Proceedings of the IASS 2024 Symposium
Redefining the Art of Structural Design
August 26-30, 2024, Zurich Switzerland
Philippe Block, Giulia Boller, Catherine DeWolf,
Jacqueline Pauli, Walter Kaufmann (eds.)

Performance-Based Design of a Bending Active Hardwood Glulam Beam-String: a Form-Finding Paradox.

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Abstract

In search of a viable tropical hardwood glulam hangar structure for the South American context, where the timber industry faces challenges in scaling up to compete with the well-established steel and concrete industries, this paper presents a form-finding design process rooted in the pursuit of efficiency, simplicity, and clarity of thought.

In a bending active beam-string, the effort generated during pre-assembly stems from both the inertia of the beams and the system's height. This marks a departure from conventional understanding, as these parameters become simultaneously the challenge and the solution for structural safety, leading to a conflicting design process.

By integrating context-sensitive design strategies into form-finding processes, the manipulation of geometry and rigidity led to a feasible solution with notably low material consumption and efficient production and assembly processes. This design outperformed typical steel and concrete alternatives, enabling the production and assembly of the first two 1300m² units. Such an approach may provide a suitable methodology for the performance-based design of sustainable structures that are both feasible and sensitive to the socio-environmental context of South America.

Keywords: bending active, hardwood, timber, beam-string, form finding, performance-based, optimization, Beaver, Karamba

Figure 1: first beam-string assembled on site.

1. Introduction

1.1. Global and local context

In the Global North, where there's a strong emphasis on low-carbon structural solutions, the recurrent development of performance-based design methods has led to the rise of increasingly complex timber structures. However, in the Global South, this approach is less prevalent as the imperative for functional and accessible construction prevails. This contrast highlights the critical need for context-sensitive design strategies that not only align with the construction realities of the Global South but also enhance feasibility and sustainability within those contexts.

In South America, where the timber industry faces challenges in scaling up to compete with the well-established steel and concrete industries, such circumstances may overshadow the pursuit of experimental innovation. However, as Philippe Block pointed out, designing within limitations has often led to groundbreaking innovations [1]. This paper suggests how form-finding design methods might offer significant advantages in this context if we fully integrate the practical constraints and inherent challenges into the foundation of the design process.

1.2. Architectural and client demand

The opportunity to conduct this research arose during the execution of a project for a complex of buildings in São Paulo, Brazil, designed by the architectural office Aflalo Gasperini. This extensive project aimed to develop various facilities, including a hotel, gym, restaurant, equestrian center, and nautical support facilities near a reservoir, totaling more than 10 timber structures.

Within this context, we faced the challenge of exploring the feasibility of constructing boat hangars. Initially, those hangars were not considered within the timber structures scope since the developer and architects favored prefabricated concrete or steel structures for perceived less prestigious areas. Despite this, we initiated a study at no cost to evaluate the possibility of designing the boat hangars in timber. Notably, the architects expressed keen interest in this prospect, affording us significant design freedom.

During this study, two primary constraints emerged. Firstly, it was important to adhere to typical boat hangar dimensions, including a main span of 20 meters, a smaller span of 6 meters, and a minimum free height of four meters. Secondly, the hangars' roofs should be constructed with flat thermoacoustic tiles to maintain consistency with other buildings in the complex and optimize construction efficiency.

1.3. ITA's tropical hardwood glulam

ITA Engenharia em Madeira, responsible for the structural design and construction of these buildings, was founded in 1980 by the engineer Hélio Olga. Over nearly three decades, the focus has been on constructing structures using native timber. However, recognizing the challenges of sourcing timber from sustainable forest management operations certified by the Forest Stewardship Council (FSC), and with a commitment to minimizing environmental impact, a strategic reevaluation was crucial approximately 15 years ago [2].

This led to the initiation of developing and producing eucalyptus glulam, a decision motivated by the abundant availability of eucalyptus reforestation in east Brazil due to the thriving cellulose industry. This pivotal transition not only tackled logistical challenges but also reduced transportation emissions by shortening the distance between the timber source and construction market.

Eucalyptus exhibits remarkable strength properties when compared to Pine. However, safely gluing tropical hardwood requires specific techniques on harvesting, drying, grading, and processing. Eucalyptus Grandis, known for its diversity, is carefully selected for ITA's production, focusing on the highest quality heartwood with an average density of $\rho_{mean} = 650 kg/m^3$ and a specific density variation. While it may not match the maximum strength of Eucalyptus Grandis, where certain parts of the tree can exceed $1000 kg/m^3$, we adhere to a density threshold that ensures secure bonding.

Figure 2 Eucalyptus Grandis heartwood.

Through this process, a glulam material with exceptionally high mechanical properties is achieved, boasting a modulus of elasticity $E_{0,mean}$ = 16,500 MPa and axial tensile and compressive strength of approximately 60N/mm². Despite its structural prowess, this premium-grade timber incurs a significant cost, underscoring the imperative of structural efficiency for the viability of our projects.

2. Goals

The methodology of this work is rooted in the pursuit of three goals: efficiency, simplicity, and clarity of thought.

Efficiency entails using material only where necessary. However, unlike concrete and steel, which can be both molded and merged — therefore excelling in topological optimizations — wood is worked through assembly. Consequently, achieving efficiency in timber structures involves determining how independent pieces can effectively come together to form a cohesive whole, such as a truss, efficiently.

Simplicity is achieved by avoiding unnecessary complexities. This requires periodic reassessment of fundamentals questions: does the structure precisely fulfill its intended function and context? This evaluation must consider its production, usage, and long-term durability. Guided by the principle of addressing practical needs and local context constraints, we can significantly enhance our resource management and reduce our environmental footprint, thereby fostering a development approach that is both more sustainable and feasible.

Clarity of thought arises from the pursuit of clear answers to the fundamental questions. To avoid constructing confusing structures, it is essential to steer clear of deviations and inconsistencies in reasoning, striving to understand and articulate the role of each structural element in pursuit of didactic clarity.

In this project, this triad unfold into three primary objectives: minimum volume of timber, minimum volume of steel in connections, and maximum production efficiency, considering gluing, machining, pre-assembly, and assembly.

3. Form-finding workflow: Grasshopper3D, Karamba3D and Beaver.

Before delving into the specific case study, it is necessary to elucidate the tools employed in the design process. After conceptually defining the desired arrangements to be studied, Grasshopper is used to construct a geometry grounded in variable parameters that will provide research freedom to the algorithm. Subsequently, the geometry serves as the basis for a Karamba3D [3] finite element analysis (FEA) model, with cross-sections also serving as variable parameters. Integrated with this analytical model, Beaver [4] performs all structural safety checks according to *Eurocode 5: Design of Timber Structures* (EC5) [5] and provides the necessary Ultimate Limit State (ULS) and Serviceability Limit States (SLS) results to allow structural safety evaluations.

Beaver, developed by Marcio Sartorelli, Renan Prandini, and the author (João Pini), is an open-source tool [6] designed to streamline timber engineering procedures within computational design workflows. It performs Eurocode 5 safety checks for both timber elements and fastener-type connections.

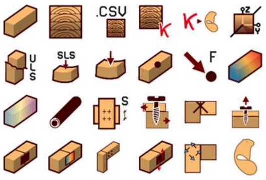

Figure 3 Beaver tools.

With its Grasshopper version [7], now fully integrated with Karamba3D, Beaver allows direct management of Karamba's data from the backend. This seamless integration empowers Beaver to conduct live limit state evaluations, generating comprehensive reports and visualizations of design ratios for each load case and failure mode. Furthermore, it provides critical check visualizations, highlighting failure modes alongside the timber elements. All design ratios and critical check visualizations presented in this study were generated by Beaver.

This outcome provides sufficient data to enable the pursuit of structural efficiency with evolutionary solvers like Galapagos [8]. Furthermore, managing data from the structural model enables the consideration of production efficiency in the design process. However, it's important to acknowledge that decisions made during the conceptual phase of geometry parameterization, which define the exploration degrees of freedom, have a greater impact than any subsequent optimization efforts.

4. Structural hypothesis: timber beam-strings

4.1. Building an algorithm upon intuition

In a recent project, we undertook the design and construction of a forty-meter-span timber truss [9], with a height accounting for 5% of the span. Despite extensive optimization efforts, the need for incorporating steel was evident due to the significant forces and numerous nodes inherent in trussed structures.

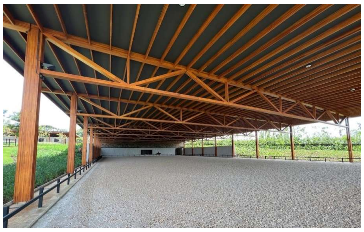

Figure 4 forty-meter span roof truss.

Reflecting on this experience and relying on intuition, an exploration was initiated aimed at minimizing nodes, considering the feasibility of transporting the entire twenty-meter beam from the factory to the site. This prompted consideration of beam-strings as a promising avenue for further design research.

To address the challenge effectively, an algorithm was developed to generate various timber beam-string arrangements, relying on three primary parameters: beam-string height, span subdivision and cross-sections. Given Brazil's snow-free environment and the lightweight roof system, wind emerged as the most critical load factor. Accordingly, a wooden lower chord was chosen over a steel cable for its ability to withstand compression forces, along with its environmental benefits. Despite the asymmetry of wind actions, simple posts were chosen instead of diagonals to minimize material consumption, provided the system proved stable enough. Additionally, aligning posts and lateral bracings with the frequency of secondary elements aimed to minimize unnecessary transitional bending in the beams.

4.2. Exploring possibilities and convergence

From the outset, Beaver demonstrated that by adding material – via cross-sections – it would be feasible to accommodate all possible variations of subdivision and height. As usual, adding material via cross sections and system height will lead to safer structures. Naturally, the optimal arrangements tend to be those that can approach the structural safety limits by evenly distributing safety responsibility throughout the structural elements, ensuring that no material remains without accountability.

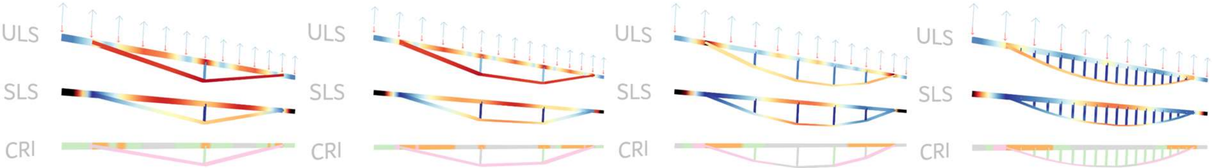

Figure 5 Multiple possible arrangements with cross-sections adjusted for acceptable ULS and SLS design ratios.

Regarding subdivision variations, it's noteworthy that scenarios with fewer posts lead to larger buckling lengths, while more posts allow for slimmer cross-sections albeit increasing the number of elements in the arrangement overall.

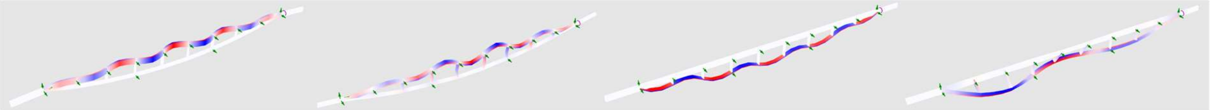

Figure 6 Local buckling lengths to be considered in Beaver EC5 safety checks will follow geometric variations.

Given the higher frequency of lateral bracing in the upper chord, there's a tendency for it to possess lower lateral inertia when compared to the lower chord, which was initially conceived with two lateral braces. Also, it's worth noting that as fewer subdivisions result in obtuse angles in the lower chord, leading to joints, while opting for a continuous chord will require less steel in connections.

Regarding height variations, lower beam-strings are governed by serviceability analysis, especially under wind loading, whereas higher beam-strings allow cross-sections to be determined by the ultimate limit state (ULS) limits.

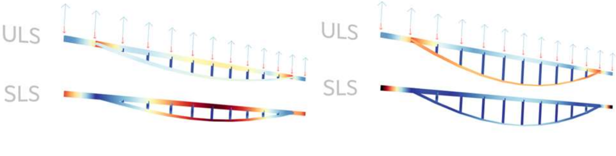

Figure 7: SLS governs (left); ULS governs (right).

As the form-finding process progressed, now with revised inputs and asymmetrical wind loads, the genetic algorithm easily revealed the most efficient beam-string, balancing height, number of subdivisions, and cross-sections while simultaneously approaching the limits of serviceability and failure. Regarding structural efficiency, this arrangement resulted in a remarkably low timber consumption of 0.022m³/m², while a traditional trussed solution would demand about 0.04m³/m² of Eucalyptus Glulam. Additionally, a lower chord built of a curved glulam without segmentations would also be successful in reducing steel usage in connections.

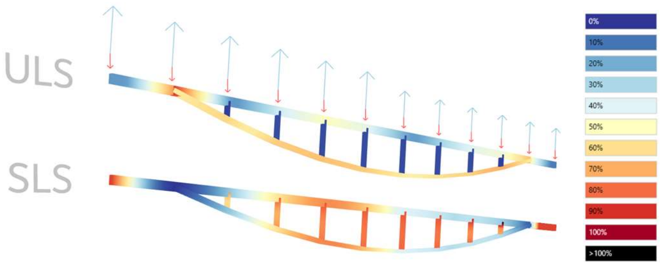

Figure 8 Optimal solution.

The success of this hypothesis stems from the accurate functions of each element, designed in pursuit of clarity of thought. However, it's crucial to recognize that the structural arrangement lacks simplicity, as it overlooks the production context. It's worth noting that Brazil's timber industry faces limitations, lacking automated presses capable of handling curved beams, and operating CNC machines restricted to processing straight beams. These considerations shed light on a new, simpler structure: Could the beam be composed of straight chords, bent during pre-assembly?

5. A new hypothesis: bending active beam-string

In this revised hypothesis, the length of the posts will dictate the spacing between chords, while their rigidity will determine the relative deformations, resulting in a 'pre-deformed' beam-string. Now, all parameters significantly influence the geometry of the structure; the stresses and deformations of the individual elements are intricately connected to the overall geometry, and vice versa.

In essence, the bending moment generated during pre-assembly arises from the inertia of the beams – cross sections – and the height of the system. This marks a departure from conventional understanding, as these parameters now present challenges rather than solutions, as previously observed.

5.1. Enhancing algorithm complexity

When two beams are bent towards each other, their deformation is directly proportional to their rigidity. To define the beam-string geometry, essential for the overall analysis, a preliminary analytical model was constructed. This preliminary model applies to the beams proportional imposed deformations that follow parabolic curvatures, representing the posts lengths. The deformed geometry resulting of their interaction will be the new geometric foundation for the beam-string chords. Subsequently, the analytical model needs to account for the fact that these curves originate from bent straight pieces. To achieve this in Karamba3D, a method analogous to that required for bending active gridshell analysis was employed, with necessary adjustments to account for the specific interrelation between the beams [10].

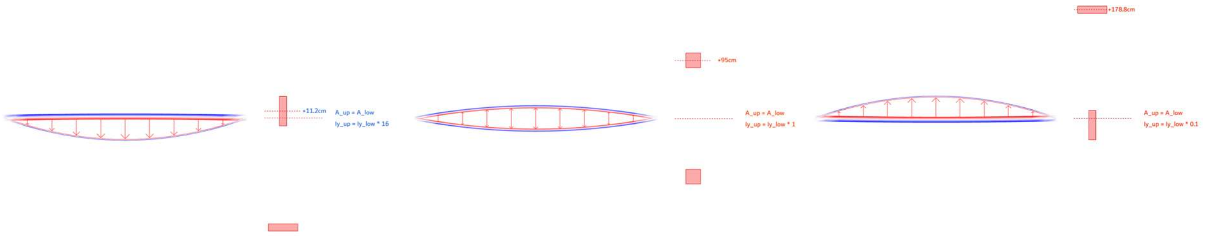

Figure 9: equal posts lengths impose different assembly deformations, depending on the inertia ratio between cross-sections.

For reasons of construction cost-effectiveness, the flat thermoacoustic tile was chosen for the entire complex of buildings. Therefore, it wouldn't be advisable to allow for a highly pronounced curvature in the upper beam, as it would require the use of a distinct tile, deviating from the standard. Consequently, alongside the regulatory limits assessed by Beaver, a restriction was imposed on the curvature of the upper beam to ensure seamless compatibility with the chosen tile. While it may not be consistent with the context, it's worth noting that allowing for the free deformation of the beams would offer benefits to the structure itself.

5.2. A form-finding paradox

This study highlights the necessity of fully understanding the conflicting dynamics presented by the structural hypothesis, where parameters influence analyses in contrasting ways. Due to the region's strong winds, the beam system must exhibit ample rigidity to safely resist failure and endure acceptable deformations without risking dangerous resonance frequencies. Thus, the short-term Ultimate Limit State (ULS) and the overall Serviceability Limit State (SLS) analysis will lead the algorithm to higher beam-strings and bigger cross-sections, as usual.

Concerning the long-term Ultimate Limit State (ULS) analysis, in the other hand, the focus lies on the bending moment induced during pre-assembly for two reasons: first, the substantial effort required to accommodate a significant curvature, and second, the lasting permanent effect it imparts. This combination is particularly critical in timber structures, where resistance considerations must account for the duration of actions via the modification factor (k_{mod}). Paradoxically, addressing this structural requirement implies minimizing beam-string height and cross sections, deviating from the previous strategy employed to maximize the overall system rigidity for SLS and short-term ULS. After developing an algorithm capable of evaluating all scenarios, finding a solution that met all requirements simultaneously appeared impossible.

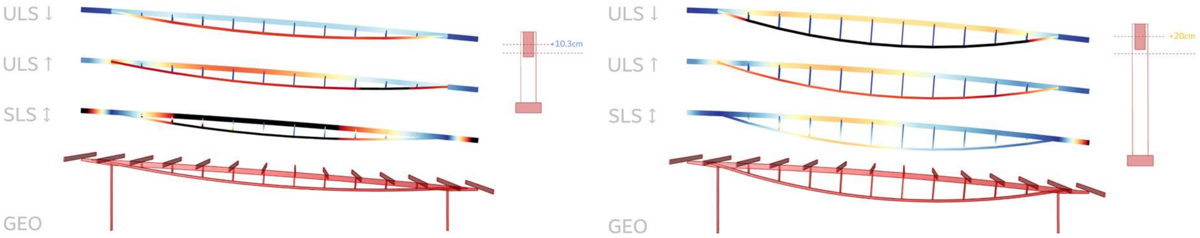

Figure 11: the impact of beam-string height in long-term ULS (k_{mod}=0.6), short-term ULS (k_{mod}=0.9), and SLS, respectively.

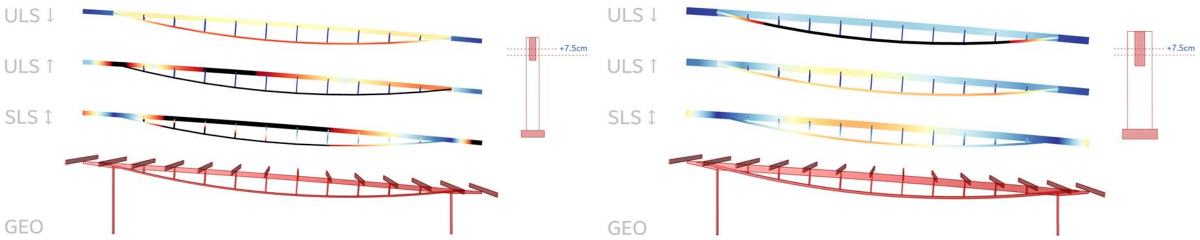

Figure 11: the impact of cross-sections increments in the evaluations, with fixed bending inertia ratio between beams.

5.3. Investigating references

Exploring solutions involves studying how engineers have benefited from bending active solutions in other timber structures. Engineer Jürg Conzett's method for the Mehrzweckhalle, designed by architect Gion Caminada, is notable. The "cable" of the beam-string comprises thin timber lamellas under tension, reflecting minimal flexural effects due to their low rigidity. Despite minimizing bending effects in lower chord, this solution would be unsuitable for our work: the significant compressive forces require collective action between the lamellas to prevent local buckling.

By layering thin, flexed lamellas, the distinguished engineer Julius Natterer implemented ribbed gridshells that span large distances, as seen in the Hannover Expo Roof. Alone, the lamella's inertia is insufficient to cause failure due to on-site bending, while the successive layers, predominantly fastened together and occasionally glued [11], act collectively to resist the compression efforts. Despite this great example, the most efficient method to impose curvature on thin lamellas and solidify them, in our context, would be to produce curved glulam beams, a practice we aim to avoid.

Figure 14: Mehrzweckhalle, Vrin

Figure 14: Hannover Expo Roof

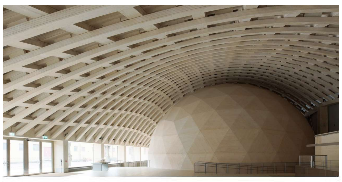

Figure 14: Wisdome Stockholm

A remarkable recent example of imposed bending on larger laminated-veneer-lumber (LVL) lamellas is the Wisdome Stockholm, engineered by SJB Kempter Fitze, Hermann Blumer, and Design to Production. "The lower layer acts as falsework and is assembled from pre-curved, CNC-machined beams, while the upper four beam layers are bent on-site" [12]. This gridshell also faces significant compression forces, alongside on-site bending and torsional efforts. A deeper exploration of this work suggests that the key to success might be the precision of curvatures, local buckling lengths, and loads.

5.4. A feasible solution

At this stage, it became crucial to meticulously review the calculation parameters for a more refined analysis. A detailed wind analysis was conducted using computational fluid dynamics (CFD) software, considering various wind scenarios and potential obstructions, to precisely identify critical pressures. Furthermore, a thorough examination of the nodes and local buckling lengths in the structure was undertaken to determine the optimal placement of bracings and incorporate the semi-rigidity of desired connections into the calculations. Through multiple iterations and subtle enhancements in the analysis, coupled with refinements in the form-finding goals, the genetic algorithm eventually converged to a feasible solution that met all requirements for failure, serviceability, and the limitation of curvature in the upper beam.

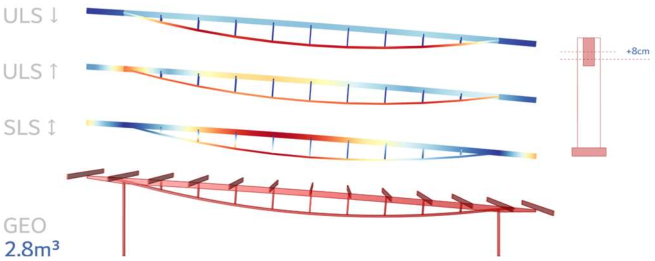

Figure 15: the optimal solution.

Convergence was achieved with a lower chord measuring 9 by 36 centimeters, a horizontal glulam beam that resembles Gion Caminada's wooden "cable" from the elevation. Despite its similar appearance, this beam possesses ample strength to withstand compressive forces due to buckling restrictions. In terms of axial load resistance, despite the significant discrepancy in local-Y inertia between the upper and lower chords, crucial for curvature requirements, they exhibit comparable areas, providing similar axial rigidity. Regarding serviceability, the beam-string demonstrates sufficient overall rigidity to meet acceptable standards without implying an excessively powerful bending moment during pre-assembly.

The final structural design yielded a timber consumption rate of 0.024m³/m², a slight increase from the initial hypothesis of 0.022m³/m². Nonetheless, it showcased unparalleled production efficiency compared to the initial hypothesis, while also offering lower consumption rates than a traditional trussed solution, which typically require around 0.04m³/m² of eucalyptus glulam. Despite the complex analysis and form-finding process that successfully addressed the relationship among all variables in space, the result is *efficient*, *clear*, and *simple*. This proposal proved sufficient to render the project feasible, enabling us to outcompete steel and concrete alternatives, and securing contracts for the first two 1300m² boat hangars. It also paved the way for further research, enabling a more thorough exploration the curvatures, connections, and production processes.

6. The optimal curvature

To expedite the form-finding process, curvatures derived from parabolas were chosen as the base to the preliminary analytical model that generates the beam-string geometry. In a preliminary model, these curvatures are introduced through imposed displacements in the beams, representing posts heights, and the beams' deformations define the geometry of the overall beam-string model.

While analyzing the bending moment diagrams generated by these curvatures, a moment peak is observed at the first posts, near supports. This happens because a parabolic curvature might not be optimal for distributing posts' forces smoothly since they impose punctual loads and not evenly distributed loads. Therefore, given the already defined cross-sections and subdivisions, a secondary form-finding process was conducted to determine the optimal curvatures in the beam-string.

As the pre-assembly bending moment significantly impacts the structural arrangement, optimizing the curvatures resulted in a 7% increase in the safety margin at the ultimate limit state (ULS) analysis, without incurring any material addition or production disadvantages.

Figure 16: bending moments derived from the preliminary curvature, two genetic algorithm variations, and final curvature.

7. Connection efficiency and shop drawings

As pursued since the inception of this process, the designed beam-string embodies only two critical nodes, those that transmit forces directly between the upper and lower chords. Due to their proportions – the upper beam is 12 centimeters wide while the lower one is 36 – it was feasible to study a timber-to-timber connection, avoiding self-penetrating dowels and steel plates. Fortunately, in consequence of the flattened curvature that results a long intersection where the lower chord embraces the upper chord, it was possible to address forces with full threaded crossed screws only.

This final milestone marked the attainment of the desired efficiency in connection design, as exclusively timber-to-timber connections were employed throughout the whole beam-string. Timber-to-steel connections were only utilized between the column and the beam-string, attributed to the restricted load distribution area available for fasteners at the column, along with the bracings and the connections with the foundations. In a traditional trussed solution, we expected about 1kg/m^2 of steel usage, while in this project we achieved an average of 0.1kg/m^2 of steel employment.

For joint machining and pre-drillings, the curvatures were flattened to enable CAD-CAM communication with the CNC, prompting a reorganization of the connections in the production model. At this stage, constructing a 1:1 prototype became imperative to obtain measurements of the assembled beam-string and compare them with the theoretical model. This step would ensure that the expected deformations were in line with reality and enable adjustments in the machining processes.

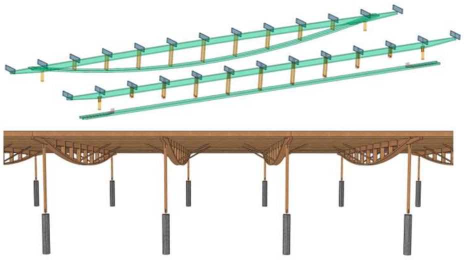

Figure 17: production models.

8. Prototype

The beam-string components were glued in an automated vertical press and machined in a Hundegger K2 CNC machining. In order to pull one beam towards the other, for the prototype assembly, special wooden clamps were crafted. The assembly process was conducted with the beam system laid horizontally to prevent the supports position to influence curvature's perfection, since there would be no room for further adjustments in the curvature once the chords were fixed together.

The process proceeded smoothly; we pulled the beams together from the center to the ends, resulting in good alignment between the machined joints. Upon measuring the curvature, we found a satisfactory outcome: the deflection of the upper chord, estimated at 80mm, reached 74mm. This variation could stem from differences in the young modulus of the beams or friction during assembly. We identified possible improvements in machining clearances and clamp design, prompting a project review before commencing the production of the 18 beam-string units that would structure the two contracted hangars.

Figure 18: Prototype assembly and measurements.

9. Pre-assembly, transportation, and assembly

The manufacturing and pre-assembly of the structure proceeded with remarkable efficiency. A team of two carpenters was able to assemble an average of two beam-strings per day, completing the task in 9 working days. With the assistance of a specialized truck and careful supervision, the transportation of the 24m beam systems proceeded as planned.

The assembly proceeded smoothly as the beam-strings were fully pre-assembled and weighed only 1 ton each, they were lifted without significant challenges using dedicated loader crane equipment. As a result, a team of three experienced carpenters successfully completed the assembly of the first hangar within three weeks of work.

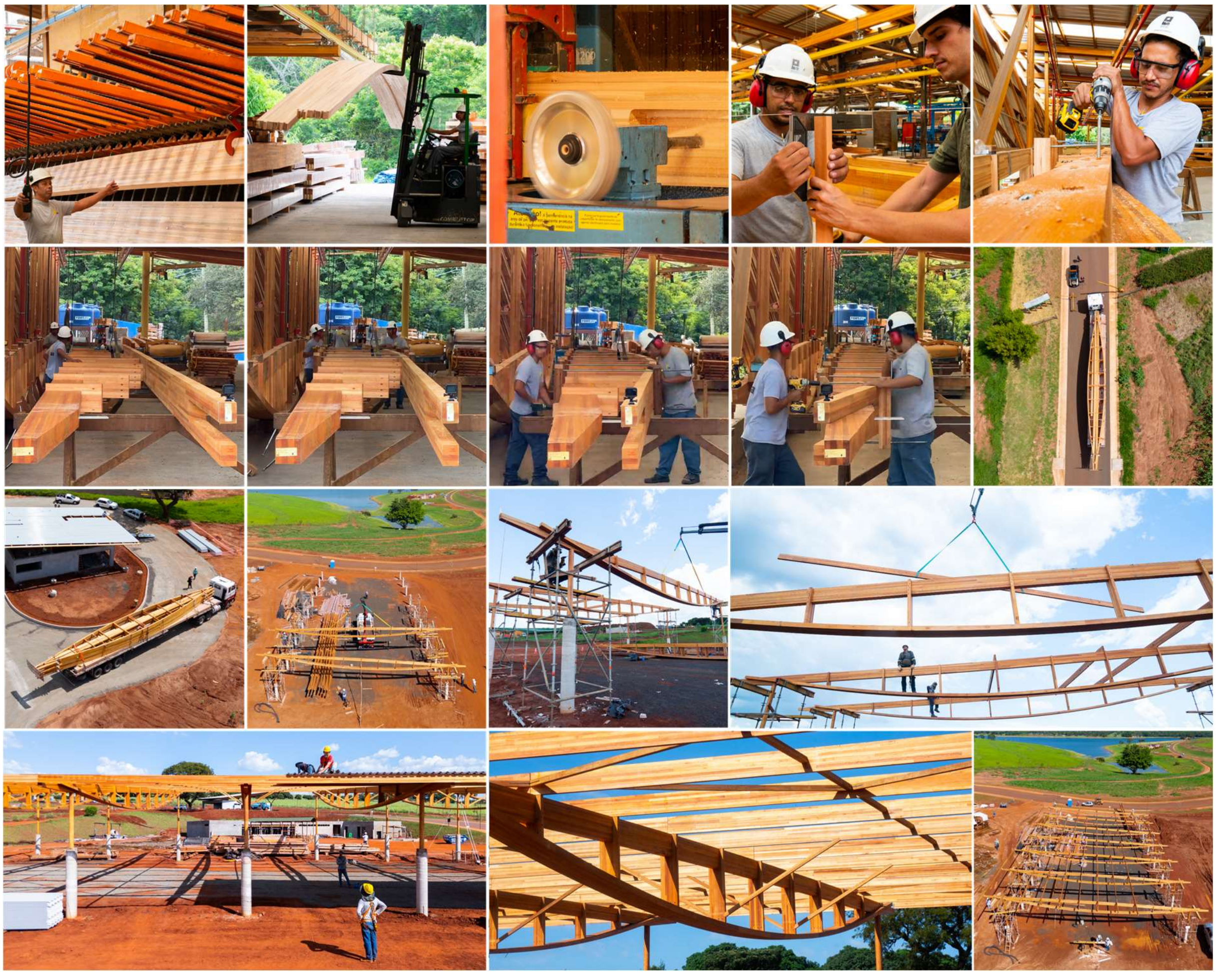

Figure 19: factory production, pre-assembly, transportation, and assembly processes.

10. Conclusions: a complex search for a simple design

While groundbreaking technological advancements have led to structures of unimaginable complexity in the global North, replicating similar solutions in places like Brazil - where challenges include limited manufacturing capabilities and the demand of cost-effective construction methods - may not align with local needs.

By considering the strengths and limitations of the chosen material within its specific context, this project delved deeply into optimizing the interplay between geometry and rigidity, striving to develop a feasible sustainable alternative to the available concrete and steel solutions. Its success suggests that integrating practical constraints and needs into form-finding processes can significantly benefit the design of sustainable structures that are both feasible and sensitive to the socio-environmental context of Brazil.

The primary outcome of this work is the physical realization of the structure, and the secondary outcome is the algorithm, which offers potential applications beyond this project. In pursuing efficiency through performance-based design, clarity of thought by seeking an elementary and didactic design, and simplicity by avoiding unnecessary expenditure of effort, resources, and environmental impact; we embrace David Billington's established principles of Efficiency, Economy, and Elegance [13], as well as contemporary considerations of Ecology and Ethics [1]. This perspective may offer a suitable approach for rethinking structural design in the Global South.

Figure 20: Finishing assembly and starting roofing process.

Acknowledgements

Acknowledgements to Clemens Preisinger for his essential support in allowing a seamless Beaver-to-Karamba integration. Special thanks to Barbara Camucce, Beatriz Nobumoto and the whole ITA's team for their invaluable contributions. Gratitude to Mariana Caires for her immense support.

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