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## **Coupling effect on cumulative damage model and hysteretic model of steel beam-column**

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### **Abstract**

The application of cumulative damage behavior is crucial for accuracy of seismic analysis and risk evaluation on structural members. However, the simulation performance is greatly influenced by the selection, the calibration and the application ways of cumulative damage models. The existing studies in this field are mainly about the development of new analytical models, but have paid limited attention on the simulation difference of various models. In this paper, a total of 4 cumulative damage models were calibrated under a series of different calibration criterion, and then incorporated into a trilinear hysteretic model on steel beam-column. The influencing laws of model type and calibration criterion on the simulation accuracy of hysteretic relation with considerable cyclic deterioration is thoroughly studied. It is found that the damage index solely includes a deformation-enhanced cumulative energy-dissipation term can provide the best simulation results among the models considered. Moreover, the hysteretic curves up to strength deteriorates to almost zero should be selected when performing calibration of model constants. The consideration of cumulative damage in hysteretic model is significantly necessary.

**Keywords:** cumulative damage model, hysteretic model, cyclic deterioration, earthquake action, seismic analysis

### **1. Introduction**

The considerable amplitude and duration of repeated earthquake actions can produce a severe cyclic deterioration in the load-bearing performance of structural steels and steel members[1-3]. This plastic cumulative deterioration behaviour should be incorporated into the seismic numerical analysis to increase the simulation accuracy, especially for the cases with long-duration and large-amplitude actions. During the past decades, the seismic cumulative deterioration behavior has attracted increasing attention when performing structural analysis under severe earthquakes and aftershocks[4-6]. The most important part within this field refers to the development of a series of cumulative damage models (CDMs). This values evaluated by such analytical models usually range from 0 to 1.0, illustrating a process from undamaged structure to a completely damaged structure. Thereafter, the CDM is needed to be incorporated into the performance evaluation models, such as strength-deformation hysteretic model[6], to reflect the cyclic deterioration of structural components. The description performance is considerably influenced by the selection of CDMs. A inappropriate choice may cause a unfavourable simulation accuracy even though the damage behaviours has already been considered[7, 8]. Besides, the way how CDM is incorporated into hysteretic model can also produce remarkably different simulation performance. It is needed to illustrate the influencing laws on selection, calibration and incorporation way of CDMs on the hysteretic simulation performance.

This paper is aiming to study the influence of several representative CDMs on the hysteretic performance of structural member. A total of four CDMs were selected and incorporated into a trilinear hysteretic model. The model constants were calibrated through a FEA hysteretic curve under different calibration criterions. The influences of calibration criterions, model selection on the trilinear hysteretic performance were thoroughly clarified.

## 2. Damage-based hysteretic model

### 2.1. Theoretical models

The hysteretic curves of structural members can be normalized by characteristic indexes, such as yield strength and yield deformation. For instance, the moment-curvature hysteretic curves ( $M-\varphi$ ) of steel beam-columns can be normalized by the yield moment  $M_y$  and yield curvature  $\varphi_y$ . To relieve the influence of structural details, all hysteretic curves studied in this paper refer to the ones normalized by the yield points. The damage-based hysteretic model consists of two main parts, including the undamaged hysteretic model and the CDM. As shown in Figure 1, the undamaged hysteretic model is taken as a trilinear form herein to ease the numerical computation. During the loading phase, the hysteretic strength-deformation curve consists of a linear elastic stage, a transition plastic stage and a linear plastic-hardening stage. The stiffnesses of the three stages are respectively described by  $E_n$ ,  $E_t$  and  $E_h$ . The breakpoints between the three stages are defined by two sets of controlling lines  $PQ(P'Q')$  and  $XY(X'Y')$ . Specifically, the intersections of linear elastic lines and the  $PQ(P'Q')$  lines describe the breakpoint between the linear elastic stage and transition plastic stage, whilst the intersections of transition lines and  $XY(X'Y')$  lines represent the breakpoint between the transition plastic stage and linear plastic-hardening stage. Note that the slopes of these controlling lines are all equal to  $E_h$  when CDM is not considered. The interceptions of these lines are described by  $y_{PQ}$  and  $y_{XY}$ . As for unloading phase, the hysteretic relation is unloaded with the linear elastic stiffness  $E_n$  when CDM is not considered.

Once the cumulative damage behaviour is considered, the crucial description parameters in the hysteretic model are correlated with the damage index  $D$ . In this paper, the three stiffness parameters  $E_n$ ,  $E_t$  and  $E_h$  are linearly deteriorated as  $D$  increases. The correlations between the deteriorated stiffness parameters  $E_{n,D}$ ,  $E_{t,D}$  and  $E_{h,D}$  are respectively expressed as from Eq.(1) to Eq.(3). On the other hand, the interception parameters  $y_{PQ}$  and  $y_{XY}$  can influence the strength amplitude of hysteretic curve. To reflect the cyclic deterioration in strength, such two parameters are also correlated with the CDM as indicated in Eq.(4) and Eq.(5). Among these equations, the parameter  $\zeta$  denotes the ultimate deterioration proportion when  $D$  reaches 1.0. Clearly, different CDMs must yield different evolutionary laws on these stiffness and strength parameters.

$$E_{n,D} = E_n(1 - D\zeta_n) \quad (1)$$

$$E_{t,D} = E_t(1 - D\zeta_t) \quad (2)$$

$$E_{h,D} = E_h(1 - D\zeta_h) \quad (3)$$

$$y_{PQ,D} = y_{PQ}(1 - D\zeta_{PQ}) \quad (4)$$

$$y_{XY,D} = y_{XY}(1 - D\zeta_{XY}) \quad (5)$$

Currently, there exists a series of different CDMs. The most representative ones include the deformation-based CDMs, the energy-based CDMs and the combined deformation-energy-based CDMs. The existing CDMs may possess considerably complex mathematical expressions, leading to a considerable inconvenience in calibration of model constants. In this study, several simplified CDMs were adopted to describe the cyclic deterioration behaviour of hysteretic relations. The exact expressions are shown from Eq.(6) to Eq.(9).

$$D_{\text{LSDCD}} = \frac{\delta_{m,p}}{\delta_{u,p}} + \beta_{\text{LSDCD}} \frac{\sum \delta_{i,p}}{\delta_{u,p}} \quad (6)$$

$$D_{\text{LSDCE}} = \frac{\delta_{m,p}}{\delta_{u,p}} + \beta_{\text{LSDCE}} \frac{\sum W_{i,p}}{F_y \delta_{u,p}} \quad (7)$$

$$D_{\text{LSDDCE}} = \frac{\delta_{m,p}}{\delta_{u,p}} + \beta_{\text{LSDDCE}} \frac{\delta_{m,p}}{\delta_{u,p}} \frac{\sum W_{i,p}}{F_y \delta_{u,p}} \quad (8)$$

$$D_{\text{DCE}} = \beta_{\text{DCE}} \frac{\delta_{m,p}}{\delta_{u,p}} \frac{\sum W_{i,p}}{F_y \delta_{u,p}} \quad (9)$$

Among the aforementioned equations,  $\delta_{m,p}$  denotes the maximum plastic deformation ever experienced under the cyclic loading,  $\delta_{u,p}$  describes a standard maximum plastic deformation,  $\delta_{i,p}$  represents the plastic deformation accumulated in the  $i$ th cyclic reversal,  $W_{i,p}$  denotes the plastic work (hysteretic energy dissipation) accumulated in the  $i$ th cyclic reversal. The first damage index  $D_{\text{LSDCD}}$  is expressed as a linear superposition of a deformation term and a cumulative deformation term (LSDCD), the second damage index  $D_{\text{LSDCE}}$  refers to a linear superposition of a deformation term and a cumulative energy-dissipation term (LSDCE), the third damage index  $D_{\text{LSDDCE}}$  is a linear superposition of a deformation term and a deformation-enhanced cumulative energy-dissipation term (LSDDCE), while the final damage index  $D_{\text{DCE}}$  solely possesses a deformation-enhanced cumulative energy-dissipation term (DCE). The parameters  $\beta_{\text{LSDCD}}$ ,  $\beta_{\text{LSDCE}}$ ,  $\beta_{\text{LSDDCE}}$  and  $\beta_{\text{DCE}}$  are respectively the model constants of the four damage indexes and need to be calibrated by using experimental and numerical hysteretic curves.

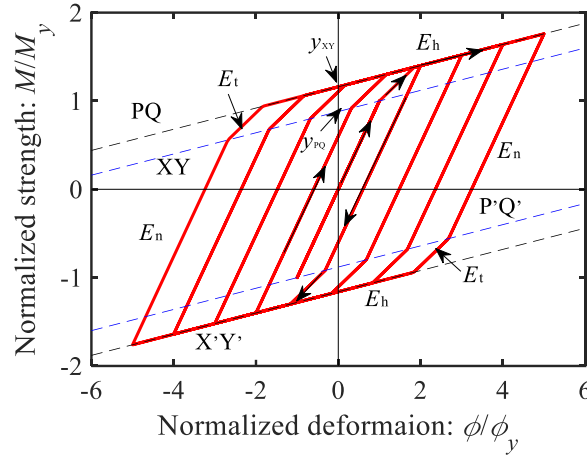


Figure 1: Trilinear hysteretic model without existence of cumulative damage

## 2.2. Calibration of model constants

The first task is to define the description parameters of hysteretic curve. Since experimental hysteretic curves are usually terminated when strength decreases to 85% of ultimate strength, the scope of deteriorated curve is quite limited and hence insufficient to yield a wider range of calibration results on model constants. In this study, a normalized hysteretic curve in FEA steel beam-column is adopted as the real curve to perform the subsequent constant calibration and influencing analysis. Unlike that safety issues may occur when experimental structural member is severely damaged, the FEA hysteretic curve can even be allowed to develop a strength deterioration up to 100% ultimate strength. Thus,

The calibration of model constants can be performed by setting the damage indexes as 1.0. Due to a relatively simple mathematical expression, all the model constants in Eq.(6) to Eq.(9) can be calculated explicitly. However, the values of influencing parameters such as  $\delta_{m,p}$ ,  $\sum \delta_{i,p}$  and  $\sum W_{i,p}$  depend on the scope of hysteretic curves. When performing experimental and numerical hysteretic analysis, the evolution of hysteretic curves is usually terminated when strength deteriorates to a certain proportion of ultimate strength. For instance, the cyclic loading tests are usually terminated when strength decreases to 85% ultimate strength. If the cyclic loading is continuously applied, the strength tends to continuously deteriorate to almost zero. Such hysteretic curve up to zero strength refers to a so-called full-range curve herein. Thereafter, different target curves can be extracted from the full-range curve by taking different termination criterions of hysteretic curve. The difference in target curves certainly leads to a difference in calibration results on model constants of CDMs.

In this study, the termination criterion of hysteretic curves refers to a strength deterioration proportion criterion, which is evaluated by a reduction factor compared to the ultimate strength ( $R_s$ ). By setting different  $R_s$  values, a series of target hysteretic curves can be extracted from the full-range curve. One representative target curve is compared to the full-range curve and peak points in Figure 2 (a). The model constants of CDMs were then calibrated by using different target curves whose  $R_s$  value ranges from 0 to 1.0 with an increment of 0.05. The relationship between  $R_s$  value and model constants  $\beta$  are schematically depicted in Figure 2 (b). It can be acknowledged that the increase in  $R_s$  lead to a continuous decrease in  $\beta$ . This is because a greater  $R_s$  indicate a greater allowable reduction of strength in hysteretic curve, leading to a wider scope of target hysteretic curve and a larger energy accumulation amount. The increase in accumulative term tends to decrease the calibration value of model constants. Among the four damage indexes, the model constants of DCE are apparently greater than that of other damage indexes within the full range.

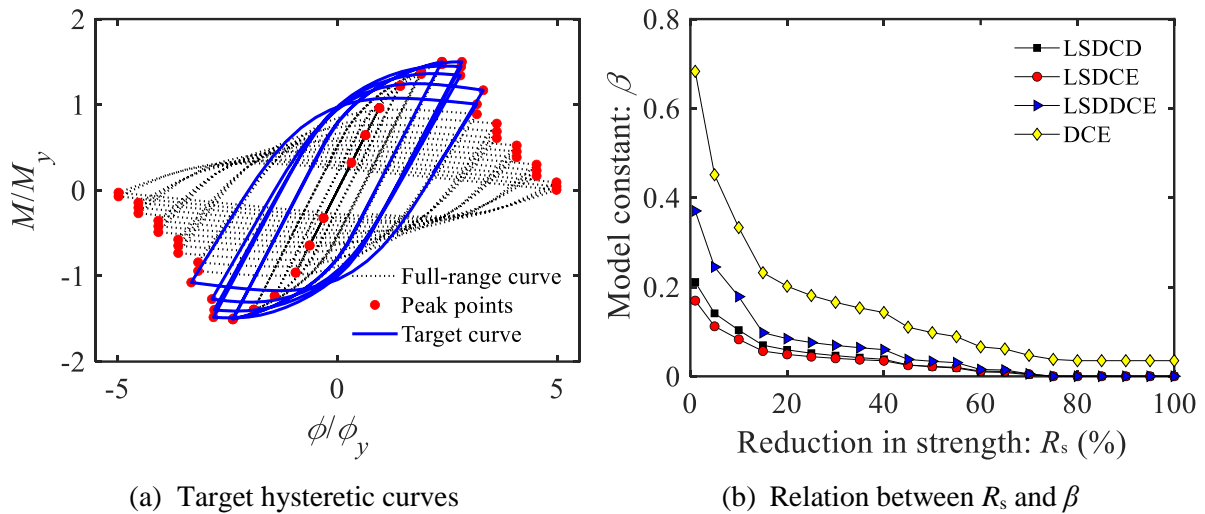


Figure 2: Calibration of model constants  $\beta$

### 3. Parametrical analysis

#### 3.1. Influence of target hysteretic curve

With regard to different  $R_s$  values, the hysteretic curves between the normalized strength and normalized deformation, the relationship between the normalized cumulative deformation and normalized hysteretic energy, the normalized skeleton curves, the relationship between the normalized cumulative deformation and normalized strength, are schematically depicted in Figure 3. It is found that the hysteretic curves continuously approach to the real hysteretic curve as  $R_s$  value increase. Correspondingly, the other three types of curves exhibit a larger magnitude in Y-axis. Overall, the evolutionary curves with greater  $R_s$  value are more closed to the real curves (FEA). When  $R_s$  reaches a magnitude of 95%, the calibration model constants can produce an accurate simulation curve compared to the FEA curve. The evolutionary curves of hysteretic energy exhibit an initial increasing stage and a subsequent decreasing stage, respectively demonstrating a considerable strain-hardening behaviour and a cyclic deterioration behaviour.

#### 3.2. Influence of CDM type

The aforementioned four types of CDMs consist of different influencing factors and possess different mathematical expressions. Hence, the damage evaluated may exhibit considerable difference even under identical cyclic loading history, further leading to an obvious alteration in damage-based hysteretic curves. In order to clarify the influencing laws, a series of evolutionary curves were numerically simulated based upon the four different CDMs and compared in Figure 4. According to the conclusion obtained in section 3.2, all model constants of CDMs were calibrated by taking  $R_s$  as no less than 95%. Among all CDMs, the DCE model exhibits the best agreement with the real curves (FEA). The

simulation curves of the other three models are quite closed to each other. This is because the first three CDMs all exhibit a linear superposition of deformation term and accumulation term. By taking the identical target curves, the calibration results of model constants tend to make similar evaluation results on damage indexes. However, the deformation term can produce a relatively large damage value and hence result in an excessive cyclic deterioration in the very first reversals. This is the reason why the hysteretic curve of DCE, which includes no deformation term, are plumper than that of the other three CDMs which possess a deformation term.

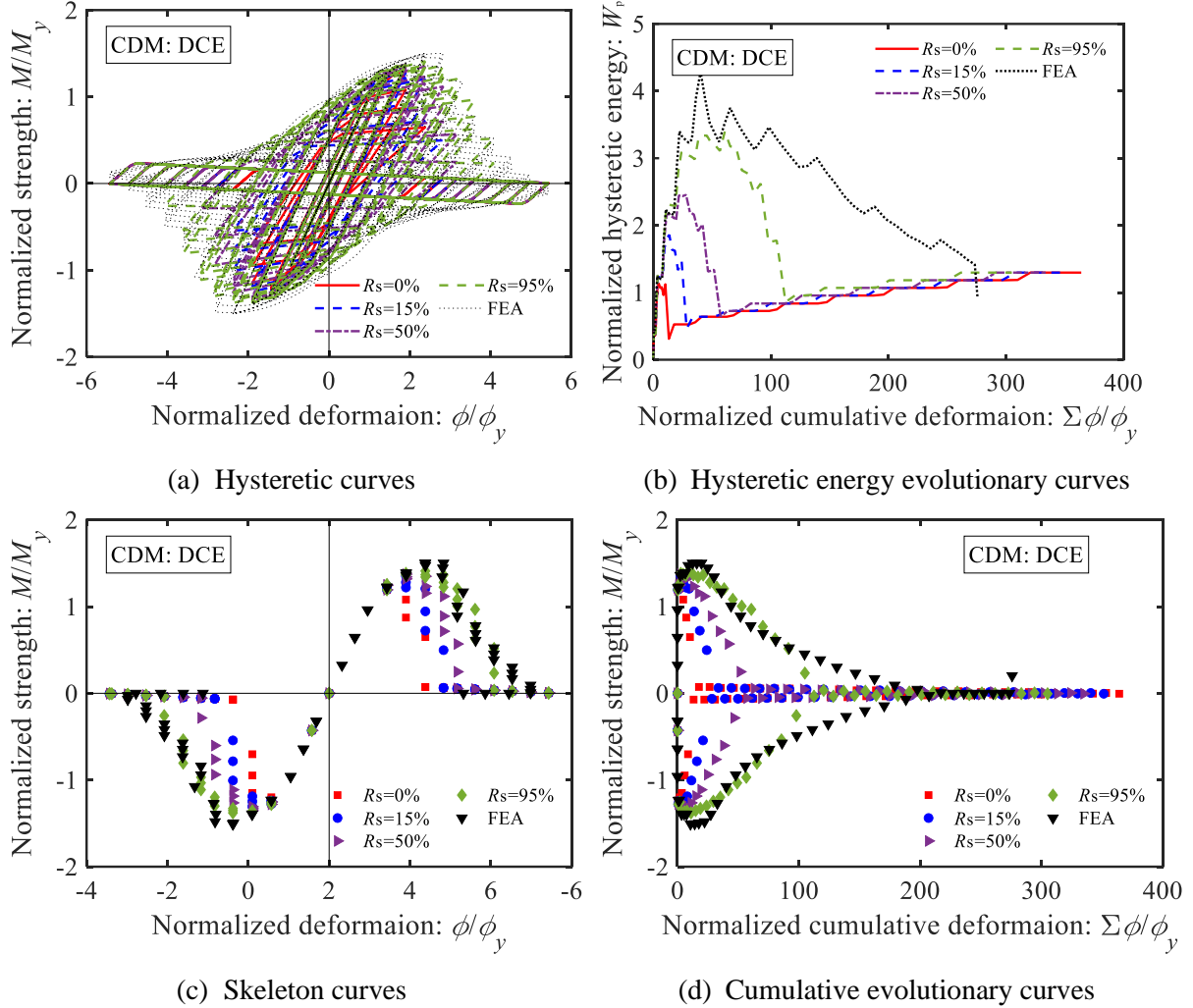


Figure 3: Comparison of evolutionary curves produced by CDM constants of different strength reduction factors

### 3.3. Influence of CDM existence

Based upon section 3.1 and section 3.2, the best way to yield an accurate cyclic behaviour is to incorporate the DCE model into the hysteretic model. Moreover, the model constant of DCE model should be calibrated by using the target curve whose strength decreases to at least 95% of the ultimate strength ( $R_s \geq 95\%$ ). In order to clarify the influencing magnitude of CDM, the hysteretic model with and without CDM are respectively generated numerically and compared to the real curve (FEA) in Figure 5. It can be seen that both the damaged curve and undamaged curve exhibit promising agreement with the real curve. However, the undamaged curve tends to considerably overestimate the hysteretic performance and energy-dissipation ability once the normalized deformation exceeds 2.0. The difference between the undamaged curve and damaged curve is continuously increased as cycle number and deformation amplitude accumulate. As cyclic loading process is terminated, the  $W_p$  value of undamaged curve exceeds 10.0, whilst the  $W_p$  value of damaged curve and real curve is closed to 1.0, indicating a unacceptable simulation gap. Even though the first cycle of undamaged curve is closed to the damaged

curve when deformation amplitude slightly exceeds 2.0, the difference between the two curves is still increased as cycle number accumulates with identical deformation amplitude. Since the cyclic loading protocol studied herein solely possesses 3 cycles for each deformation level, the gap between the undamaged and damaged curve is not obvious. To further clarify the influence of cumulative damage, a cyclic loading protocol whose deformation levels include 20 cycles, was applied to the computation code to obtain the hysteretic curves and energy-dissipation curves. To mainly reflect the influence of cycle accumulation, the deformation amplitude is within 2.0. According to Figure 6, a distinct gap between the damaged and undamaged curves can be found, demonstrating that the considerable cycle number can produce a severe deterioration even under relatively small deformation amplitude. The incorporation of proper CDM into hysteretic model is necessary.

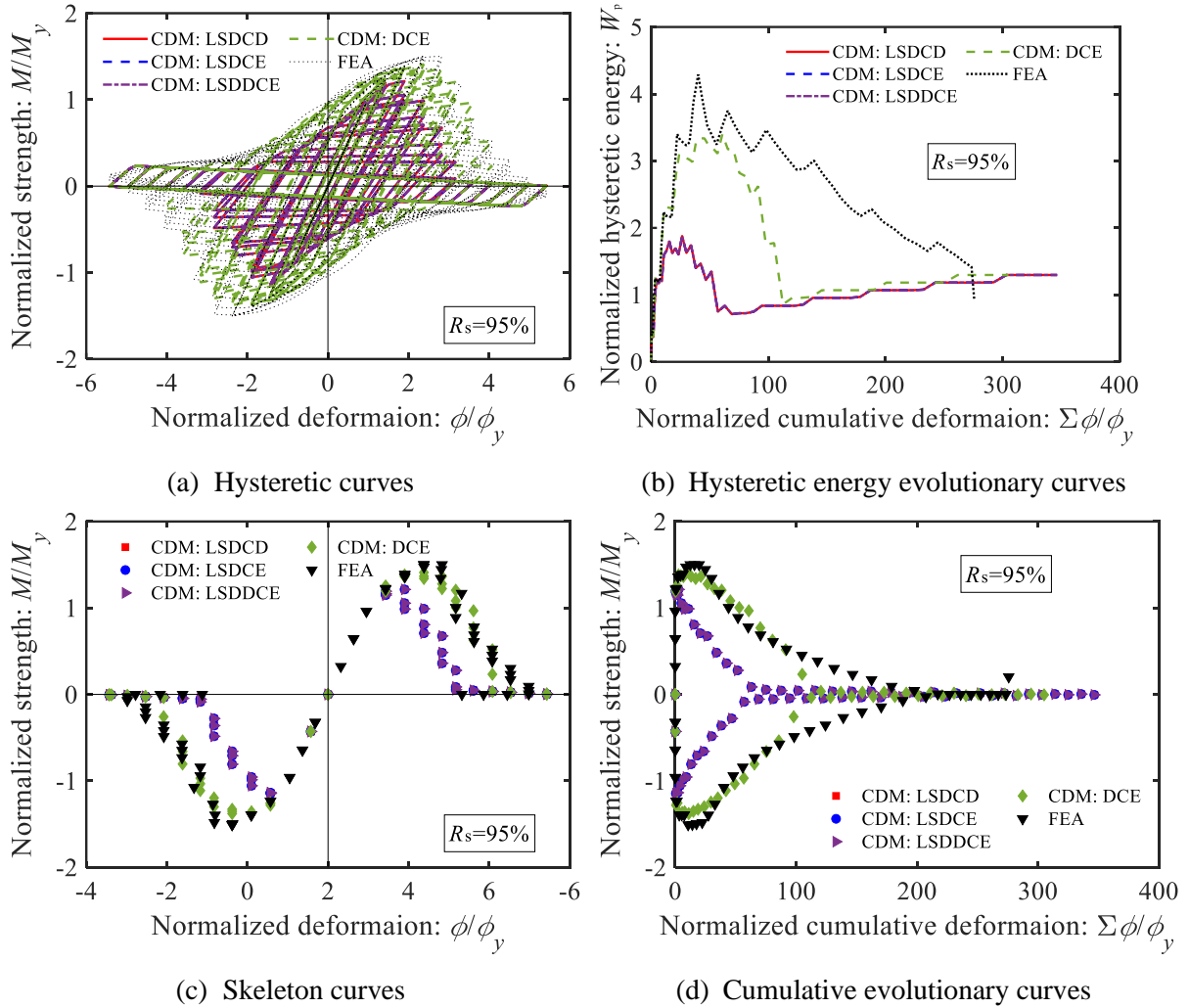


Figure 4: Comparison of evolutionary curves simulated by different CDMs



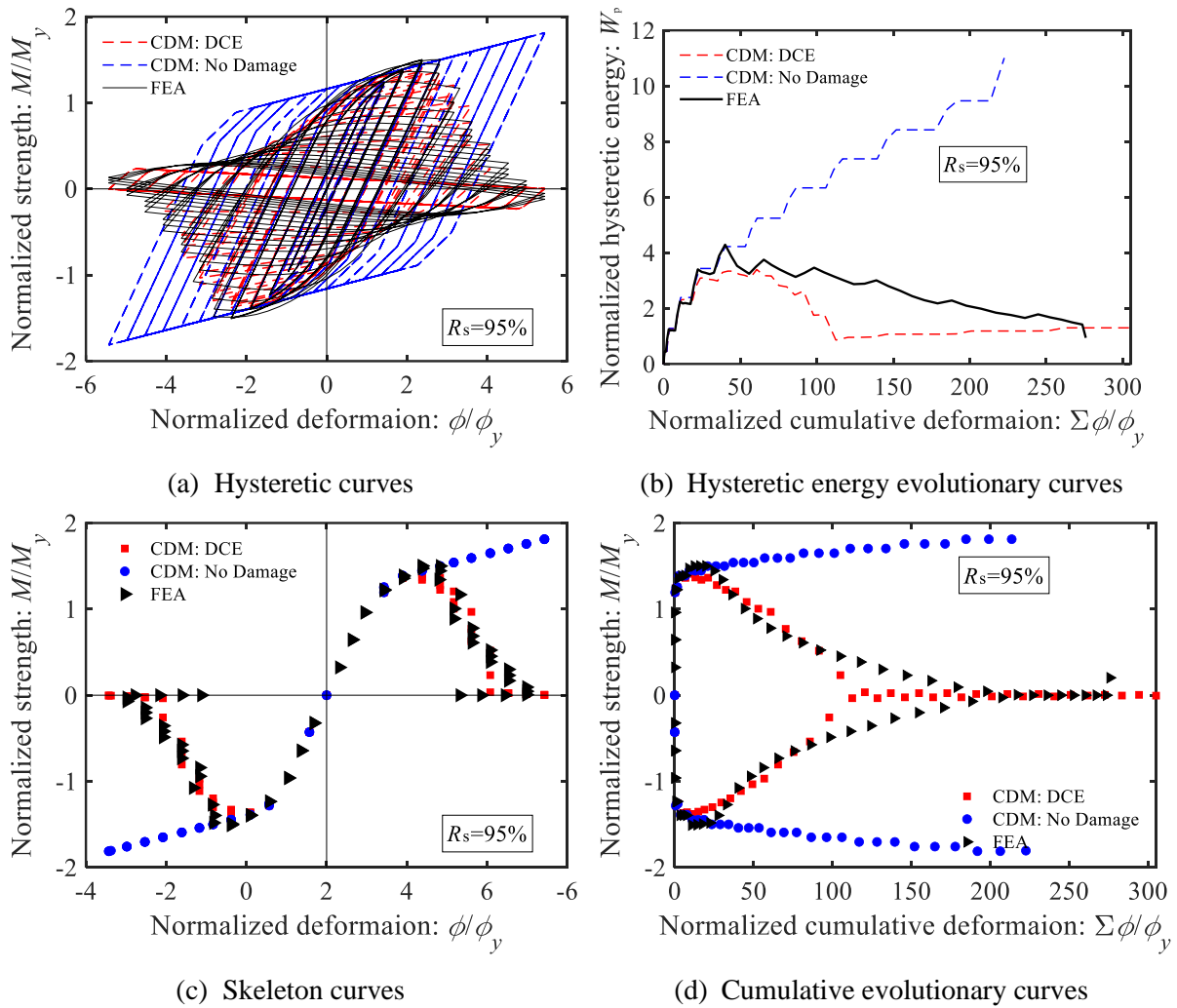


Figure 5: Comparison between damaged curves and undamaged curves

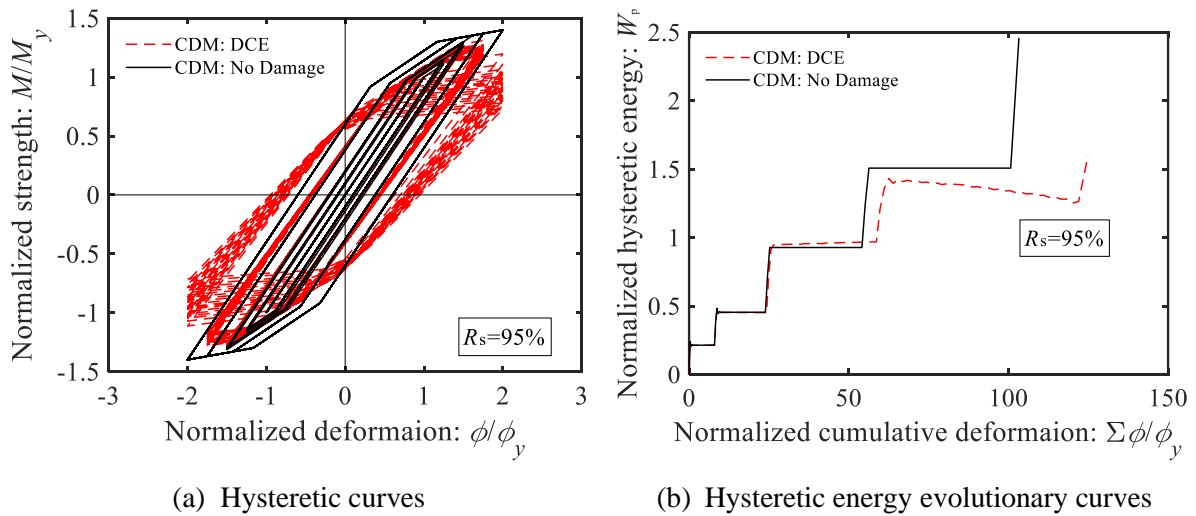


Figure 6: Comparison between damaged curves and undamaged curves

#### 4. Conclusion

This paper has performed a series of numerical simulations on hysteretic curves of steel beam-column under different CDMs and calibration ways. The influences of calibration criterion and CDM types on the simulation accuracy of trilinear hysteretic relations have been thoroughly clarified. The best CDM type and calibration criterion are determined accordingly. The following conclusions can be drawn:

The simulation performance of CDM that solely includes a deformation-enhanced cumulative energy-dissipation term is better than that of CDMs with linear superposition forms of deformation and energy terms.

The calibration results on model constants of CDMs are sensitive to the scope of hysteretic curve, in other words, the target hysteretic curve. The hysteretic curve that the strength decreases to almost zero is the best target curve for the calibration of model constant.

The hysteretic model without the existence of CDM tends to considerably overestimate the hysteretic performance and energy-dissipation ability especially under large deformation amplitude and cycle numbers. The consideration of cumulative damage in hysteretic model is significantly necessary.

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