Proceedings of the IASS 2024 Symposium
Redefining the Art of Structural Design
August 26-30, 2024, Zurich, Switzerland
Philippe Block, Giulia Boller, Catherine DeWolf,
Jacqueline Pauli, Walter Kaufmann (eds.)

An automated topology optimization framework for material minimization in concrete building structures

Simone M. PETER*, Caitlin T. MUELLER

* Massachusetts Institute of Technology
77 Massachusetts Avenue, Cambridge, MA 02139, USA
smpeter@mit.edu

Abstract

Reducing embodied carbon in residential buildings with similar multi-story typologies motivates the development of frameworks to limit the construction industry's environmental impact. One approach for optimizing material use is to locally optimize the structural geometry of individual building components. However, to more globally achieve the highest possible embodied carbon savings with such optimization methods, a holistic approach for large-scale buildings needs to be considered. Therefore, this paper proposes a topology optimization methodology to reduce the building-scale level embodied carbon of concrete structures. Based on the Julia TopOpt package, a minimum volume optimization problem formulation with a combined displacement and compliance constraint is implemented. The basis for the optimization are two-dimensional building frame structures, mimicking the standard design of multi-story buildings. Comparing the results with the material volume of well-tried building techniques shows possible concrete material reductions ranging from 10% to 50%. To make this method easily accessible, the paper develops an automated input framework to optimize standardized multi-story buildings while only requiring a small number of user-provided boundary conditions and parameters. This workflow enables a preliminary sensitivity analysis and quantification of potential embodied carbon savings on a building-scale level at an early design stage.

Keywords: building-scale optimization, embodied carbon, parametric design, structural optimization, topology optimization, automatic differentiation

1. Introduction

The building construction industry is responsible for 6% of global carbon emissions [1], which underlines the importance of reducing embodied carbon in structures. Many recently constructed residential buildings comprise multi-story structures, adhering to a similar architectural typology. One strategy for more efficient material use is the structural optimization of individual components by using, for instance, topology or shape optimization methods. However, to understand the potential material reduction across the full design space of a building, the influence of these methods should be considered holistically across the full building.

1.1. State of the art

Structural optimization is a fairly mature field of research in civil engineering and architecture, which has recently been utilized to reduce embodied carbon in structures (Ching et al. [2]). It can be divided into size, shape, topology, and multi-objective optimization (Mei et al. [3]). These optimization methods can be tailored to reinforced concrete structures (Afzal et al. [4]) and can achieve material reductions of almost 50% (Ismail et al. [5], Costa et al. [6]).

A first step toward holistic models is the simultaneous optimization of floor slabs and foundation systems, reducing the required embodied carbon by 75% compared to classical building designs (Feickert et al. [7]). This value shows the potential of further exploring even more integrated methods as it was shown that the structural geometry has a significant impact on the embodied carbon (Gauch et al. [8]). However, such methods do not fully consider the possible design space by restricting their optimization formulations to selected structural components or discrete design variables (Tamrazyan et al. [9], Kripka et al. [10]).

For the extension of the design space, the use of building-scale topology optimization is promising and can, for instance, be applied for the compliance minimization of braced frames (Stromberg et al. [11]). Another method, which is similar to the idea of this paper, is the minimization of the structural weight (Liang et al. [12]). The included compliance constraint can be replaced by a maximum displacement constraint, which is a more intuitive parameter, especially for engineers (Chen et al. [13]).

Another research topic in topology optimization, besides varying the problem formulation, is the integration of more realistic material mechanics instead of simply assuming an isotropic model. Since, with concrete, a broad range of designs can be fabricated, specific topology optimization formulations exist for this type of material (Stoiber et al. [14]).

The respective problem formulations and applications described above are rather involved, and their results cannot be easily compared to ”business as usual” structural benchmarks such as moment frames. Therefore, these methods should be made more tangible and accessible to structural designers, engineers, and architects and improve collaboration (Smith et al. [15]). This can be achieved by providing frameworks for building-scale topology optimization (Beghini et al. [16]) and for simultaneous optimization of form and function (Zegard et al. [17]). Another interesting area for such automated approaches is the estimation of embodied carbon in steel framing systems (Weber et al. [18]).

1.2. Motivation

There is a significant need to reduce the environmental impact of the construction and concrete industry. One promising research focus is optimization methods for a more efficient material use. However, there is still potential for investigation, especially by applying these methods to full-scale buildings and making them more available to practicing engineers and architects. Therefore, this paper presents a user-friendly framework for applying and testing topology optimization methods on building frames. It expands the design space using continuous variables and only requires a small number of user-provided inputs (Section 2.). The problem considers concrete material, which is, with its flexibility, best suited for the complex shapes resulting from the topology optimization procedure and is also highlighted by the United Nations Environment Programme [1]. The framework allows conducting a sensitivity analysis for different parameters (Section 3.), enabling the quantification of possible material savings compared to as-built structures at an early design stage.

2. An automated topology optimization framework for material minimization

This paper implements a minimum volume optimization problem formulation with a differentiable displacement constraint using the concept of automatic differentiation. Additionally, the formulation includes a novel scaled compliance parameter, improving the otherwise deficient convergence properties of the problem, as demonstrated in Section 2.3.. The topology optimization program uses a continuum-based finite element approach in combination with the gradient-based method of moving asymptotes. The full workflow is displayed in Figure 1, and detailed explanations of each step are provided below. The minimum compliance problem is necessary to determine a reasonable value for C_{tol} as input to the following step, which supports the goal of reducing the required material in the structure. After comparing the determined material of the benchmark example, described in Section 2.4., the framework quantifies the material savings potential of the topology optimization method.

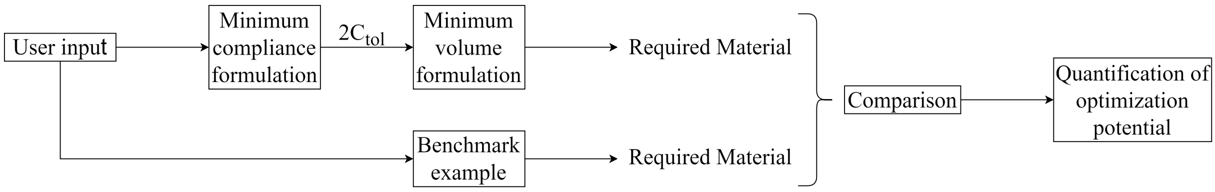

Figure 1: Automated framework consisting of user input (Section 2.1.), minimum compliance (Section 2.2.), novel minimum volume topology optimization formulation (2.3.) and benchmark comparison (Section 2.4.).

2.1. User input

The framework is based on limited user input, as specified in Table 1, excluding the nonintuitive compliance value. It is very applicable for practicing structural designers because no actual formulation of the optimization problem is necessary. Its straightforward way of creating results in a short amount of time allows for a sensitivity analysis of different parameters. The procedure is demonstrated based on a two-dimensional concrete building frame with three stories (see Figure 2 (left)), where the respective values for the input variables are specified in Table 1 and an isotropic material model is implemented. The design domain for the topology optimization is defined as shown in Figure 2 (right). It is discretized with a mesh consisting of 128 × 288 elements, which was identified as a good compromise between precision and computational time for the purpose of this paper.

Table 1: Definition of the user input parameters for the framework and values used for the example building frame structure.

Engineering parameters	Exemplary values
Width [m]	8.0
Height [m]	12.0
Young's modulus E [$\frac{kN}{mm^2}$]	27.0
Poisson's ratio ν	0.20
Gravity load [$\frac{kN}{m}$]	0.0
Lateral load [kN]	15.0
Number of stories	3
Allowable displacement [m]	0.03
Algorithmic parameters	
Compliance scaling factor c_1	1
Compliance scaling factor c_2	1
Filter radius	0.4

2.2. Minimum compliance formulation

As a next step, the inputs are used to perform a standard minimum compliance topology optimization, subject to a volume constraint. Here, the framework follows the problem formulation in Equation (1), which can be considered equal to maximizing the stiffness. The compliance is computed by multiplying the global force vector $\mathbf{F}$ with the displacements $\mathbf{d}$. The formulation aims to optimize the element densities ρ^e, defined as design variables. They are bounded by a minimum density ρ^e_{min} from below (corresponding to a void element) and a value one from above (material element). The displacements $\mathbf{d}$ are computed via a finite element analysis using the global stiffness matrix $\mathbf{K}$, which depends on the design variables and again the force vector $\mathbf{F}$. This computation is described in the state equation $\mathbf{h}$, which enforces the equilibrium condition. Additionally, a volume constraint g is implemented, ensuring that the sum of the element volumes of the material elements, computed by a multiplication of the respective element density ρ^e and the element volume v^e, does not exceed a certain maximum volume V.

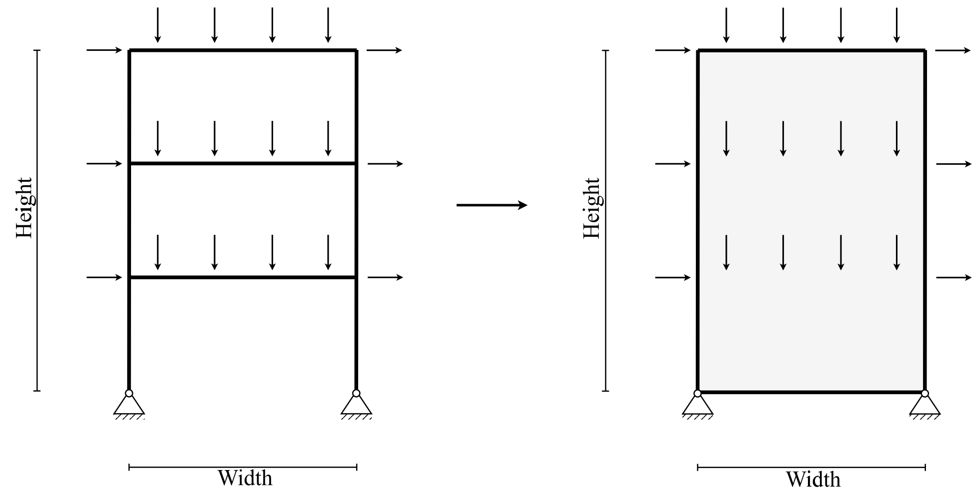

Figure 2: The geometry of the two-dimensional building frame used as a basis for the optimization (left). The respective input values are denoted in Table 1 and the design domain is defined on the right.

$$
\begin{aligned}
\underset{\rho^e}{\text{minimize}} \quad & f = \mathbf{F}^{\mathbf{T}}\mathbf{d} \\
\text{subject to} \quad & \mathbf{h} = \mathbf{K}(\rho^e)\mathbf{d} - \mathbf{F} = \mathbf{0} \\
& g = \sum_{e \in \Omega} \rho^e v^e - V \leq 0 \\
& \rho^e_{min} \leq \rho^e \leq 1 \quad \forall e
\end{aligned} \tag{1}
$$

This formulation results in the optimized structural design in Figure 3 (left) with the specified values for the volume, the maximum displacement at the top right corner of the frame, and the compliance. However, following building code requirements, the maximum allowable displacement is defined by $\frac{h}{400}$, indicating the potential to optimize the structure further.

2.3. Minimum volume formulation

The embodied carbon in the structure should be reduced instead of the volume fraction being prescribed beforehand. Therefore, the next step performs a topology optimization to minimize the material volume, which corresponds to the material area in the displayed two-dimensional case. The objective is subject to a combined displacement and compliance constraint as defined in Equation (2) (right), where this paper is, to the authors' knowledge, the first one to propose and test this formulation. Because of the mono-material assumption, the minimization of material can be considered equal to the minimization of

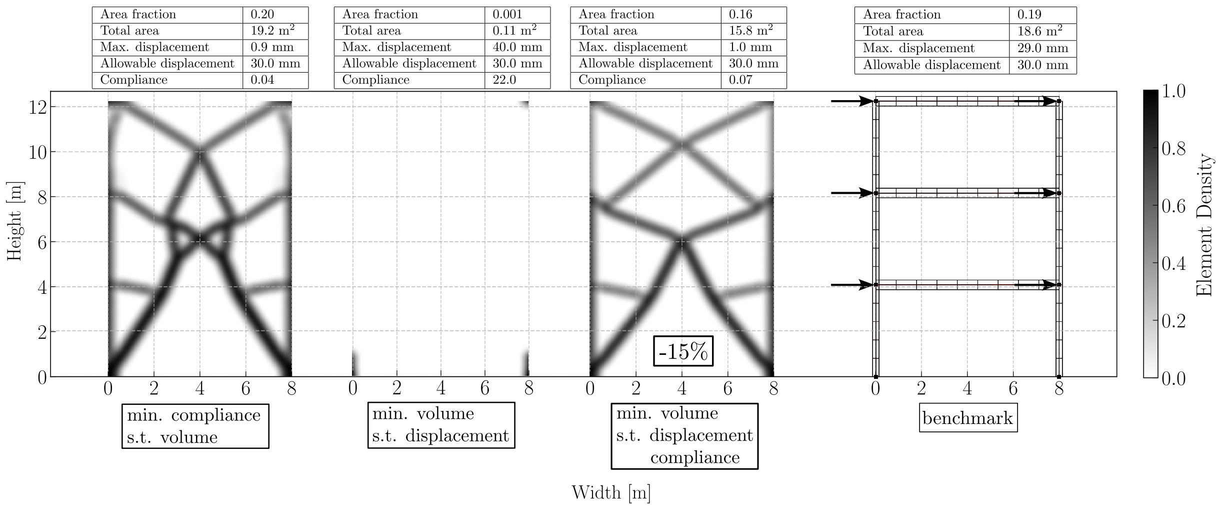

Figure 3: Frame structures resulting from the minimum compliance topology optimization (left) and the minimum volume formulations (center) in comparison with a benchmark example (right). The optimized design in the center-right achieves a material reduction of 15% compared to the benchmark.

embodied carbon. Another motivation for this is that the volume constraint is usually fully utilized for the standard formulation, which may not be the most efficient solution regarding sustainability metrics.

The engineering formulation in Equation 2 (left) containing only one displacement constraint, applied at the top right corner of the structure, which is the assumed location of maximum displacement, runs into numerical instabilities and leads to disconnected results as shown in Figure 3. Therefore, this paper uses a combined compliance and displacement constraint. In Equation 2 (right), this joint constraint is defined, where twice the value of the previously computed compliance value is used as an input for C_{tol} to avoid the user's prescription of a compliance threshold. Tarek et al. [19] have shown that compliance positively influences the numerical properties by intrinsically ensuring material placement at the load application points. The scaling factors c_1 and c_2 regulate the influence of the compliance on the overall optimization procedure. The maximum allowable displacement value d_{tol} follows the building code requirements described in Section 2.2..

$$
\begin{aligned}
\underset{\rho^e}{\text{minimize}} \quad & f = \sum_{e \in \Omega} \rho^e v^e & \qquad \underset{\rho^e}{\text{minimize}} \quad & f = \sum_{e \in \Omega} \rho^e v^e \\
\text{subject to} \quad & \mathbf{h} = \mathbf{K}(\rho^e)\mathbf{d} - \mathbf{F} = \mathbf{0} & \qquad \text{subject to} \quad & \mathbf{h} = \mathbf{K}(\rho^e)\mathbf{d} - \mathbf{F} = \mathbf{0} \\
& d_{max} \leq d_{tol} & & d_{max} + c_1 \times \mathbf{F}^T\mathbf{d} \leq d_{tol} + c_2 \times C_{tol} \\
& \rho^e_{min} \leq \rho^e \leq 1 \quad \forall e & & \rho^e_{min} \leq \rho^e \leq 1 \quad \forall e
\end{aligned}
\tag{2}
$$

The resulting design is shown in Figure 3 which, except for small details, results in a similar form to the minimum compliance formulation. The structure shows a lower area than the previous example, supporting material and embodied carbon reduction, and the maximum displacement is still conservatively contained in the range defined by the building code.

2.4. Benchmark comparison

The result is compared with a benchmark example to assess the potential material savings for the topology-optimized structure. The standard design displayed in Figure 3 (right) represents the frame. A finite element analysis is conducted on the parametrically modeled structure. To estimate the required material to sustain the applied load and keep the maximum displacement in the prescribed range of allowance, the width of the columns w_c and the height of the beams h_b are set as variables. To simplify the problem, they are put in relation to each other by $h_b = 1.5 \times w_c$. The remaining single design variable w_c gets modified until the structure shows a displacement of approximately d_{tol}. Finally, the total material area and area fraction, which corresponds to the volume fraction in two dimensions, are computed by summing up the contributions of each structural component. After evaluation, the optimized structure in Figure 3 shows a 15% decrease in material, indicating the potential impact of using topology optimization methods on a building-scale level compared to as-built techniques.

2.5. Implementation

The implementation is performed using the Julia TopOpt package [20], which enables evaluating the gradients by automatic differentiation. The solutions are computed using the method of moving asymptotes with a convergence tolerance of 10^{-4}. For the above results, a filter radius of 0.4 and compliance scaling factors $c_1 = c_2 = 1$ are used. However, the algorithm is highly sensitive to these parameters, as demonstrated in the following section.

3. Sensitivity analysis

The framework's setup allows for a sensitivity analysis of not only algorithmic but also engineering parameters. This enables an early-stage assessment of the influence of certain parameters on the overall structure and the required material. Each result is generated in the order of minutes, where the minimum compliance optimization takes significantly longer than the volume minimization. However, it becomes visible that the element densities do not take clear binary values. Future work could study how to circumvent this behavior, and the current results can be seen as a compromise between a conceptual and a physically meaningful result.

3.1. Engineering parameters

This section focuses on parameters related to the design and visual appearance of the structure as well as the engineering mechanics behind it. Another aspect that is not further elaborated in the scope of this paper is the change in the applied loading. The effects in saving are lower for a gravity and a combined load case, indicating a better performance of the moment frame in cases where the vertical displacement is governing.

3.1.1. Young's modulus

The first parameter that is varied is the Young's modulus, with variables lying in the general range for concrete material. One can observe from Table 2 that the stiffer the material, the less the material savings of the optimized structure compared to the benchmark example. This result is not surprising, as, in general, the strength of structural optimization methods lies in low-strength material.

Table 2: Possible material savings for varying Young's modulus E. With increasing E, the optimization potential decreases.

Young's modulus E $\left[\frac{kN}{mm^2}\right]$	Material savings [%]
27.0	15.0
35.0	9.0
44.0	8.0

3.1.2. Number of stories

The number of stories is highly influential on the saving potential. As shown in Figure 4, an increase in height allows for larger savings. This results from the fact that the topology-optimized structure follows the load-bearing principles of a truss and, therefore, shows improved performance compared to a moment frame. However, the available computational resources could not process samples with more than eight stories.

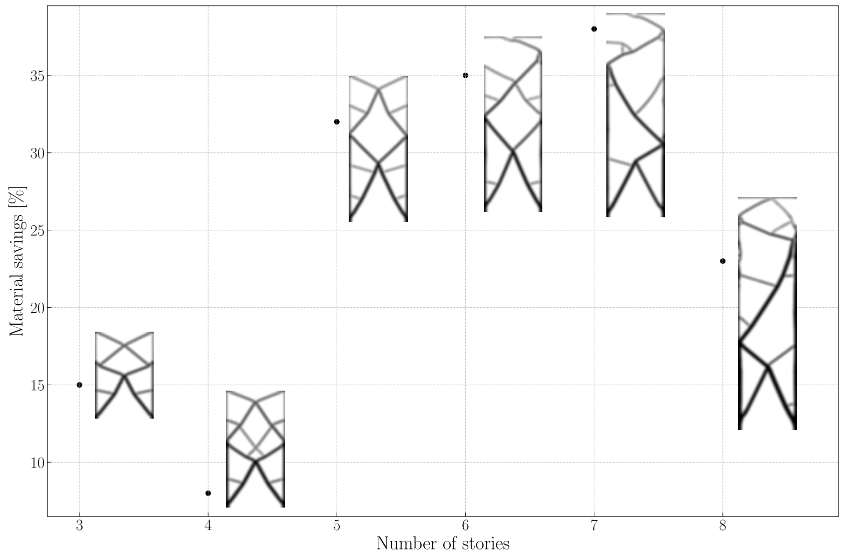

Figure 4: Relation between the number of stories and the possible material savings of the topology-optimized structures compared to the benchmark. Overall, an increase in height allows for larger savings.

3.2. Algorithmic parameters

This section briefly overviews the results' sensitivity to the compliance scaling factors and the filter radius. It becomes clear that the optimization formulation does not yet show stable performance and is strongly dependent on the chosen parameters, as displayed in Figure 5. Another aspect to analyze is the influence of the initialization, where a similar pattern with a high dependence on the result can be observed. This behavior is likely related to the problem formulation's strong non-convexity.

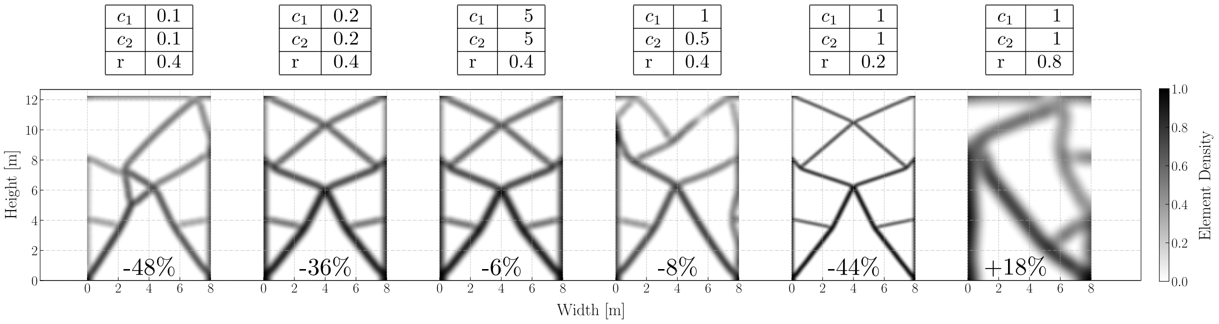

Figure 5: Optimized designs based on variations in compliance scaling factors and filter radius. The resulting savings strongly depend on the exact choice of the values, indicating an unstable algorithm behavior.

4. Discussion and conclusions

This paper provides a framework that quantifies the material-saving and embodied-carbon-reduction potential with topology optimization methods. It is applied to concrete building frames at an early design stage to provide a method for comparison to as-built techniques and designs. Given that it only requires a limited number of input variables, such as geometry, Young's modulus, and allowable displacement, it is especially applicable to practicing engineers without the need to be familiar with the details of the optimization formulation. Standard building code formulas can be applied for the prescribed displacement while the framework provides the compliance value. This novel formulation operates on a carbon-emission-relevant metric, as, for mono-material structures, it can be assumed to be proportional to the material usage. This workflow allows an easy sensitivity analysis and an assessment of the influence of certain parameters on the final design, the required material, and the possible savings. This straightforward way of creating many different results using automatic differentiation could also serve as a basis for creating a training dataset for a predictive machine-learning pipeline for generating topology-optimized building frames.

In the examples presented, substantial possible material reductions ranging from 10% to 50% were shown, indicating that significant savings are possible when considering a larger design space. Although topology optimization methods have only recently been applied to building structures, the results demonstrate its potential. For most structures, the displacement constraint was satisfied very conservatively, indicating the capacity to optimize the structures further and increase the savings even more. However, it has to be noted that additional postprocessing and accounting for other failure mechanisms, such as buckling, stress, and strength, would be necessary to ensure constructability and achieve a realistic estimate, which might be smaller than the ones presented in this paper. As shown in Section 3., the results are strongly sensitive to the algorithmic parameters, making future tuning necessary to ensure stable results. The next steps will include improving the implementation to account for larger structures than the ones presented in this paper. The model will be extended to three dimensions to examine out-of-plane effects in future work. Finally, the material mechanics should consider high-fidelity structural mechanics instead of an isotropic material law.

In summary, the framework proposes a strategy to make topology optimization more accessible to practicing engineers. The quantification of possible material and embodied carbon savings motivates the usage of structural optimization methods at an early design stage, taking a step towards reducing the construction industry's environmental impact.

Acknowledgments

The authors would like to thank the developers of TopOpt.jl, in particular Mohamed Tarek, for their support in developing and implementing the optimization formulations.
Funding for this research was provided by the Louis Berger, the Friesecke (1961), and the Pennell (1962) Fellowship, Massachusetts Institute of Technology.

References

[1] United Nations Environment Programme, "Building materials and the climate: Constructing a new future," 2023.

[2] E. Ching and J. V. Carstensen, "Truss topology optimization of timber–steel structures for reduced embodied carbon design," *Engineering Structures*, vol. 252, p. 113 540, 2022. DOI: 10.1016/j.engstruct.2021.113540.

[3] L. Mei and Q. Wang, "Structural optimization in civil engineering: A literature review," *Buildings*, vol. 11, no. 2, p. 66, 2021. DOI: 10.3390/buildings11020066.

[4] M. Afzal, Y. Liu, J. C. P. Cheng, and V. J. L. Gan, "Reinforced concrete structural design optimization: A critical review," *Journal of Cleaner Production*, vol. 260, p. 120 623, 2020. DOI: 10.1016/j.jclepro.2020.120623.

[5] M. A. Ismail and C. T. Mueller, "Minimizing embodied energy of reinforced concrete floor systems in developing countries through shape optimization," *Engineering Structures*, vol. 246, p. 112 955, 2021. DOI: 10.1016/j.engstruct.2021.112955.

[6] E. Costa, P. Shepherd, J. Orr, T. Ibell, and R. Oval, "Automating concrete construction: Digital design of non-prismatic reinforced concrete beams," in *Second RILEM International Conference on Concrete and Digital Fabrication*, F. P. Bos, S. S. Lucas, R. J. Wolfs, and T. A. Salet, Eds., Cham, 2020, pp. 863–872. DOI: 10.1007/978-3-030-49916-7_84.

[7] K. Feickert and C. T. Mueller, "Thin shell foundations: Quantification of embodied carbon reduction through materially efficient geometry," *Architecture, Structures and Construction*, 2023. DOI: 10.1007/s44150-023-00101-z.

[8] H. L. Gauch, C. F. Dunant, W. Hawkins, and A. Cabrera Serrenho, "What really matters in multi-storey building design? a simultaneous sensitivity study of embodied carbon, construction cost, and operational energy," *Applied Energy*, vol. 333, p. 120 585, 2023. DOI: 10.1016/j.apenergy.2022.120585.

[9] A. Tamrazyan and A. Alekseytsev, "Strategy for the evolutionary optimization of reinforced concrete frames based on parallel populations evolving," *IOP Conference Series: Materials Science and Engineering*, vol. 869, no. 5, p. 052 019, 2020. DOI: 10.1088/1757-899X/869/5/052019.

[10] M. Kripka, J. T. Boscardin, and S. D. Casteli, "Topology Optimization of Reinforced Concrete Plane Frames," *Revista Sul-americana de Engenharia Estrutural*, vol. 13, no. 2, 2016, Number: 2. DOI: 10.5335/rsaee.v13i2.6343.

[11] L. L. Stromberg, A. Beghini, W. F. Baker, and G. H. Paulino, "Topology optimization for braced frames: Combining continuum and beam/column elements," *Engineering Structures*, vol. 37, pp. 106–124, 2012. DOI: 10.1016/j.engstruct.2011.12.034.

[12] Q. Q. Liang, Y. M. Xie, and G. P. Steven, "Optimal topology design of bracing systems for multistory steel frames," *Journal of Structural Engineering*, vol. 126, no. 7, pp. 823–829, 2000. DOI: 10.1061/(ASCE)0733-9445(2000)126:7(823).

[13] Z. Chen, K. Long, X. Wang, J. Liu, and N. Saeed, "A new geometrically nonlinear topology optimization formulation for controlling maximum displacement," *Engineering Optimization*, vol. 53, no. 8, pp. 1283–1297, 2021. DOI: 10.1080/0305215X.2020.1781106.

[14] N. Stoiber and B. Kromoser, "Topology optimization in concrete construction: A systematic review on numerical and experimental investigations," *Structural and Multidisciplinary Optimization*, vol. 64, no. 4, pp. 1725–1749, 2021. DOI: 10.1007/s00158-021-03019-6.

[15] M. S. Smith, D. Fang, C. Mueller, and J. Carstensen, "Reducing embodied carbon with material optimization in structural engineering practice: Perceived barriers and opportunities," *Journal of Building Engineering*, p. 109 943, 2024. DOI: https://doi.org/10.1016/j.jobe.2024.109943.

[16] L. L. Beghini, A. Beghini, N. Katz, W. F. Baker, and G. H. Paulino, "Connecting architecture and engineering through structural topology optimization," *Engineering Structures*, vol. 59, pp. 716–726, 2014. DOI: 10.1016/j.engstruct.2013.10.032.

[17] T. Zegard, C. Hartz, A. Mazurek, and W. F. Baker, "Advancing building engineering through structural and topology optimization," *Structural and Multidisciplinary Optimization*, vol. 62, no. 2, pp. 915–935, 2020. DOI: 10.1007/s00158-020-02506-6.

[18] R. E. Weber, C. Mueller, and C. Reinhart, "Generative structural design for embodied carbon estimation," *Proceedings of IASS Annual Symposia*, vol. 2020, no. 8, pp. 1–12, 2020.

[19] M. Tarek and T. Ray, "Adaptive continuation solid isotropic material with penalization for volume constrained compliance minimization," *Computer Methods in Applied Mechanics and Engineering*, vol. 363, p. 112 880, 2020. DOI: 10.1016/j.cma.2020.112880.

[20] Y. Huang and M. Tarek, "TopOpt.jl: Truss and continuum topology optimization, interactive visualization, automatic differentiation and more," *Proceedings of the 14th World Congress of Structural and Multidisciplinary Optimization*, 2021.

Proceedings of the IASS 2024 Symposium
Redefining the Art of Structural Design
August 26-30, 2024, Zurich Switzerland
Philippe Block, Giulia Boller, Catherine De Wolf,
Jacqueline Pauli, Walter Kaufmann (eds.)

Copyright Declaration

Before publication of your paper in the Proceedings of the IASS Annual Symposium 2024, the Editors and the IASS Secretariat must receive a signed Copyright Declaration. The completed and signed declaration may be uploaded to the EasyChair submission platform or sent as an e-mail attachment to the symposium secretariat (papers@iass2024.org). A scan into a .pdf file of the signed declaration is acceptable in lieu of the signed original. In the case of a contribution by multiple authors, either the corresponding author or an author who has the authority to represent all the other authors should provide his or her address, phone and E-mail and sign the declaration.

Paper Title: An automated topology optimization framework for material minimization in concrete building structures

Author(s): Simone M. Peter, Caitlin T. Mueller

Affiliation(s): Massachusetts Institute of Technology

Address: 77 Massachusetts Avenue, Cambridge, MA 02139, USA

Phone: +1 (617) 308-1435

E-mail: smpeter@mit.edu

Printed name: Simone Peter Signature: Peter Simone

Location: Cambridge, MA, USA   Date: July 1st, 2024