for disassembly, lightweight structure.

1. Introduction
A timber gridshell can be understood as a grid of thin timber laths that extend over a reference surface of the structure. These laths, when curved tangentially to the surface, generally adopt a combination of stresses due to torsion and bending in the two axes of inertia of the cross-section because of the geodesic torsion, normal curvature and geodesic curvature of the underlying network of curves over the reference surface. Some patterns lack some of these components, such as geodesic curves, with vanishing geodesic curvature; asymptotic curves, in which the normal curvature is zero; or principal curvature lines (PCL), whose geodesic torsion is null. The imposition of these conditions can allow, at the expense of a certain formal restriction, to simplify the manufacturing and construction process [1], in contrast with free form structures whose elements present curvature in both axes of inertia and torsion.

Traditionally, two main strategies have been used for permanent elastic timber gridshells meshing. On the one hand, the compass method used by Frei Otto, which results in a equidistant nodes grid that, once deployed and erected, presents three components of curvature so squared cross-section laths are frequently used. On the other hand, the so-called Julius Natterer's ribbed shells were meshed using geodesic curves so constant distance between nodes was not possible and laths were frequently wider than thicker due to the lack of geodesic curvature. Furthermore, several research have been carried out within the use of asymptotic curves in elastic - timber or aluminum - gridhsells as shown in [2] and some pre-curved timber gridshells have been made featuring PCLs as Solemar Therme in Bad Dürrheim.

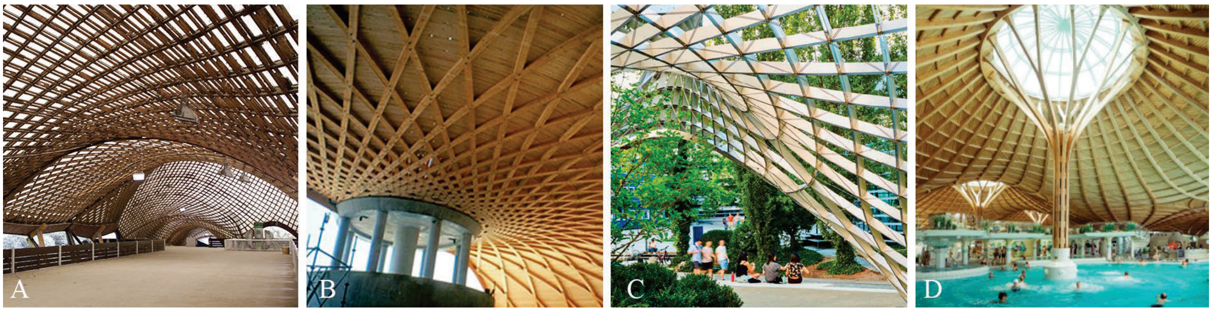

Figure 1. A) Mannheim Multihalle. Frei Otto. B) Swimming pool at St. Quentin. Julius Natter C) INSIDE/OUT pavilion. Eike Schling [2] D) Solemar Therme in Bad Dürrheim. Geier and Geier.

However, the potential of using PCLs in elastic timber gridshells remains unexplored yet. They constitute a network of orthogonal and conjugate curves aligned with the maximum and minimum normal curvature, which has implicit advantages from the point of view of fabrication-aware design of actively bent gridshells. It allows for perpendicular torsion free nodes, which has great advantages for multi-layer systems [3] and being conjugate curves implies that, by taking the tangents of one family at points along the other one, they form a developable surface. On an infinitesimal scale, two very close tangents are supposed to be coplanar so in a discrete version, faces of a principal mesh are flat quadrilaterals [1]. This operation can be carried out with normal and binormal vectors at points of the conjugated family resulting developable surfaces too. These, together with tangential surfaces, form three families of orthogonal surfaces generating structures with triple orthogonality [4].

The generation of this type of networks can be approached, on the one hand, by a post-rationalisation (top-down) process of a pre-defined target surface [5]. Oberbichler et al. [1] extract PCL from a surface by searching, from a defined point, trajectories whose geodesic torsion is zero. Even though this method is open to a wide variety of free forms, the mesh density control is still quite complex. Chai et al. [6] propose a strategy to reach that goal, among others, by segmenting the surface through PCL drawn from umbilical points to generate a discrete surface of developable strips. This method does not allow for an exhaustive control over the curvature of the mesh laths, which makes it difficult to translate these shapes into actively bent structures. On the other hand, PCL can be generated from pre-rationalisation (bottom-up) strategies. Mesnil et al. [3] employ moulding surfaces, a specific case of Monge surfaces [7], to generate isogonal moulding surfaces: single layer structures with straight elements with a very high degree of repeatability. The surface originates by the sweep of a curve (generatrix) that is oriented along another curve (directrix) laying on its rotation minimizing frame. The result is intrinsically a PCL mesh being the generatrix curves geodesic curves of the surface. Moreover, these structures are easily discretisable in PQ meshes and features torsion free orthogonal nodes. Besides, the two-curve generation system allows precise control over the curvature of, at least, one of the families, which is very useful for the generation of actively bent structures where this curvature conditions the shape. This generation strategy, analogous to scale-trans-surfaces [8], makes it possible to replicate numerous architectural shapes from a graphic point of view, what is easily accessible to architects and engineers.

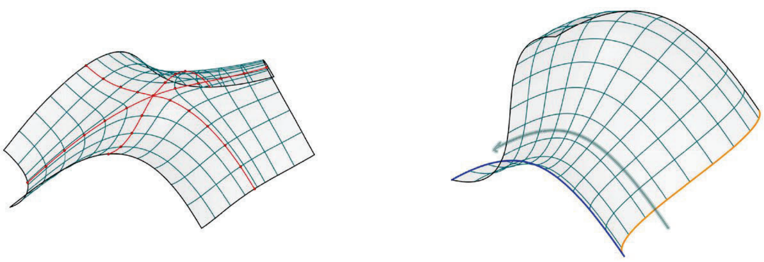

Figure 2: top-down PCL generation from an umbilic point using Grasshopper plugin Bowerbird [1] (left) and bottom-up PCL generation using Monge surfaces (right) from a directrix (blue) and a generatrix (orange).

Regarding to its construction, traditional timber gridshells have historically presented a high degree of craftmanship. Although, digital fabrication and industrialization have been widely adopted in pre-curved timber gridshells as shown in La Seine Musicale, among others, where complex technology such as 5-axis robotic arm machining has been implemented notably increasing economic costs. However, to the authors' knowledge, this level of development has been just implemented in the field of strained gridshells in the design and construction of the Wisdome Stockholm gridshell [9], which combines a pre-curved initial layer and on site bent thin laths connected by dowels-like joints.

In this work, a novel design and fabrication strategy is addressed to get strained timber gridshells design and construction closer to digital fabrication and industrialization logic thanks to the exploration of how to integrate PCLs patterns, 3-axis CNC machining and demountable CNC carpentry joints. The proposed methodology allows for a high degree of accuracy using an accessible 3-axis CNC router for structural boards machining, what has been tested by the construction of a full-scale demonstrator (Figure 3**)**.

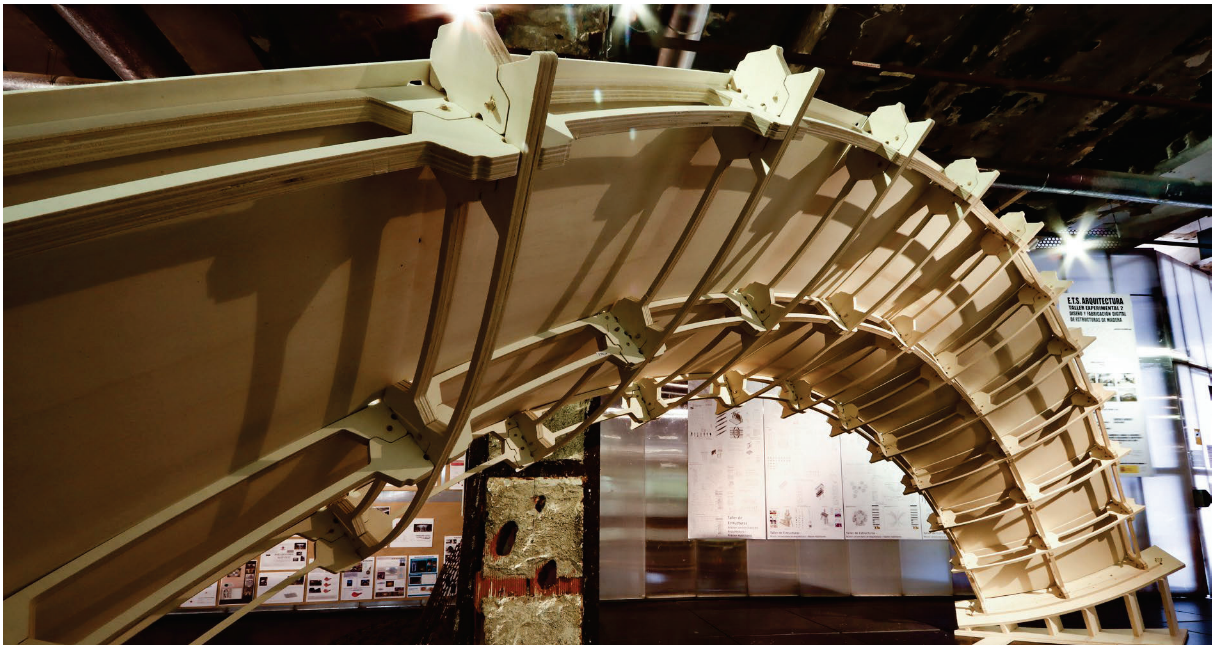

Figure 3: Full-scale prototype.

2. Joinery and disassembly

Gridshells have historically been used for the construction of singular architectures, so a possible alternative to its design and construction is explored in this research through a digital, agile and accessible fabrication relying on two main ideas: 1) the use of structural LVL panels, which significantly reduces defects in wood leading to a much more efficient product that can be easily machined by CNC milling and offers great freedom when designing carpentry joints (Section 2.1). In the present work, standard plywood panel measuring 2,44 m × 1,22 m are used for they can accommodate long strips of small cross-section easily although a splice joint has been developed to obtain laths longer than 2,44 m; 2) The implementation of **3-axis CNC machining** for being an accessible and affordable method. It offers high manufacturing precision and it is suitable for machining pieces joining at 90 degrees due to the milling direction being perpendicular to the board.

These aspects require the use of meshes with orthogonal joints, which is fully consistent with the use of principal meshes. Furthermore, in this type of constructions, the use of several layers it is generally required to provide a competent structural response to medium and large spans so joint design is key when it comes to the erection of the structure and has a significant impact on the overall stiffness of the structure, as well as on the assembly and erection times.

2.1. CNC Orthogonal connection system

In elastic timber gridshells, the connection system is one of the main design aspects. It must resolve the encounter between the different lath directions (nodal joint), generate a composite section with high inertia (multilayer system) while maintaining the thickness of the individual laths (which is limited by bending radii) and facilitate the assembly process. In traditional elastic gridshells, the nodal connection is articulated, generally resolved by means of a bolt placed at the intersection of the lamellas to allow free rotation between them, since the angle they form is different at each point. In these structures, the multilayer connection system between the different timber layers is based on the use of mechanical fasteners that produce semi-rigid joints with sliding in two shear planes [10]. This connection must transmit shear forces and can be made in different ways: at the nodes using bolts, as in the Mannheim Multihalle (Fig. 3 left); at the members by means of timber shear blocks fixed with screws to the laths, as in the Savill gridshell (Fig. 3 right); or at both places, as in the Downland gridshell (Fig. 3 middle), in which shear blocks at members and steel patented connection at nodes were used [11]

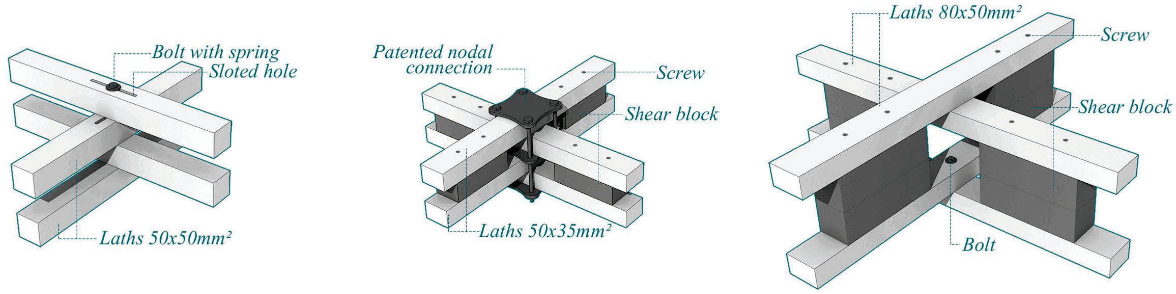

Figure 4: Connection system in traditional elastic timber gridshells: Mannheim Multihalle gridshell (left), Weald and Downland gridshell (middle) and Savill Garden gridshell (right)

In this work, and as a result of the 3-axis CNC logic in which milling direction is perpendicular to the board, an interlocking CNC orthogonal carpentry joint is designed to resolve both nodal connection and multilayer system. In contrast with traditional solutions, the lath height of the proposed geometry does not remain constant being higher at the nodes, which provides two great advantages to the system: on the one hand, the drilling of the lath for bolts and screws is omitted, avoiding the loss of section and the subsequent risk of breakages at the time the disassembly and reuse are favoured; and, on the other hand, the shear forces are transmitted by wood-wood contact in a single shear plane, which qualitatively improves the stiffness of the joint. The solution proposed in this work features a half lap joint whose shape allows for the approximation of one lath over the corresponding one (it is to say, the parallel one). Furthermore, in order to prevent the pieces from separate, wooden dowels are used. Figure 5 show how the successive elements are interlocked and fixed.

Figure 5: Joint interlocking sequence. Splice joint (left), orthogonal pieces (middle) and full joint (right).

To overcome length limitations of using standard plywood boards, the laths feature a longitudinal splice joint at some nodes. Figure 4 left shows a solution based on the concept of a half lap dovetail joint. In this case, to ensure easy dismountability, the notch angle is parallel to the grain. The wooden dowels provide the tensile strength, while the tenons ensure bending forces transmission by wood-wood contact. The assembly of the joint alternately causes one piece to block the other generating a self-stable system that resists out-of-plane moment as well (Figure 4 right).

3. Design workflow

The aforesaid concepts are put together through a workflow that consists of the following phases: 1) A design stage addressed by a Basic model containing the main features of the structure 2) a Reference model (multilayer wireframe geometry) and bending stress analysis calculation; 3) a Detailing model that renders thicknesses and details from which basement and cladding are determined; and 4) a 2D model containing deployed geometry which will be manufactured in a fabrication stage before assembly.

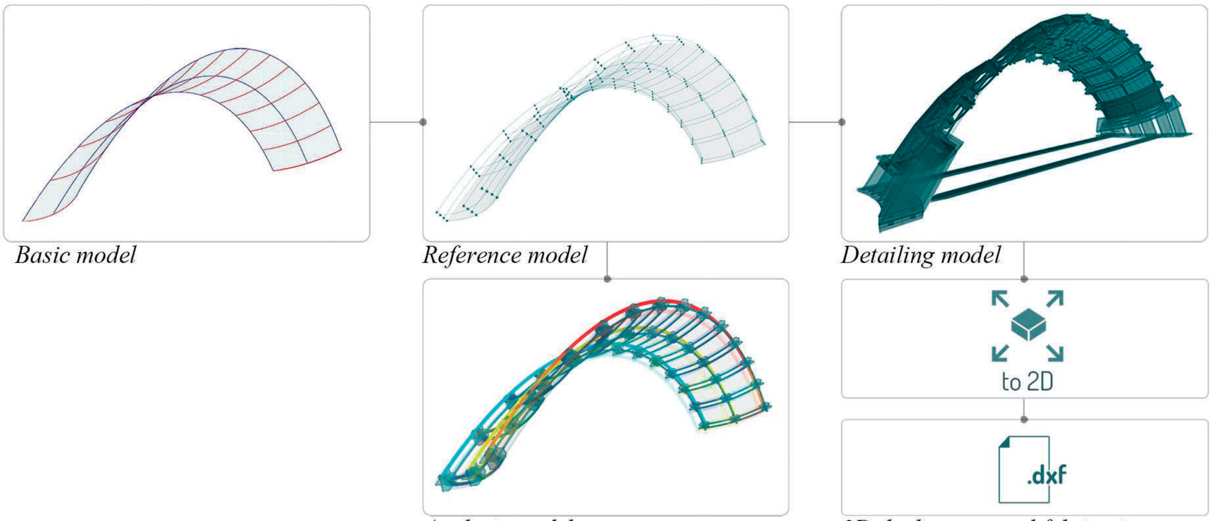

Figure 6: Design workflow.

3.1. Basic model

It consists of an agile model in which the geometry of the structure (span and proportions), the grid layout (surface meshing and curvature) and support conditions (angle and height at the meeting with ground plane) are defined. As a result, a model composed of a PCL network and its underlying surface is extracted as exemplified in Figure 7. The curvature radius (R) to lath thickness (t) ratio is considered for this preliminary design stage, which ranges from R=150t to R=300t. Auto-intersections between generatrix curves are also checked before generating the Reference model.

The specific geometry of the prototype is inspired by the vaults of Eladio Dieste and starts from a flat parabolic directrix and an s-shape generatrix that is oriented along the rail curve resulting in Figure 7 middle. Due to logistics and available means, a part of this geometry was selected to be developed (Figure 7 right).

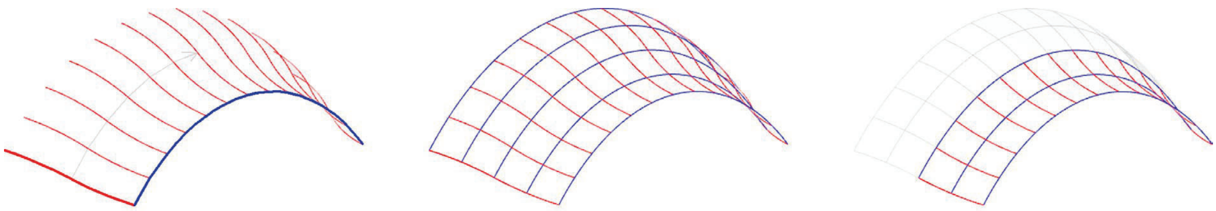

Figure 7: Generation of reference grid. Displacement of the generatrix along the directrix (left), PCL network (middle), selected piece for fabrication (right).

3.2. Reference model

For the construction of the prototype a 25 mm thick poplar plywood board was used, and a lath height of 15 mm was adopted. To avoid lath deformation at the joint, its height was almost doubled obtaining a “S” distance of 44.8 mm between layers. From the Basic model geometry, nodes are computed at the intersection of both PCL families. To generate the multilayer wireframe structure (Reference model) those nodes are moved along reference surface’s normal axes (evaluated at the very nodes) a distance “S” at a time resulting on a total of four consecutive layers. Reference surfaces of each layer are calculated as an offset of the original one at a distance "S" as well. Figure 8 shows this process for a single node (N0) of the reference surface obtaining N1, N2 and N3 at a multiple of the distance "S" between layers. This Reference model will be the initial input for the upcoming models.

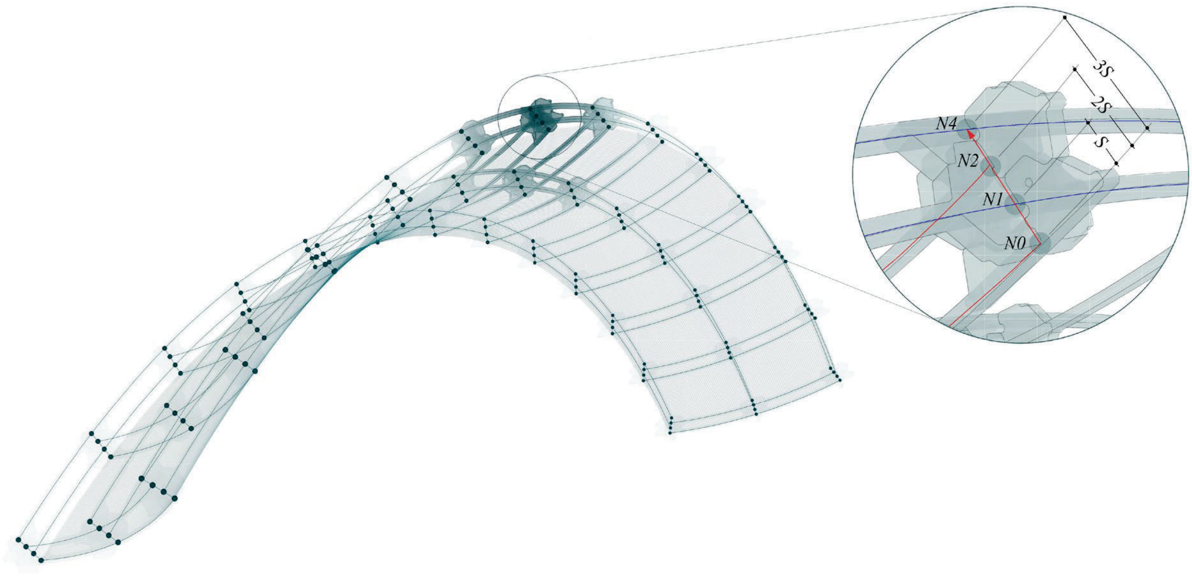

Figure 8: Reference model generated from joint geometry.

3.3. Curvature analysis model

Once the reference geometry is set, it is essential to determine bending stresses, what is addressed in this work in a geometrical way [12]. For the given networks, the curvature vector ($\vec{k}$) at each point of the curves can be extracted discretely along its entire length and projected according to the directions normal and binormal to the surface, obtaining the normal ($\vec{k}_N$) and geodesic ($\vec{k}_G$) curvatures (Figure 9).

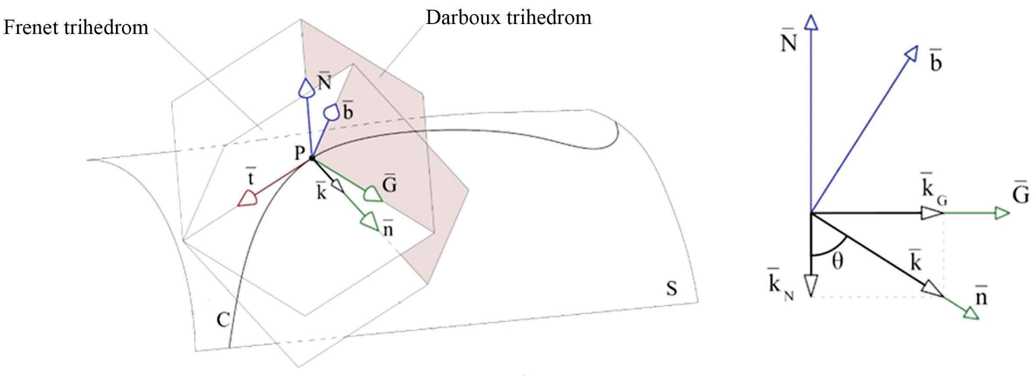

Figure 9: Relationship between curvatures in space (left) and in 2D projection (right).

Denoting the y-axis and the z-axis as the bending axes of the lath's cross-section, tangent and perpendicular to the surface of the gridshell respectively, and "b" and "h" as the cross-sectional dimensions associated with each axis, the corresponding maximum initial bending stress due to normal and geodesic curvature, $\sigma_{m,y,0}$ and $\sigma_{m,z,0}$ respectively, can be calculated as indicated below being E_{edge} y E_{flat} the corresponding Young modulus of the plywood board tested edgewise and flatwise, respectively.

$$\sigma_{m,y} = k_N E_{edge} \frac{h}{2} \tag{1}$$

$$\sigma_{m,z} = k_G E_{flat} \frac{b}{2} \tag{2}$$

For the final verification of the structure due to the elastic bending of the lamellas, it must be checked that the final bending stresses do not exceed the board's bending strength [13]. Since plywood is an orthotropic material, its strength varies depending on the load direction. Eq. (3) is used in this work for the bending verification of the cross-section, being $f_{m,0,Edge,d}$ and $f_{m,0,flat,d}$ edgewise and flatwise design bending strengths, respectively, and being $\sigma_{m,y}$ and $\sigma_{m,z}$ bending stresses due to active bending.

$$U = U_N + U_G = \frac{\sigma_{m,y}}{f_{m,0,edge,d}} + \frac{\sigma_{m,z}}{f_{m,0,flat,d}} < 1 \quad (3)$$

The following mechanical properties of the plywood panels have been considered in this work: $f_{m,0,edge}$= 27.30 MPa, $E_{m,0,edge}$= 6165 MPa, $f_{m,0,flat}$ = 29.80 MPa and $E_{m,0,flat}$ = 4557 MPa.

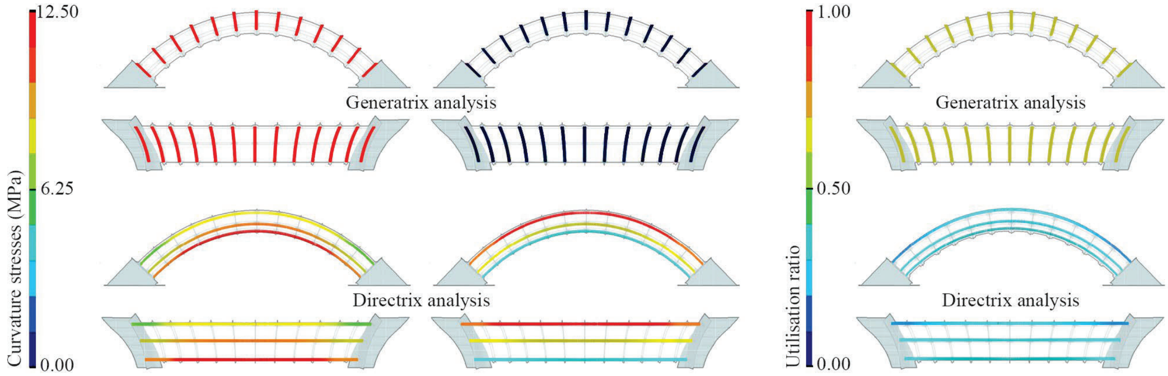

Figure 10: Stresses due to normal curvature (left) geodesic curvature (middle) and utilization ratios (right).

3.4. Detailing model

From the Reference model, a Detailing model (Figure 11) containing all the geometry necessary for the subsequent design of supports and covering of the structure is obtained. Once the cross-section size is known, the laths geometry is obtained by an offset of the lath's neutral axis according to the cross-section axes (section's y-axis and z-axis), which coincide with the normal and binormal vectors of the surface evaluated along the lamellas. Furthermore, the geometry of the joints in space can be obtained by means of a local planes system. Joints are modelled in the global XY plane, with the origin of this plane being located on the lath's neutral axis, and then reoriented in the structure local planes formed by the nodal points, the normal vector to the surface at that point and the tangent vector to the curve.

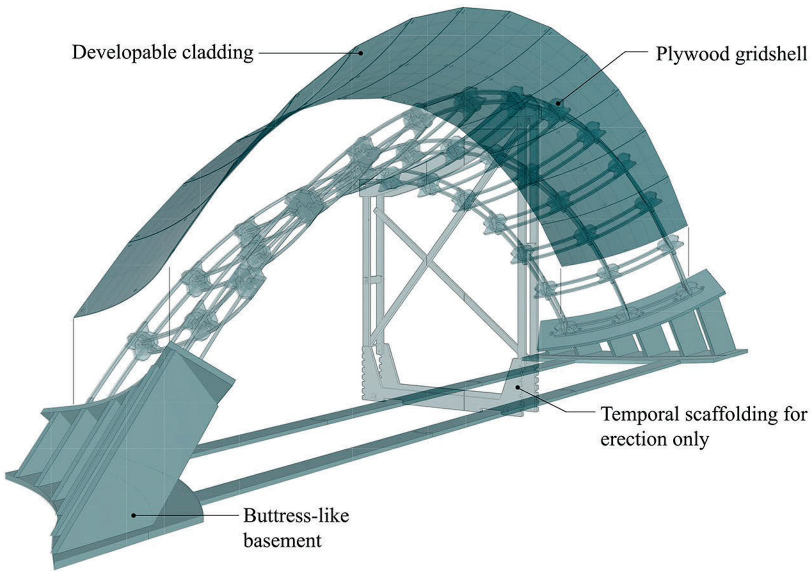

Figure 11: Detailing model including scaffolding for erection.

Thanks to the planar condition of generatrix curves, a buttress-like basement is designed to resolve the transition with the floor. This basement gathers the first and last generatrix, which are machined as solid pieces, to fix them in place through tenons emerging from the transversal components that cross the base. Several dowels reinforce the joint of the generatrix to the supporting deck too. In order to ensure dismountability, sliding components fixed by wedges are designed for the basement, so that the structure can be easily dismantled once the wedge is removed.

For the covering, a semi-discrete version of the surface is taken generating developable stripes between two consecutive generatrix curves (that are also geodesic curves of the surface). The cladding is materialised in a 3 mm poplar plywood board that is curved elastically and fixed to the grid by means of L-shaped tabs incorporated in the joints (Figure 11 left).

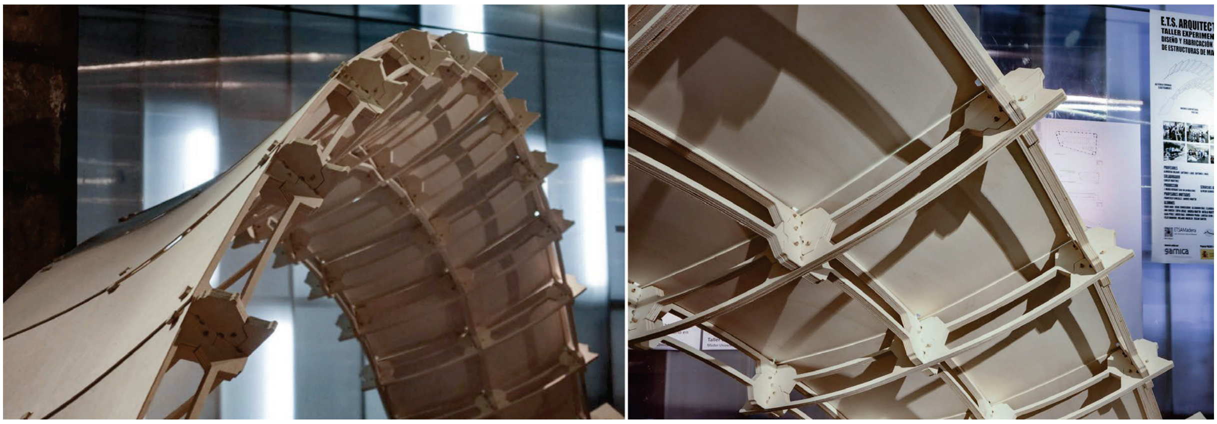

Figure 12. Prototype close-up.

The erection of the pavilion begins with the location of the bases in their definitive position thanks to a pair of braces that determine the separation between buttresses. All the grid members are assembled flat on the floor and a mounted lath-by-lath during the erection process. The first and last solid generatrixes are fixed to the basement in order to accommodate the first family of longitudinal laths (directrix), which spans from one basement to another with an intermediate support on a CNC milled scaffolding for this purpose. Then, upper transversal laths are located first in those nodes where the lower directrix laths are segmented to avoid them to separate. Once this is achieved, the remaining upper generatrix laths are mounted so the structure can be completed by adding upper directrix and lower generatrix laths.

4. Lath segmentation and 2D deployment

In contrast to pre-curved timber gridshells, which are generally machined by means of a 5-axis robotic arm, the manufacturing of the prototype is carried out using 2D 3-axis CNC data. On the basis of the Reference model, a deployment strategy is proposed based on length measuring so PCL and nodes on the Reference model are used to make the translation into 2D according to local parameters. Each curve of the networks is divided according to the points where it is decided to segment the laths for manufacturing resulting in several segments per curve subdivided simultaneously by the nodes. As shown in Figure 13a, for each segment, lengths between nodes (P_0, P_1 and P_2) are evaluated being this length ($L_{0,1}$ and $L_{1,2}$) the parameter used for 2D deployment as straight lines and nodes oriented along the global X-axis. Incorporating lath's height and the geometry of the joint, the CNC milling geometry is obtained (Figure 13b). The rest of the components (basement and cladding) can be easily deployed for them being planar o developable surfaces, respectively.

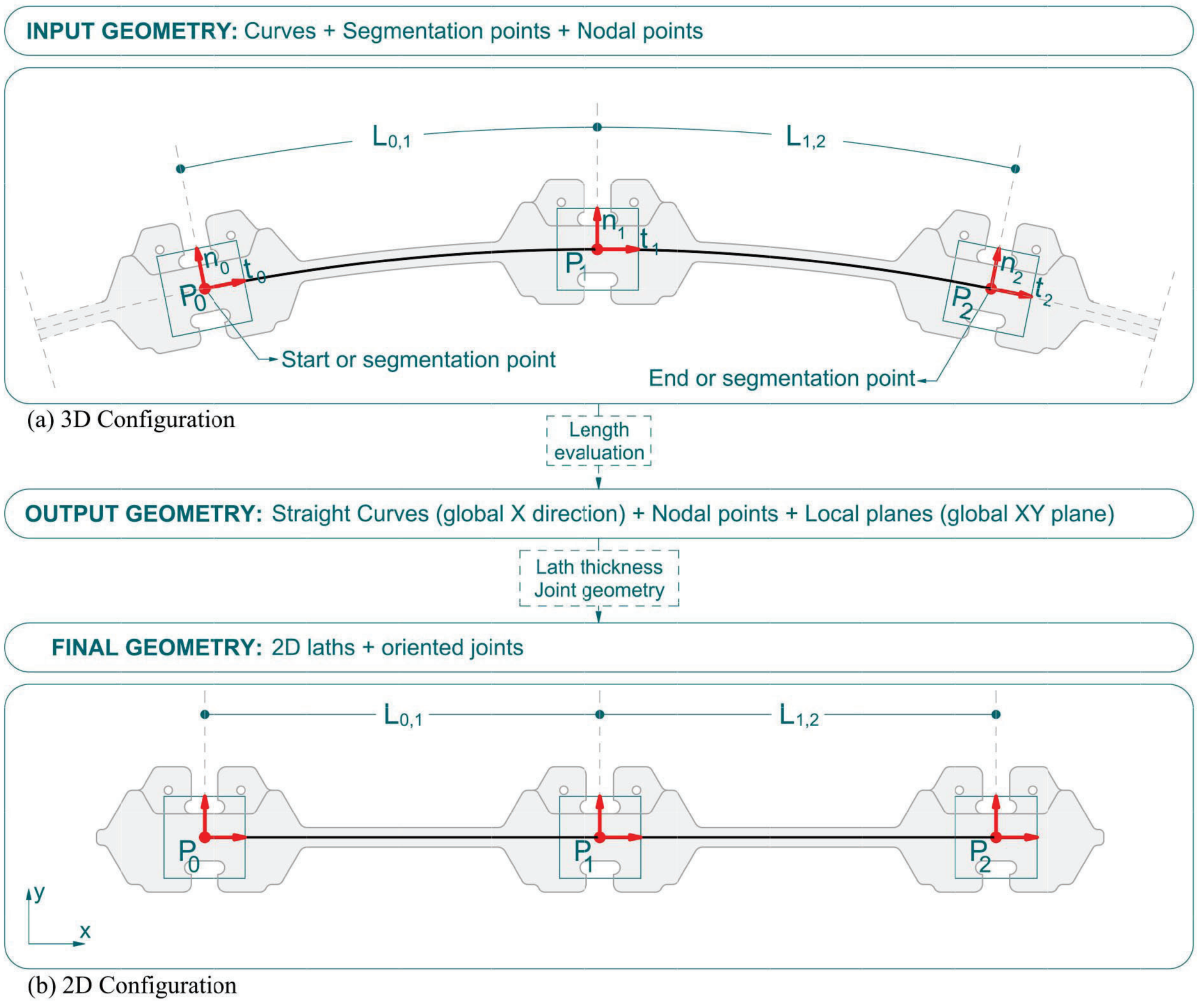

Figure 13: Lath 2D deployment.

5. Conclusions

This work provides an alternative to traditional design of elastic timber gridshells by combining the advantages of active bending and 3-axis CNC machining what, due to the vertical milling process, makes sense when using orthogonal patters such as PCLs and right-angled carpentry joints. This combination results in an accurate, coherent and attractive system using exclusively single-tool machined structural boards what allows for a high degree of industrialization. The proposed carpentry joints, which address nodal and shear connection at the same time, avoid lath perforation and the subsequent loss of cross-sectional area what reduces lath breakages. Where necessary, a demountable splice carpentry joints has been implemented too to overcome length limitation without using traditional methods such as glue scarf joints or overlapping bolted joints. The system allows for a later disassembly and reutilisation of the structure or its components. The strategy has been successfully validated through the construction of full-scale mock-up. Further research of interest is the exploration of extend the formal freedom of the system by using not only PCLs but to orthogonal networks too. Furthermore, the structural analysis of the loaded structure and characterisation of the connection system are key aspect to explore in the future.

Acknowledgements

This work was undertaken within the framework of the R&D project PID2020-112954RA-I00, financed by MCIN/AEI/10.13039/501100011033. We would like to sincerely thank Mr. José María Herranz for his collaboration in the machining and assembly of the prototype. Also, thanks are due to Grupo Garnica Plywood S.A.U. for the supply of the material.

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