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Lightweight Design of Tensegrity V-Expander Structures

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Abstract

The V-Expander tensegrity modules stand out for their deployability and ease of assembly, offering a novel approach to constructing large-scale and complex structures like masts, towers, and robotic arms. While existing research on V-Expanders has concentrated mainly on structural design, actuation methods, and prestress strategies, this paper focuses on their fundamental lightweight properties under three principal engineering mechanics loads: tension and compression. The study begins by outlining the design of various V-Expander topologies and their clustering methods. In tensegrity structures, clustering refers to combining individual strings into a single, continuous string that navigates through pulleys or loops at node points. To facilitate a minimal mass design, we introduce a lightweight design optimization algorithm. This algorithm avoids member failure while optimizing the structure's complexity to bear external loads effectively. By studying the V-Expanders, the paper illustrates the design concept and the versatility of this approach across varying levels of complexity. Our findings highlight the V-Expanders as highly efficient in mass across diverse structural forms and loading scenarios. Furthermore, the optimization method presented here is versatile and can be applied to other tensegrities, trusses, membrane structures, and various systems, including terrestrial, aerial, and underwater, to achieve optimized lightweight designs.

Keywords: tensegrity, minimal mass design, nonlinear optimization, lightweight structures

1. Introduction

Tensegrity structures have demonstrated their effectiveness in creating lightweight [1, 2], deployable [3, 4], and soft robotic systems [5, 6]. Similar to conventional structures in civil engineering, these can be modularly configured into various forms like columns, plates, and shells for constructing complex assemblies. The literature introduces various tensegrity modules, including tensegrity octahedrons [7], prismatic tensegrities [8], X-frames [9], T-Bar and D-Bar systems [10], and n-strut cylindrical booms [11]. This paper focuses on V-Expander tensegrities [12], which are noted for their lightness and ease of assembly, originally conceptualized by Raducanu and Motro in 2002 [13].

Current research on V-Expanders has predominantly concentrated on aspects such as morphology [14], self-stress design [15], identification [16, 17], static deformation [18], deployability [19], and actuation speeds [20]. Our study focuses on the underexplored area of V-Expander tensegrity cells' loading analysis. Given their significant loading potential, a deeper examination of their mechanical behavior is warranted. The application of V-Expander tensegrity structures in adaptive or deployable systems offers exciting opportunities in structural engineering; analyzing their loading capacities is crucial for advancing our understanding of these structures.

Research on clustered cable actuation within tensegrity structures is limited but evolving. Ali et al. developed a finite element analysis for static clustered tensile structures, accounting for friction from sliding [21]. Chen et al. examined how different clustering strategies affect the minimal mass required for tensegrity structures [22]. Ge et al. introduced a machine learning method to quantify uncertainty and manage probability in the deformation of flexible clustered tensegrity structures [23]. Ma et al. [24] formulated statics equations for these structures, considering pulley sizes. Despite these advances, research into selecting proper cables for optimal mass remains scarce. This study enhances the capability to create lightweight, clustered V-Exaonder tensegrity structures by introducing flexible clustering strategies. It offers a comprehensive method for designing structures with minimal mass while considering static equilibrium, stiffness, and potential failure modes.

This paper is structured as follows: Section 2. explores the V-Expander typologies and clustering strategies. Section 3. examines the statics of the entire structure. Section 4. outlines the mass formulation and gravity forces for the clustered tensegrity system and introduces a minimal mass design approach via nonlinear optimization. Section 5. showcases two numerical examples to verify the accuracy and efficacy of the minimal mass design theory for clustered tensegrity systems. Finally, Section 6. provides a summary of the conclusions.

2. The V-Expander tensegrity

V-Expander tensegrity cells offer multiple advantages, including geometric symmetry, ease of assembly, deployability, and adjustability. Our research focuses on their mass-to-strength ratio, specifically aiming to optimize loading capacities with minimal mass. Building upon the work of [12, 13, 15], we establish the topology of a single V-Expander cell in both two and three dimensions and introduce parameters to measure the complexity of its configuration.

2.1. The V-Expander unit

Definition 2..1 (The Three-Dimensional V-Expander Cell)**.** *A three-dimensional V-Expander cell is a V-shaped tensegrity structure with $2p$ struts, where p is the cell complexity, divided into two groups of p equally length compressed struts. Its integrity is upheld by a network of cables consisting of one vertical, p bottom horizontal, p top horizontal, and $2p$ diagonal cables, as shown in Figure 1.*

From the above definition, we know the geometry of the cell is defined by three parameters: h, which denotes the height of the pyramid formed by a group of p struts; r, the radius of the circle enclosing the cell nodes in the horizontal plane; and d, the length of the vertical cable. The nodal coordinates of the V-Expander cell of the upper nodes are specified as follows: $\boldsymbol{n}_1 = \begin{bmatrix} 0 & 0 & h-d \end{bmatrix}^T$ and $\boldsymbol{n}_2 = \begin{bmatrix} 0 & 0 & h \end{bmatrix}^T$, etc. For the lower nodes, labeled j ($j = 1, \cdots, p$), the coordinates are $\boldsymbol{n}_{j+2} = \begin{bmatrix} r\cos(2j\pi/p) & r\sin(2j\pi/p) & 0 \end{bmatrix}^T$. For the upper nodes, labeled k ($k = 1, \cdots, p$), the coordinates are $\boldsymbol{n}_{k+p+2} = \begin{bmatrix} r\cos(2k\pi/p) & r\sin(2k\pi/p) & 2h-d \end{bmatrix}^T$. The different values of the cell complexity parameter p result in various configurations of the V-Expander tensegrity cell.

2.2. Clustered V-Expander unit

In this study, we examine four distinct types of clustered actuation mechanisms for cases involving the lightweight design.

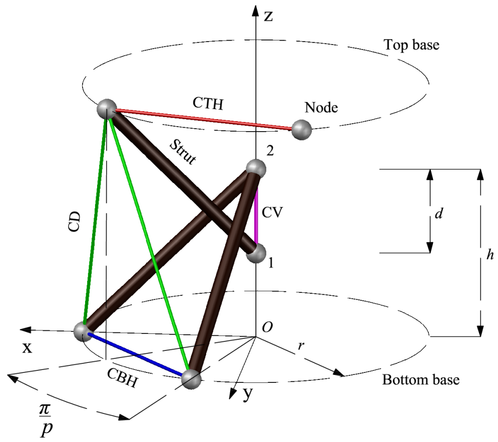

Figure 1: The V-Expander tensegrity cell: connectivity of the elements. The thick black lines are bars. The thin magenta, red, green, and blue lines are CV, CTH, CD, and CBH cables. Light grey spheres are ball joints.

Definition 2..2 (The Clustered V-Expander Cell)**.** *Through an analysis of the geometry and connectivity of the V-Expander tensegrity cell, we classify its components into specific groups: vertical cables (labeled as CV), bottom horizontal cables (CBH), top horizontal cables (CTH), and diagonal cables (CD), as shown in Figure 1.*

3. Clustered Tensegrity Statics

This analysis assesses how clustering members influence structural characteristics such as stiffness and load-bearing capacities. It is crucial to equip engineers with the ability to modify clustering strategies according to specific requirements. To facilitate this, we introduce the clustering matrix and elaborate on its properties, static behavior, mass formulation, and other pertinent aspects.

3.1. Clustering matrix and elements

Definition 3..1 (Clustering Matrix)**.** *The clustering matrix $\boldsymbol{\mathcal{S}} \in \mathbb{R}^{n_{ec} \times n_e}$ (n_e represents the number of elements before clustering, and n_{ec} indicates the total number of elements after clustering.) is introduced to record the connectivity of clustered cables:*

$$[\boldsymbol{\mathcal{S}}]_{ij} = \begin{cases} 1, & \textit{if the ith clustered element is composed of the jth non-clustered element.} \\ 0, & \textit{otherwise.} \end{cases} \quad . \tag{1}$$

The force density of a structural member is the ratio of its axial force to its actual length. For both clustered and non-clustered tensegrity structures, the force density vectors are defined as:

$$\boldsymbol{x}_c = \hat{\boldsymbol{l}}_c^{-1}\boldsymbol{t}_c,\ \boldsymbol{x} = \hat{\boldsymbol{l}}^{-1}\boldsymbol{t},\ \boldsymbol{t} = \boldsymbol{\mathcal{S}}^T\boldsymbol{t}_c, \tag{2}$$

where element length vectors $\boldsymbol{l}_c \in \mathbb{R}^{n_{ec}}$ and $\boldsymbol{l} \in \mathbb{R}^{n_e}$, $\boldsymbol{t}_c$ and $\boldsymbol{t}$ represent the internal forces in the structural members, and $\boldsymbol{v}^{-1}$ is a vector with each entry being the reciprocal of the corresponding entry in $\boldsymbol{v}$.

3.2. Clustering tensegrity statics equations

Theorem 3..1 (Tensegrity Statics)**.** *The static equilibrium of clustered tensegrity in terms of the force density vector $\boldsymbol{x}_c$ can be written as:*

$$\boldsymbol{E}_a^T \boldsymbol{A}_{1c} \boldsymbol{x}_c = \boldsymbol{E}_a^T \boldsymbol{f}_{ex}, \tag{3}$$

where the stiffness matrices $\boldsymbol{A}_{1c} \in \mathbb{R}^{3n_n \times n_{ec}} = \left(\boldsymbol{C}^T \otimes \boldsymbol{I}_3\right) \boldsymbol{b.d.}(\boldsymbol{H}) \hat{\boldsymbol{l}}^{-1} \boldsymbol{\mathcal{S}}^T \hat{\boldsymbol{l}}_c$, and $\boldsymbol{C}$ is the connectivity matrix for structural elements. The matrix $\boldsymbol{H} = \boldsymbol{N}\boldsymbol{C} \in \mathbb{R}^{3 \times n_e}$, with $\boldsymbol{N}$ as the nodal matrix, represents the structure element matrix. The function $\boldsymbol{b.d.}(\bullet)$ denotes the block diagonal operator, and $\boldsymbol{f}_{ex}$ is the vector of total external load forces. $\boldsymbol{E}_a$ is an index matrix to take free nodes $\boldsymbol{n_a}$ from total nodes $\boldsymbol{n}$, satisfies $\boldsymbol{n_a} = \boldsymbol{E}_a^T \boldsymbol{n}$.

Proof. The static equations of clustered tensegrity structures can be described using three equivalent representations: nodal vectors, force density, and force vectors. Detailed proof of this equivalence is provided in [22]. □

4. Minimal Mass of the Clustered V-Expander

Definition 4..1 (The Minimal Mass)**.** *The minimum mass of a specific tensegrity structure is reached when all its components fail (either buckle or yield) at the same time.*

Theorem 4..1 (Minimal Mass Function)**.** *The minimal mass of a tensegrity structure, considering both buckling and yielding as modes of bar failure, can be expressed as follows:*

$$M = \boldsymbol{\Gamma} \boldsymbol{x} + \boldsymbol{\Lambda} \boldsymbol{x}^{\frac{1}{2}}, \tag{4}$$

where $\boldsymbol{\Gamma}$ and $\boldsymbol{\Lambda}$ are constant coefficient matrices:

$$\boldsymbol{\Gamma} = \begin{bmatrix} \frac{\rho_s}{\sigma_s} \left(vec(\lfloor \boldsymbol{S}^T \boldsymbol{S} \rfloor)\right)^T \widehat{|\boldsymbol{C_s e_i}|} & \frac{\rho_b}{\sigma_b} \left(vec(\lfloor \boldsymbol{B}^T \boldsymbol{B} \rfloor (I - \boldsymbol{Q}))\right)^T \widehat{|\boldsymbol{C_b e_i}|} \end{bmatrix}, \tag{5}$$

$$\boldsymbol{\Lambda} = \begin{bmatrix} \boldsymbol{0} & 2 \frac{\rho_b \left(vec(\lfloor \boldsymbol{B}^T \boldsymbol{B} \rfloor^{\frac{5}{4}} \boldsymbol{Q})\right)^T}{\sqrt{\pi E_b}} \widehat{|\boldsymbol{C_b e_i}|} \end{bmatrix} \tag{6}$$

and $\boldsymbol{Q} \in \mathbb{R}^{n_b \times n_b}$ is a diagonal matrix that identifies the mode of failure for bars, with diagonal elements as follows:

$$\boldsymbol{Q}_{jj} = \begin{cases} 0 & \boldsymbol{\lambda}_j \geq \dfrac{4\sigma_b^2 ||\boldsymbol{b}_j||}{\pi E_b}, \; \textit{Yield} \\ 1 & \lambda_j < \dfrac{4\sigma_b^2 ||\boldsymbol{b}_j||}{\pi E_b}, \; \textit{Buckle} \end{cases}. \tag{7}$$

Proof. The derivation is straightforward. The first and second parts of Eq. (4) represent the mass of strings and bars subject to yielding, and the mass of bars prone to buckling, respectively. A detailed discussion can be found in [22]. □

4.1. Minimal Mass of V-Expander

Theorem 4..2 (Minimal Mass CTS). *Considering the topology ($\boldsymbol{N}$, $\boldsymbol{C}$, $\boldsymbol{\mathcal{S}}$), external forces $\boldsymbol{f}_{ex}$, and predefined prestress in the strings $\boldsymbol{\epsilon}_c$, the minimal mass V-Expander under different loading conditions can be determined by solving the subsequent nonlinear programming problem:*

$$\begin{cases} \underset{\boldsymbol{x}_c}{\textit{minimize}} & M \\ \textit{subject to} & \boldsymbol{E}_a^T \boldsymbol{A}_{1c} \boldsymbol{x}_c = \boldsymbol{E}_a^T \boldsymbol{f}_{ex},\ \boldsymbol{x}_c \geq \boldsymbol{\epsilon}_c\ (\boldsymbol{\epsilon}_c \geq 0) \end{cases}. \tag{8}$$

where $\boldsymbol{\epsilon}_c \geq 0$ ensures that all strings remain under tension and all bars stay in compression.

Algorithm 1: Minimal mass of the V-Expanders.

1) Given structure topology ($\boldsymbol{N}$, $\boldsymbol{C}$, $\boldsymbol{\mathcal{S}}$) subject to various complexity p, free nodal index matrix $\boldsymbol{E}_a$, external force $\boldsymbol{f_{ex}}$, force density computational tolerance μ.
2) Assumes all bar buckles, $\boldsymbol{Q} = I^{n_b \times n_b}$, and μ = 1e-6.
while $\min\{eig(\boldsymbol{K}_{Tc})\} < eig(\bar{\boldsymbol{K}}_{Tc})$ **do**
  while $\boldsymbol{Q}_{i+1} \neq \boldsymbol{Q}_i$ *or* $max|\ \boldsymbol{x}_{c,i+1} - \boldsymbol{x}_{c,i}\ | \geq \mu$ **do**

$$\begin{cases} \underset{\boldsymbol{x}_c}{\text{minimize}}\ M \\ \text{subject to}\ \boldsymbol{E}_a^T \boldsymbol{A}_{1c} \boldsymbol{x}_c = \boldsymbol{E}_a^T \boldsymbol{f}_{ex},\ \boldsymbol{x}_c \geq \epsilon_0. \end{cases}$$

    Compute force densities $\boldsymbol{x}$ from $\boldsymbol{x}_c$:
    $\boldsymbol{x} = \hat{\boldsymbol{l}}^{-1} \boldsymbol{\mathcal{S}}^T \hat{\boldsymbol{l}}_c \boldsymbol{x}_c$.
    Take $\boldsymbol{\lambda}$ out of $\boldsymbol{x}$, check Eq.(7), update $\boldsymbol{Q}$.
    $i \leftarrow i + 1$.
  end while
end while

5. Numerical examples

In this section, we examine two numerical examples to validate the proposed method: a tensegrity V-expander with varying complexities and clustering strategies, subjected to compressive and tensile loads. The two numerical examples analyzed do not have self-stress in their initial configurations. Aluminum bars and UHMWPE strings are used as the material in all cases.

5.1. Compressive loads

The relationship between mass and complexity (p = 1, 2, $\cdots$, 20) in the V-Expander tensegrity cell, considering different clustering strategies under compressive loads, is depicted in Figure 2. The graph shows that mass increases monotonically as complexity rises for the three clustering strategies.

5.2. Tensile loads

The relationship between mass and complexity (p) for the V-Expander tensegrity cell under tensile loads, with varying clustering strategies, is illustrated in Figure 3. The graph indicates a monotonic decrease

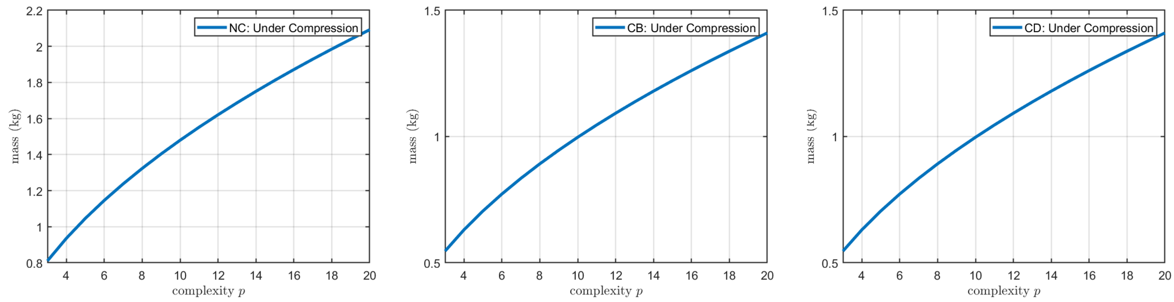

Figure 2: The mass versus complexity (p) relationship for the V-Expander tensegrity cell with various clustering strategies subject to compressive loads.

in mass with increasing complexity for the NC case. The other two cases show fluctuations in mass at different complexities. Specifically, the CD case exhibits less variation in mass compared to the BT case.

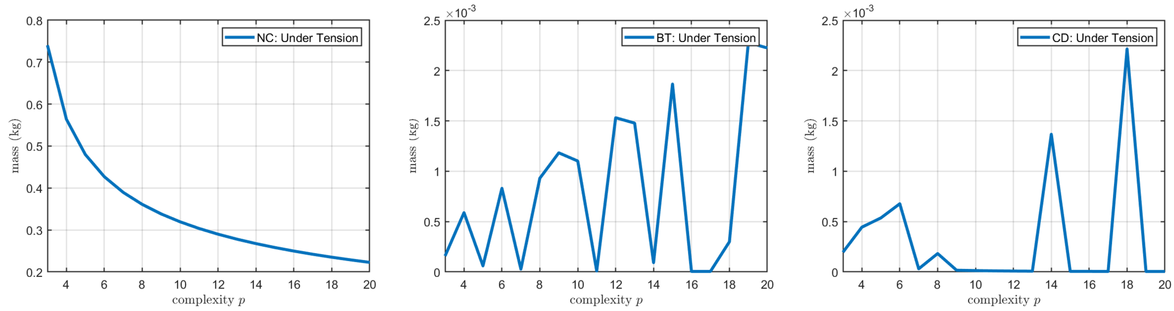

Figure 3: The mass versus complexity (p) relationship for the V-Expander tensegrity cell with various clustering strategies subject to tensile loads.

6. Conclusions

The V-Expander tensegrity modules are notable for their deployability and ease of assembly, providing innovative solutions for constructing large-scale and complex structures such as masts, towers, and robotic arms. While existing studies on V-Expanders have focused on structural design, actuation, and prestress strategies, this paper explores their inherent lightweight properties under three key engineering mechanics: load, tension, and compression. We start by detailing the design of various V-Expander topologies and their clustering methods, where clustering involves integrating individual strings into a continuous configuration through pulleys or loops at nodes. To promote minimal mass design, a lightweight design optimization algorithm is introduced to prevent member failure and enhance the structure's capacity to withstand external loads. This study not only demonstrates the design flexibility of V-Expanders across different complexities but also presents an optimization method applicable to other structural systems, including tensegrities, trusses, and membrane structures in various environments, ensuring optimized lightweight solutions.

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