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Form finding by shape optimization with implicit splines and vertex morphing

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Abstract

The procedure of form finding of mechanically loaded structures is an art. The resulting shape is the goal, the paths to that goal are numerous as well as the tools applied. Everything is possible. The handles which control the process are plenty in number, many are rationally explained, others are heuristic in nature, intuitively chosen or even hidden in the complexity of procedures, tools and their interaction. Even when principally using the same method, the final structural and esthetical result of form finding still tells much about the mind behind. Tools such as computational methods should be designed to support the individual understanding of form finding and the aesthetics, allowing the largest possible design space while being efficient and controlled by a minimum of effort. We present numerical shape optimization with vertex morphing as the solution: Numerical optimization as a rational technique to guide the process and vertex morphing as a most flexible and easiest to be applied method of shape control with arbitrarily large numbers of degrees of freedom. The methodological kernel are so-called implicit splines which don't need fixed control meshes as standard splines do. Together with so-called filters which again may be defined explicitly or implicitly they are related to variants of subdivision splines allowing for great varieties of solutions to the form finding of structures. The paper gives an illustrative demonstration of the method together with various examples of optimal shell design and some other applications.

Keywords: Fom finding, shape optimization, Vertex Morphing, implicit splines, nonlinear numerical optimization, shells, solids

1. Form finding of shells: Design noise and the infinity of design space

The principal challenge of form finding can briefly be explained by an illustrative example. The task is to design the stiffest structure made from a piece of paper which is able to act as a bridge carrying load. The solution is well known. As the piece of paper is unable to act in bending stiffeners have to be introduced by folding the paper. However, there exists an infinite number of solutions which all of them do the job creating stiff solutions of at least similar quality which is by far better than the quality of the initially flat piece of paper. Surprisingly enough, even an arbitrary pattern of random folds appears to be a possible solution, Fig. 1. The figure of the randomly crinkled paper is an ideal paradigm for the infinity of the design space or, more ostensive, the "design noise". As for the actual example the crinkled paper can be understood as the weighted combination of all possible stiffening patterns one can easily think of a procedure to derive any of the individual, basic solutions of distinct stiffening patterns by applying suitable "filters" to the design noise. It is clear that the kind of "filter" as well as its size, i.e. the filter radius, can be freely chosen as a most important design decision which guides the form finding process and the final solution.

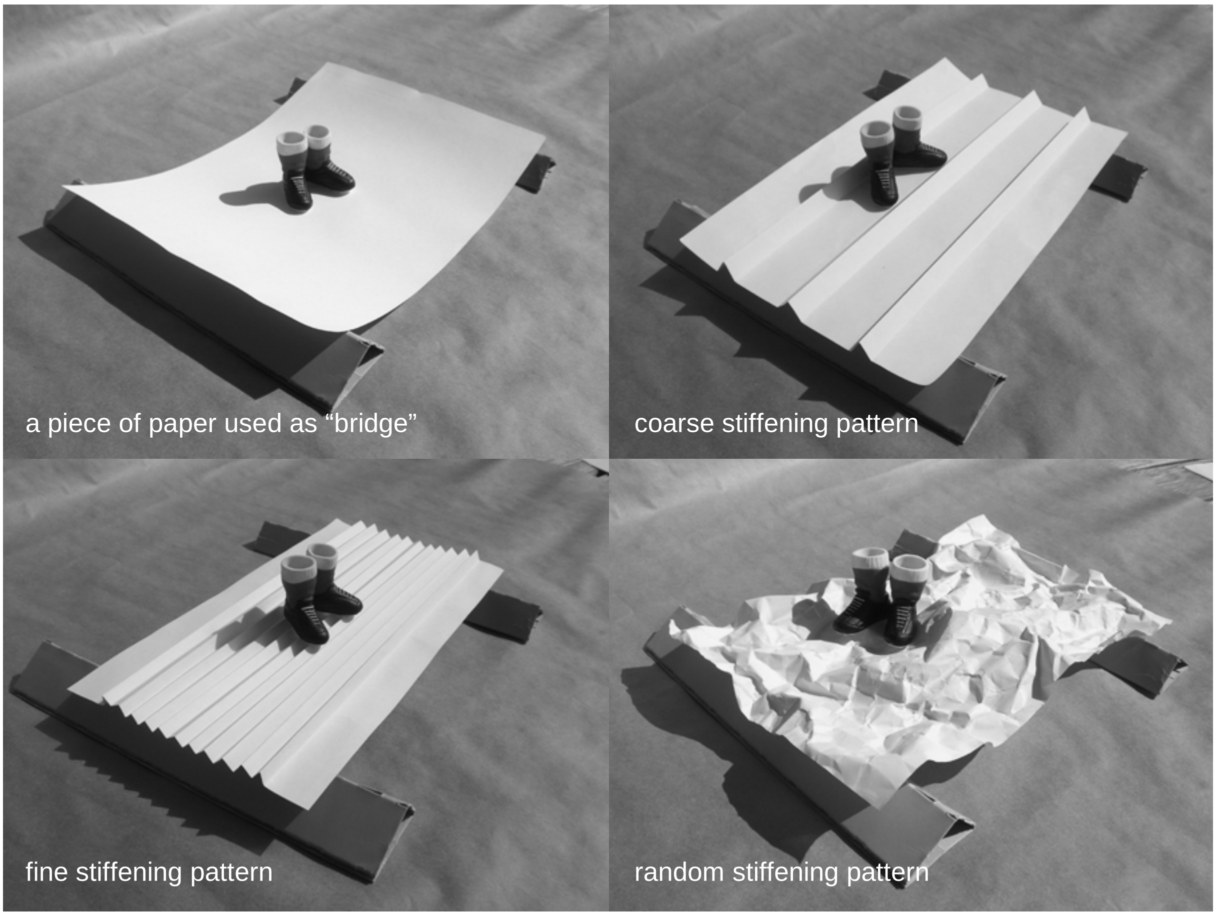


Figure 1: Stiffened shell structures made from folded paper.

2. Regularization of the "design noise" by implicit splines and vertex morphing

Following the above example, we take the crinkled paper as what we call the control field s of our procedure. From the control field s we derive the geometry x by applying a low pass filter A, Fig. 2. The formula reflects the convolution of s with the material surface coordinates ξ and the filter function A of radius r which is centered at ξ_0 generating the geometry x from the control field s:

$$x = \int_{\Gamma} A s \, d\Gamma = \int_{\xi_0 - r}^{\xi_0 + r} A(\xi, \xi_0, r) s(\xi) d\xi \tag{1}$$

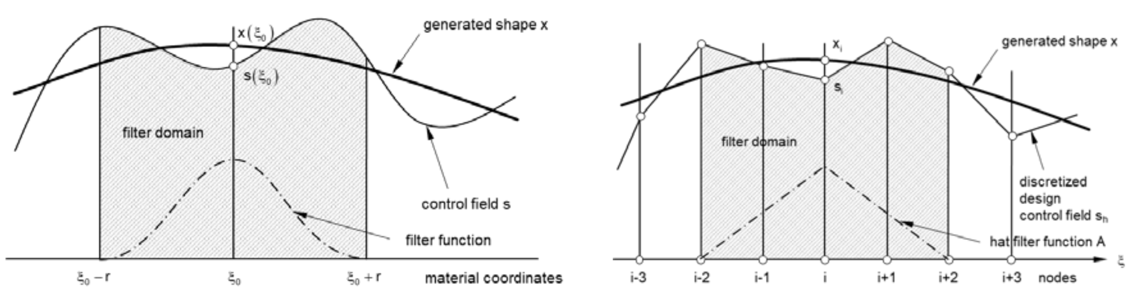


Figure 2: Applying filter A to generate shape x from control field s, continuous (left), discretized (right).

As may be anticipated from the discretized version of the filter process, Fig. 2 right, the control field s is the continuous equivalent of the control polygon s_h as it is known from standard splines. As an example, a cubic B-spline is created by filtering a linear hat function by a linear hat filter, refer to Fig. 2. That explains the well-known definition of subdivision splines [1]. Its generalization we call vertex morphing [2-7]. The vertices of the discretized control field s_h are the nodes of the underlying finite element mesh which has been created in advance by any standard pre-processor. When implementing vertex morphing the discrete control handles s_i can be condensed out if the control field is discretized on the same mesh as the geometry x which is the standard case. Then, the discrete values of the control field must not be determined, although they are the crucial part of the formulation. The nice consequence for the application is that the control field must not be displayed at any stage of the form finding procedure. The designer directly deals with the generated shape only. Which is the subject of interest.

Still, the kind of filter function A may be freely chosen. The simplest variant is the linear hat function as introduced above along a line. On a surface it may be extended to a cone. Filter functions may be defined on a limited support, i.e. they are zero outside of r. Or, they may span the complete structure. Most important is the filter radius r which controls the "waviness" of the generated shape. The radius defines the size of limited supports or a characteristic length of unlimited filters. With respect to numerical shape optimization, typically, the chosen radius steers the optimization to local minima of shape which waviness is characterized by that radius r. As a consequence, the choice of the filter radius is an important design parameter. For many practical applications, the input of one filter radius value is sufficient. Also, different values of the filter radius can be applied to different regions of the structure. In all cases, even the most challenging design problems, the application of filter radii is effortless and intuitive.

Fig. 3 displays a characteristic example. The optimal shape of an initially flat shell with circular plan due to distributed vertical load is determined for maximal stiffness. Left, without applying filters, an algorithm determines the crinkled "design noise", while on the right we receive the well-known dome if a simple hat filter with radius of the circular boundary is applied. From a numerical point of view, there is an additional argument for filters as distorted meshes as on the left are artificially stiff. That must be avoided and is easily done by at least taking the smallest allowable filter radius to be some multiple of the element size, typically 2 to 4. For smaller filter radii the mesh must be refined.

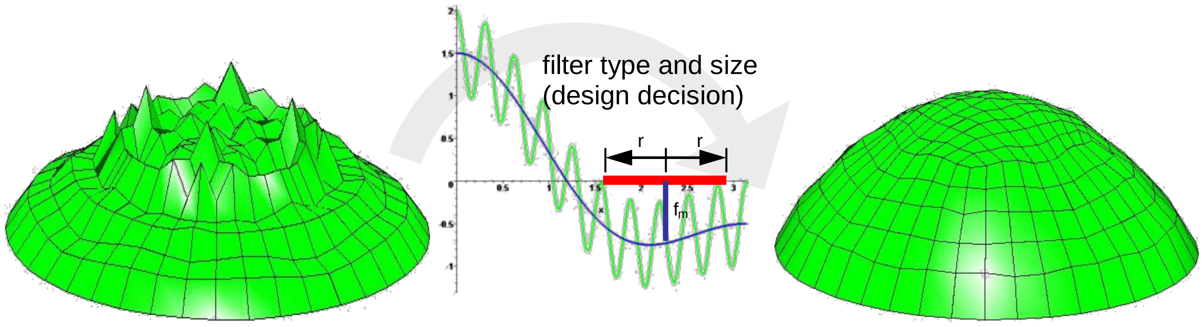


Figure 3: Direct numerical stiffness optimization and filtering of a plate subjected to vertical distributed load.

Last but least we distinguish explicit and implicit filtering. That is best explained by the discrete version of filtering, refer to equation (1). Then the filter function A is replaced by a filter matrix **A** which filters the discrete control field values collected in vector **s** to the discrete shape coordinates collected in **x**, (2, left). For the case of equal numbers of control and coordinate numbers the operation may be inverted (2, right) with implicit filter matrix $\tilde{\mathbf{A}}$:

$$\text{explicit filtering:} \quad \mathbf{x} = \mathbf{A}\,\mathbf{s} \qquad\qquad \text{implicit filtering:} \quad \mathbf{s} = \mathbf{A}^{-1}\,\mathbf{x} = \tilde{\mathbf{A}}\,\mathbf{x} \tag{2}$$

As an example the implicit filter matrix $\tilde{\mathbf{A}}$ may be defined as:

$$\tilde{\mathbf{A}} = \mathbf{I} - \varepsilon\,\boldsymbol{\Delta} \tag{3}$$

with the unit matrix **I**, the Laplace operator $\boldsymbol{\Delta}$ and some scalar ε which is equivalent to the filter radius, all together defining what is known as Helmholtz filtering or Sobolev smoothing [8, 9]. The Laplace operator $\boldsymbol{\Delta}$ applied to the geometry in terms of **x** represents the surface curvature properties. The result is the smoothing of the (unwanted) geometric "waviness" and ε, equivalent to r, as a measure of the wavelength. The convincing property of the implicit version of filtering is that boundary conditions of the generated shape **x** can be directly controlled compared to the explicit one. Without further explanation one can imagine that there also exists a continuous equivalent filter function $\tilde{A}$ to the implicit filter matrix $\tilde{\mathbf{A}}$.

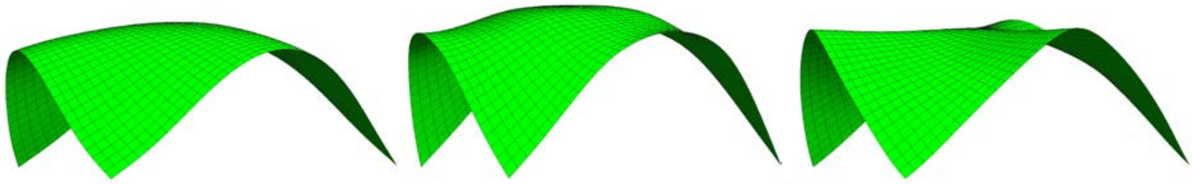
The following examples are created by using explicit and implicit filtering. For the implicit case eventually together with filter functions spanning the complete structure, so controlling the overall continuity of shape together with the boundaries. The choice of ε or, equivalently, the filter radius r is the design handle for form finding, steering the result to a respective solution.

Besides the tremendous amount of mathematics in the background of the method it is simplest and straightforward to be applied. What is necessary is a finite element model of the structure as output of common CAD preprocessors. It must be fine enough to be undistorted and able to resolve the smallest expected curvature radii of shape. Pragmatically, better choose a finer mesh than at least necessary. Every node of the mesh may move during form finding. That defines problem sizes with the number of optimization parameters as the number of mesh nodes multiplied by the number of coordinates at every node. Obviously, that gives very large numbers which, however, can easily be treated as a consequence of the indirect control of geometry by the control field s and the filter idea. Refer to Fig. 3 and estimate the size of the respective problem. What remains are the choice of filter functions (typically either explicit or implicit) and the size of the filter radii as control handles. For the following examples very fine meshes are used. Typically, they are not shown because they do not give additional information. The filter sizes are not reported in absolute numbers. They are chosen large enough to give the displayed results. Since shell structures are most effective to transfer loading by an infinite number of alternative load paths there exists a related, as well infinite number of optimal shapes. Consequently, the choice of a filter radius or, eventually, a certain distribution of filter radii (e.g. smaller at the edges, larger in the interior to allow stiffened edges with locally larger curvature) decide about the result of form finding.

The idea of the paper is to demonstrate stages of a typical design procedure: (i) Take some filter and filter radius, set up the optimization problem and receive a first optimal shape, (ii) discuss the quality of that shape with respect to its properties (e.g. its load carrying principle, geometric properties, aesthetics, manufacturability, etc.), (iii) repeat the first steps as often as necessary to explore the design space to find alternative solutions, (iv) take your final choice among the intermediate solutions found. The examples do not represent more than one of these. Since we could show any other as the result of a varied filter, the specific value of the filter radius is not important information. Typically, finding the shape of shells is a process. There are no optimal solutions without alternatives. A form finding method, such as vertex morphing, must help identify them.

Example 1: explicit filtering and varying filter radii

The shape of a three point supported shell is to be optimized for stiffness subjected to self-weight, Fig. 4. The ground plan is fixed. The shell thickness is held constant, in particular not allowing for thicker edge beams. The size of the filter radius is varied choosing smaller filter radii at the edges and larger ones in the interior shell domain as given in the figure. Playing with filters a large variety of shapes can be generated which represents well known classes of optimized shell structures which all of them do not need extra edge beams: (a) the positive curved shell with special edges treatment perhaps similar to Frei Otto’s “Segelschalen”, (b) the negative curved edge well known from many of Isler’s shells, and (c) the “Candela type” hypar-like solution [10]. Of course, the aesthetic quality is far from satisfactory due to the comparatively small number of design degrees of freedom, but the example shows very nicely how different solutions can be found in principle by extraction from the "design noise" of the chosen discretization. Nevertheless, the shapes are generalized splines as the result of applying explicit hat filters to the control field s. This field cannot be displayed although it is the methodological nucleus.



a) radius size 11:3 (interior : edge) b) radius size 7:3 c) radius size 5:3

Figure 4: Different optimal shapes of shells extracted from the design space by varying the filter size.

Example 2: The Valencia "Candela"-shell

Felix Candela is known as the master of hypar shells. He developed a principle of shape based on the negative curved hyper-paraboloids, on the same time aesthetically impressive and efficient to be designed and built. He even developed an own approach to the structural analysis of hypar shells [11]. The community has agreed that structures such as his are optimal. This example is about to further analyze the structural properties of the "Candela"-shell in Valencia and to eventually find further improvement by applying vertex morphing. The implicit filter has been applied with a filter radius large enough to maintain a smooth overall shape. The example also shows nicely that the filter works on the shape modification rather than on the resulting shape as the sum of initial shape and modification. As a consequence, the sharp gorges of the shape and the "Candela"-characteristic are maintained. The existing shell in Valencia was the last structure Candela was involved. It was designed and created by local engineers applying new technologies such as fiber reinforced concrete [12].

Fig. 5, left, shows the built structure. On the right one can see the models of the original shell (grey) and the optimized one (red). The stiffness is maximized due to distributed surface load, while the mass is held constant. As a consequence, the center of the shell is lifted and outer regions of the vaults are lowered. The cross section and detailed focus on a vault show the high quality of geometry of the optimized shell in terms of curvature and waviness. The finite element mesh is very fine and not shown.

The optimized shell significantly deforms better than the original one, Fig. 6. The maximum deformation is concentrated to the center and is reduced by factor 4. Very impressively, the deformation at the free edges is simultaneously reduced and homogenized. The localized large deflections at the edge have disappeared.

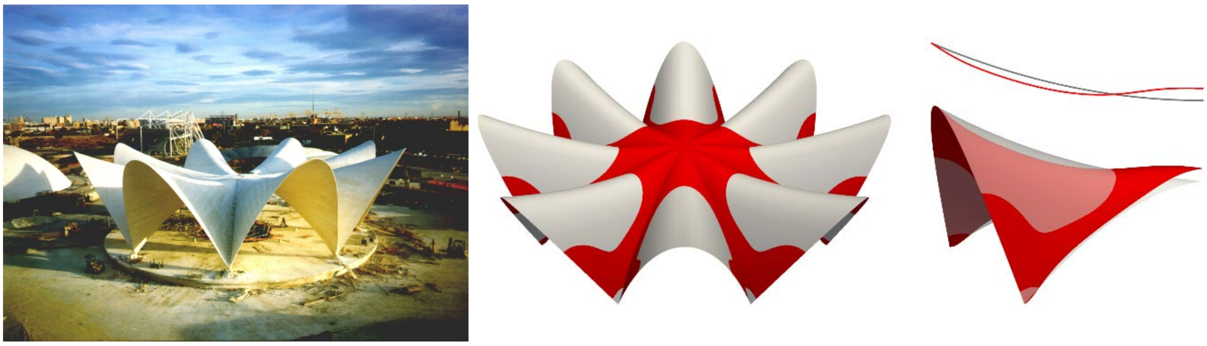


Figure 5: Left: The Valencia shell in construction; Right: the original (grey) and optimized (red) shell.

The example demonstrates how vertex morphing may be applied to study and to develop shells along accepted design principles, exploring the infinity of design space while finding as well engineering as aesthetically convincing solutions. Still, of course, it is left to everybody to search for further variants by varying the filters or modifying the optimization problem, adding constraints or simultaneously optimizing shape and thickness, Fig. 7.

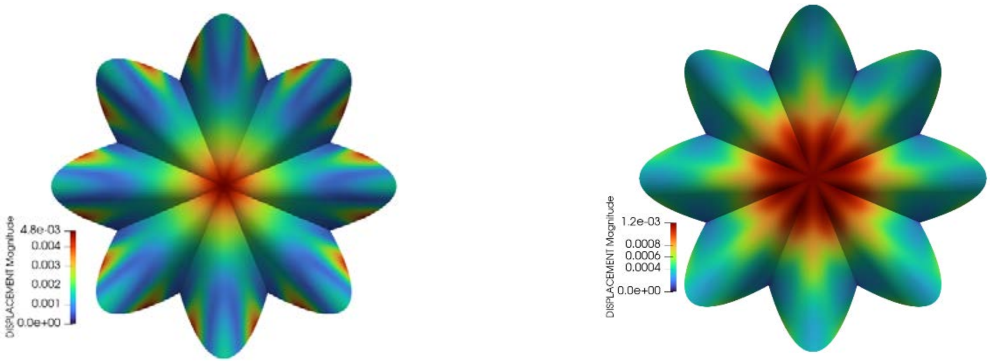


Figure 6: Magnitudes of displacement, original (left) and optimized (right).

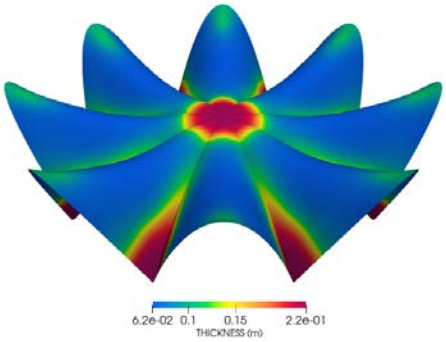


Figure 7: Candela-shell, filter technique applied find optimized thickness distribution.

Example 3: The "Torroja"-shell

Another icon of concrete shells is Torroja's shell for the market hall in Algeciras, Figs. 8, 9. Again, the shape has been further optimized for stiffness due to distributed surface load and a constraint on mass. The implicit filter with a large radius has been applied. The technique allows to maintain the characteristic kink between the "caps" of the shell and the interior region. Obviously, it appears to be a good idea to raise the apex of the arches of the edge caps. Consequently, the originally positive double curved interior dome is now transferred into a negative curved, "wavy" surface. The specific properties of vertex morphing allow to generate another, aesthetically pleasing solution. Again, further alternatives may easily be generated with varying filters and setups of the optimization problem.

Fig. 9 displays several cross sections in radial and tangential direction through the original and optimized shells from which the geometry of the newly form found vaults becomes obvious. Again, the load carrying behavior of the optimized shell is improved and much more homogeneous as before which can be anticipated from Fig. 10 which shows the vertical deflections.

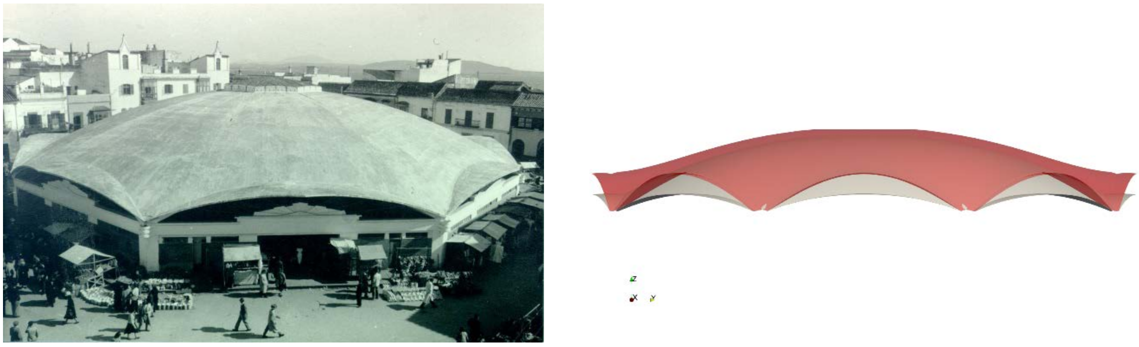


Figure 8: Left: Torroja's shell; Right: models of the original (grey) and optimized (red) shell.

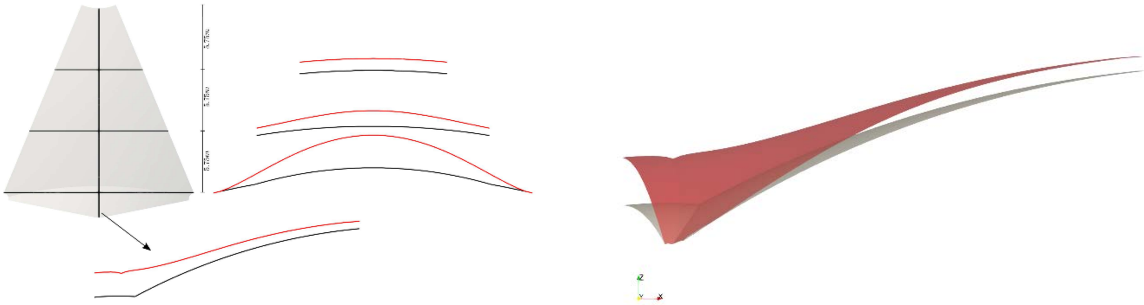


Figure 9: Left: Torroja-shell, cross sections; Right: Comparison, original (grey) and optimized (red) shell.

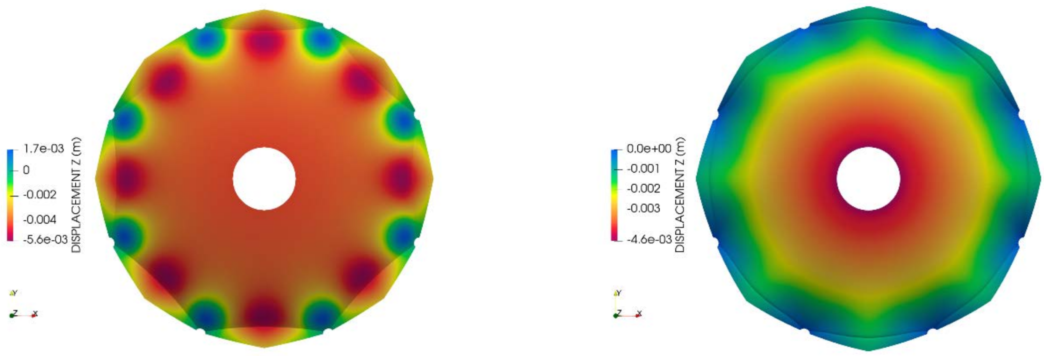


Figure 10: Torroja-shell, vertical deflections; Left: the original, Right: the optimized shell.

Example 4: Free from "hanging model"-shell

This example is another academic case of a completely free-form shell. Over a quadratic plan the shape is optimized for maximum stiffness due to vertically acting, unifom load, Fig. 11 (top). Vertex morphing is applied with implicit, Fig. 11 (middle), and explicit filters, Fig. 11 (bottom) together with large and small radii, respectively. The four corners and parts of the interior, initially circular holes are held on the ground to be the supports of the shell. Refer to the figures for further information of the model definitions. Note the fine finite element mesh. Besides the supports, each of the nodes are allowed to move freely. Again the number of optimization variables is very large but not of importance. In particular, the example nicely shows the effect of the filter radius size. With large filters one receives a very smooth and continuous surface. The properties of an implicitly defined spline surface becomes obvious. As can be seen from the result at the bottom of Fig. 11 small filters drive the shape to one with localized curvature. This may be useful if shapes with strong stiffeners shall be designed. Again, the possible range of form finding is demonstrated. Optimization is used as a tool of form finding. All shapes found are optimal solutions in equilibrium as a consequence of the chosen filter strategy.

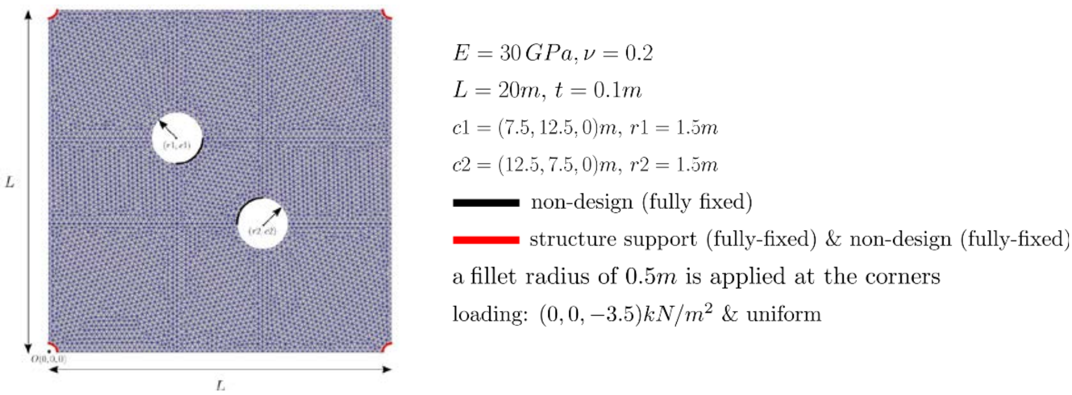
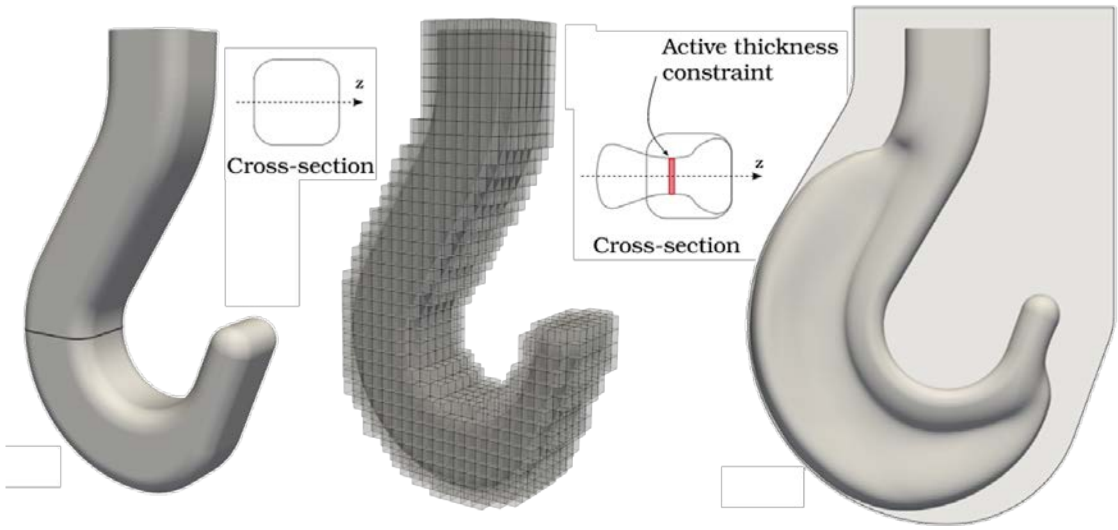


Figure. 11: Free form shell on quadratic plan. FEM mesh and model set up.

Figure. 11, continued: Free form shell on quadratic plan. Top: results with implicit filter and large radius; Bottom: result with explicit filtering and small radius.

Example 5: Shape optimization of 3D solids with embedded techniques

Vertex morphing has been combined with embedded techniques to optimize the shape of complex 3D structural objects. Shape control by vertex morphing is applied to the surface of an object that is fully embedded in a background mesh of regular, voxel-like finite elements for simulation and sensitivity analysis. The trick is how to deal with the moving free shape surface, which typically divides elements into filled and empty parts [7]. The application of shape optimization is as simple as before: select one or more filter radii that you consider suitable and start the procedure. The example shows the stiffness optimization of a hook as it is embedded in the background mesh and the optimized result, Fig. 12. Vertex morphing is applied to the trimmed structure surface within the surrounding voxels.



(a) initial geometry (b) embedded (c) final geometry

Figure 12: Shape optimized 3D hook for max. stiffness.

Example 6: Racing car

The last example gives an impression of where and how vertex morphing is used by industrial partners from other technical areas, Fig. 12. Front and rear parts of the car body as well as the rear spoiler are the subject of a shape optimization to improve lift and drag of a BMW racing car [5]. The form finding process is exactly what was described above, but applied to fluid dynamics.

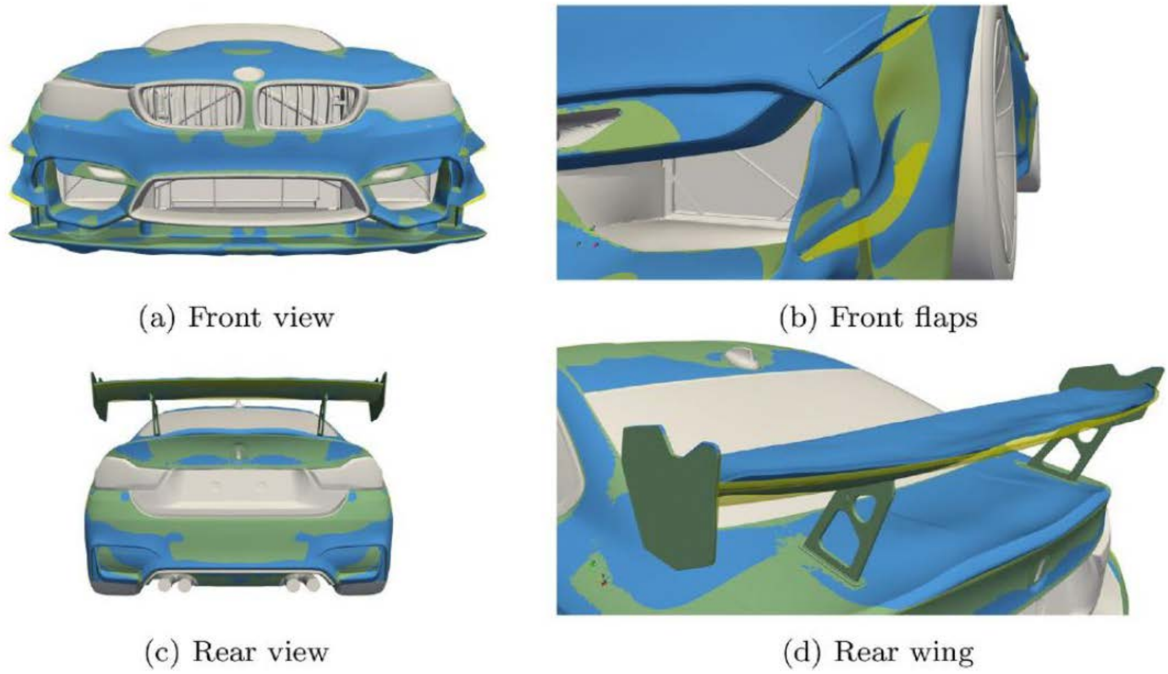


Figure 13: Form finding by shape optimization of a BMW racing car.

Conclusions

The development and application of form finding methods have a great tradition in architecture and civil engineering. Initially focused on the form finding of shells the interest has been extended to the form finding of tensile structures, the application of numerical techniques and on this basis to structural optimization in general and shape optimization in particular. In any case, the magic tool box does not exist. Form finding techniques only help to find new solutions but have to be guided with care by using the remaining procedural parameters. Form finding remains to be an art but the methods, chances, and challenges evolve with the acceptance of computational methods as modern tools. Vertex morphing appears as a very robust and successful method, intuitively to be applied in many different fields at the interception of design and engineering.

Acknowledgements

This paper summarizes some of our work and successes over the last 35 years in the fields of shape optimization and form finding. My greatest thanks go to all our collaborators and doctoral students during this time, as well as our supporters from authorities and practice. These include: DFG, EU, German and Bavarian governments, Airbus, BMW, MTU, Siemens, Volkswagen. Architecture and civil engineering are the core and inspiration for everything that drives us, and it's great to see how this works in other areas. It would be great if our friends at IASS shared our enthusiasm and found what we have achieved useful.

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