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Development and assessment of a spherical cap timber gridshell with compound beam sections

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Abstract

Elastic gridshells are an efficient family of structures. However, stability issues arise when they are employed at large scale. When stacking subsequently bent elements in superposed layers and mechanically coupling them together, a compound beam section is achieved that can provide sufficient cross-sectional capacity and structural stability to overcome these issues. The structural behaviour of such systems is highly non-linear with discrete force transfer, rendering the numerical modelling and analysis difficult. Existing strategies have proven accurate with negligible computational errors. However, these approaches are often cumbersome, and a more straight-forward approach is required especially at the early design stages. This paper follows the approach of modelling the individual lamellas and coupling them with spring elements with analytically evaluated spring stiffnesses. A structural test was performed on a spherical cap timber gridshell with compound beam sections, serving as a benchmark for the evaluation of two numerical models: one including all lamellas and one including only the continuous lamellas. The geometrical concepts that formed the basis for the design developments, the realization of the prototype as well as the structural and numerical results are presented in the paper. Finally, the results are discussed and evaluated against findings from other reference assessments.

Keywords: gridshell, spherical cap, architectural geometry, timber construction, elastic bending, structural analysis

1. Introduction

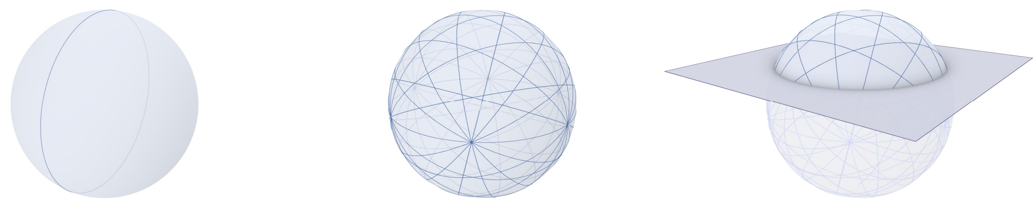
Elastic bending is a smart yet simple way of generating structural form and curvature. However, it has been shown that for elastically bent structures where the elements are laid tangentially on the design surface, these systems have structural limitations when scaled up. Due to the alignment of the elements, the final structure possesses limited bending stiffness (Mesnil and Baverel [1]), making snap-through buckling the decisive failure scenario. Increasing the second moment of inertia to compensate for these effects, as one would do in ordinary structures, is for elastically bent structures not possible, since the material strength and the bending curvature limits the admissible bending stiffness (Lienhard and Knippers [2]). These systems are therefore rarely seen as large-scale structures, and their realisation is rather confined to demonstrator-size pavilions. To go beyond experimental and temporary structures, and to be able to apply these systems at large scale, one is therefore forced to stack bent elements in superposed layers and mechanically couple them to achieve a compound section with sufficient structural capacity and stability. The structural behaviour of an elastically bent system is highly non-linear (La Magna et al. [3]). Furthermore, assessing the structural capacity of such systems is an intricate task, which gets even more complex with an increasing number of layers. These complexities make the analysis and the design of elastically bent structures highly advanced, consequently stalling their further development.

Serving as an initial benchmark for future developments, this paper presents the design and realization of a spherical cap timber gridshell and the numerical modelling for structural analysis together with performed structural tests.

2. Geometrical background and digital design

In the design of elastic gridshells that achieve their spatial form through elastic bending of initially flat planks, it is desirable to constrain the design to either a geodesic or an asymptotic curve network (Schling et al. [4]), to avoid bending around the strong axis and to ensure straight development (Mesnil and Baverel [1]).

Pioneering research on geodesic gridshells was initiated by Natterer (Natterer et al. [5]) and was later extended upon by Pirazzi and Weinand [6]. This category of structures provides both construction simplicity and design flexibility. A geodesic curve network can be created on any free-form embedding surface, unlike asymptotic curves, which are limited to surfaces with negative Gaussian curvature (Mesnil and Baverel [1]). Construction simplicity is however only guaranteed, when the torsion in the elements is kept to a minimum. Great circles are a special family of geodesic curves that fulfil this criterion, as they only exhibit normal curvature (Schling and Barthel [7]). The revolution of one great circle thus formed the base for the design surface of the spherical cap and consequently the layout of the curve network (Figure 1).

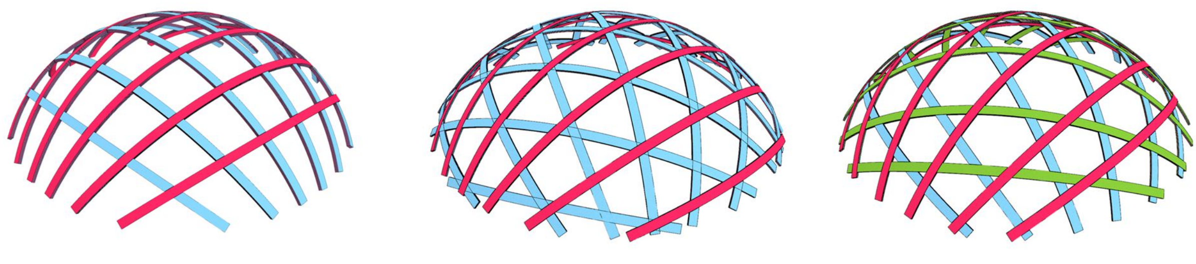


(a) (b) (c)

Figure 1: Design workflow of the gridshell; the base surface (a), the network of great circles (b) and the centre axes of the final spherical cap gridshell (c).

2.1. Curve network

The choice of curve network (Figure 2) has major implications on the structural behaviour and the construction of a gridshell. A quadrilateral grid “relies on bending stiffness of connections” (Mesnil et al. [8]) or need additional bracing to ensure structural stability, whereas a triangulated grid “benefits from a shell-like behaviour without the need for rigid connections”(Mesnil et al. [8]). However, for gridshells constructed from discrete beam elements, a triangulated grid entails more complex connection details in the nodes because of the higher node valency of six in comparison to four, as for quadrilateral grids. The lesser-known Kagome pattern is a pattern commonly found in basket weaving which has also found application in several architectural works. This curve network offers the same node valency as for quadrilateral grids but introduces triangulated segments, increasing the structural stability and stiffness and thus avoids the need for additional bracing.



(a) (b) (c)

Figure 2: The spherical cap gridshell with a quadrilateral (a), triangulated (b) and Kagome (c) beam network.

A gridshell with compound beam sections normally consists of continuous lamellas and shorter infill pieces, which are inserted sequentially after the continuous lamellas are erected. This assembly process is relatively straightforward and uncomplicated. Even though the node valency of a Kagome grid is four, the construction complexity of such a pattern with compound sections increases due to the introduction of half-continuous lamellas (Figure 3), which need to be erected in parallel with the assembly of the continuous lamellas. However, the structure benefits from the half-continuous layers, as they improve load transfer to the nodes.

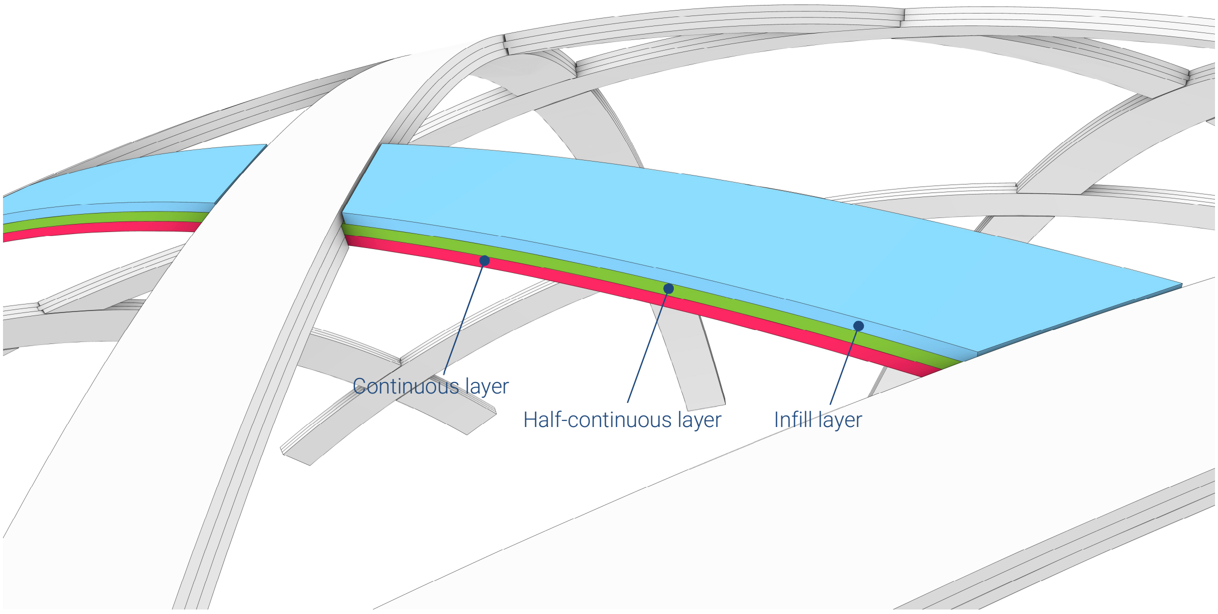


Figure 3: Compound build-up of the Kagome beam network.

3. Gridshell prototype

The presented gridshell prototype has the form of a spherical cap, with a span of 3,2 m and a height of 1 m. The gridshell comprises three main directions with a rotational offset of 60°, with four axes in each direction. Since there are three main directions, each axis needs three local layers of lamellas (one continuous-, one half-continuous- and one infill layer) to complete one global layer of the gridshell. The final gridshell consists of two global layers, i.e. six local layers of lamellas. The lamellas are produced out of silver fir, with a width of 80 mm and a thickness of 6 mm.

3.1. Construction process

The production process involved drilling the holes and cutting the lamellas to their correct length. The simplicity of the manufacturing steps made advanced machinery unnecessary. The entire production could therefore be executed using simple three-axis CNC milling. The complete set of finished lamellas could be easily flat-packed on a single pallet (Figure 4 (a)), facilitating the transportation from the production facility to the assembly site.

A base plate with supports for each axis (Figure 4 (b)) was meticulously prepared with a CNC machine to ensure precise spatial positioning of each end of each lamella, which formed the boundary of the structure. The exact positioning of the boundary was important to establish a fixed reference for the construction process. The first continuous lamellas in two of the three main directions could be easily erected. With the start of the third direction the half-continuous lamellas needed to be erected in parallel (Figure 4 (c)).

As all the continuous and half-continuous lamellas were erected (Figure 4 (d)), the infill pieces were inserted (Figure 4 (e)). Owing to the geometric layout of the Kagome grid, all infill pieces were wedge-shaped, which greatly simplified the insertion process. Lastly, the timber layers were clamped and screwed together using 4mm screws (Figure 4 (f)), with an approximate spacing of 10cm. To avoid cracks and splitting of the lamellas it was necessary to pre-drill the screw holes.

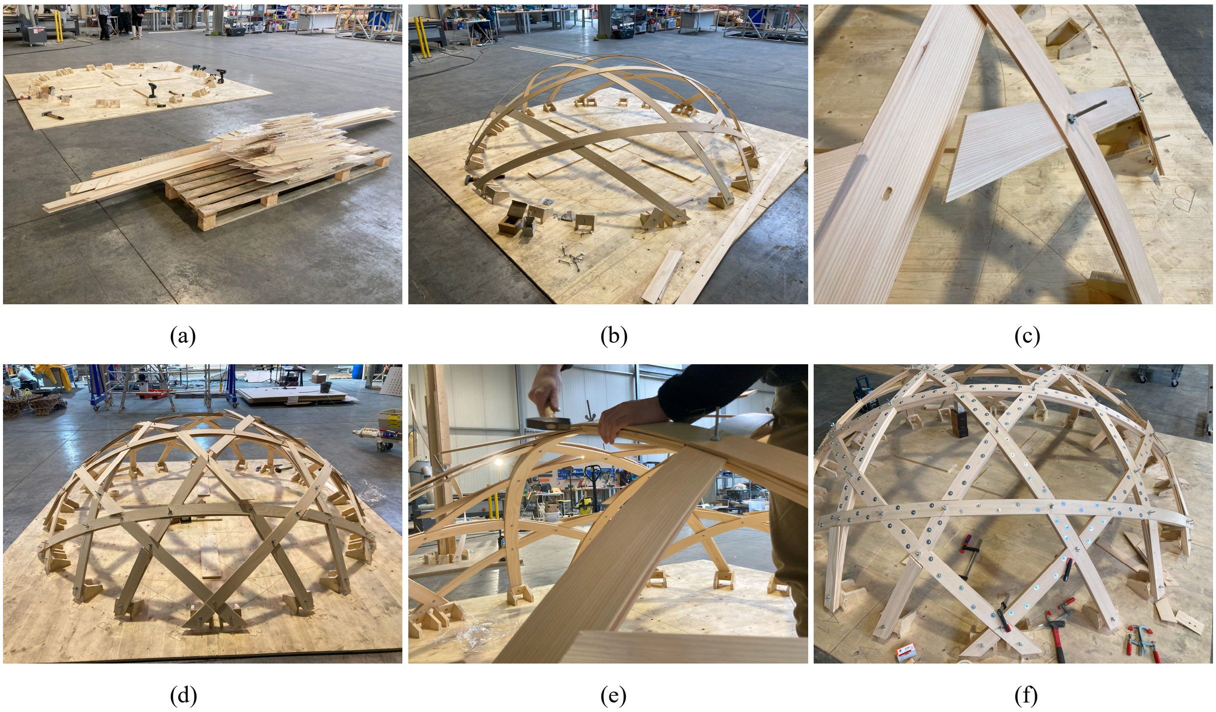


Figure 4: Construction process of the gridshell prototype.

4. Structural test

A structural test was performed on the gridshell prototype. The structural response of the gridshell was assessed by gradually applying a load to the six top-most nodes (Figure 5) while measuring the deformation. Continuous measurement of the deformation was not possible, therefore the load had to be incrementally applied with 1 kN load steps.

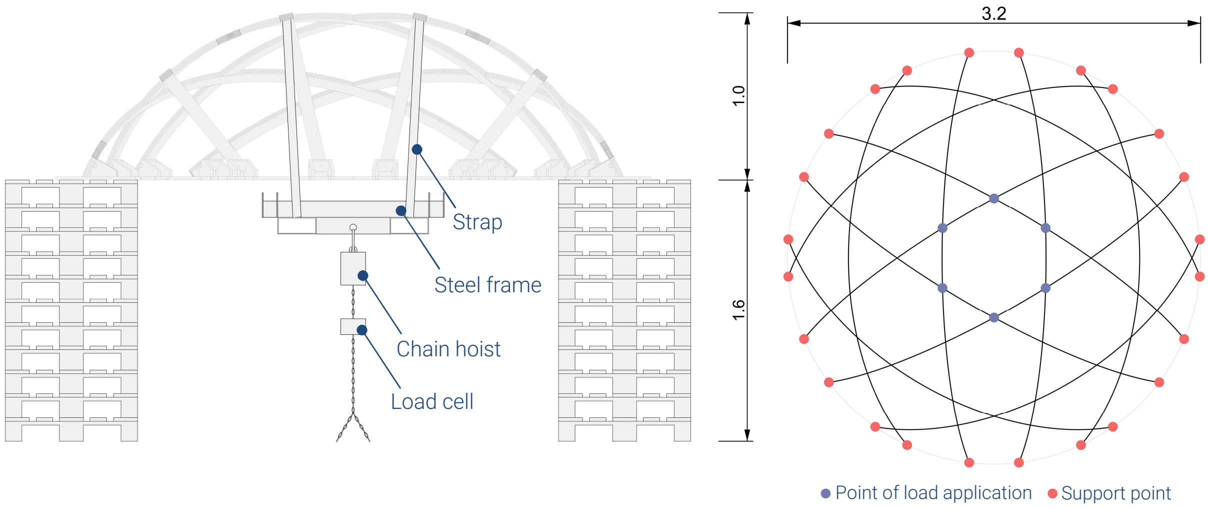


Figure 5: Diagram depicting the test setup (left) and a schematic illustration highlighting the points of load application (right).

4.1. Test setup

A chain hoist was anchored with four bolts to the ground and attached to the six top-most nodes of the structure via a stiff steel frame, ensuring a symmetric load distribution. A load cell with an external monitor was attached between the chain hoist and the ground, to be able to control the magnitude of the applied load. To ensure sufficient pulling distance for the chain hoist, the gridshell was elevated using pallets.

Figure 6: Set-up of the structural test.

4.2. Deformation measurement

To compare the gridshell's actual response with the numerical predictions, the deformation was measured by means of a scanning total station during the load test. Four signalized points attached at specified nodes of the gridshell were measured at each load step, whereas the entire gridshell was scanned at each fifth load step. Due to occlusions caused by the test set-up, two viewpoints were necessary for scanning the entire structure. To register the measurements into a joint coordinate system, a local reference frame was established by means of installed reflectors. To analyse the acquired point clouds, the individual beams were initially semi-automatically segmented. Afterwards, the data points representing individual beams were projected into their centre plane by means of a principal component analysis. Finally, the projected point clouds were approximated by means of best-fitting cubic B-spline curves (Harmening and Butzer [9], Harmening and Neuner [10]).

5. Numerical analysis

A strategy for numerical modelling of structures with compound beam sections developed by Gliniorz et al. [11], suggests representing each compound with two beam elements connected with inextensible bars. Although the strategy has proven accurate with negligible computational errors (Gliniorz et al. [11]), it involves translating the structural properties of the compound to equivalent bending and shear stiffnesses, which can be a cumbersome task.

A simplified modelling approach, where each individual lamella was modelled with beam elements and continuously coupled with linear springs, was adopted for the numerical analysis of the gridshell. To evaluate the influence of the modelling complexity against the accuracy of the structural response, two models were investigated; Model A where all six layers were modelled and Model B where only the two layers of continuous lamellas were modelled. In both models, the layers were coupled with spring elements in the position of the bolt at the overlap and along each segment in the actual position of the screws (Figure 7).

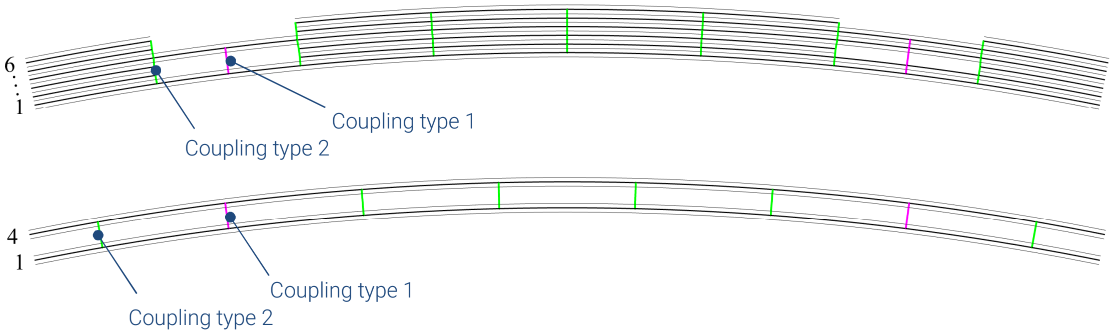


Figure 7: Modelled layers and corresponding coupling arrangement for the two models. Model A (top) included all continuous, half-continuous and infill layers from 1 to 6, while in Model B (bottom) only the continuous layers 1 and 4 were modelled.

The models were analysed in a non-linear geometric limit load routine with large deformations (SOFiSTiK A.G. [12]) where the load was gradually ramped up until the analysis diverged, to find the limit load at which the models were unstable.

The main material properties used to model the silver fir lamellas are summarised in Table 1:

Table 1 Material properties

	E_0 [N/mm^2]	E_{90} [N/mm^2]	G_{mean} [N/mm^2]	ρ_{mean} [kg/m^3]
Silver fir	9500	320	590	390

5.2. Definition of spring couplings

The couplings were defined according to the design recommendations in Eurocode 5 [13] for mechanically jointed beams. The slip modulus for dowel-type fasteners K_{ser} in N/mm per shear plane per fastener was calculated accordingly:

$$K_{ser} = \rho_m^{1,5} \frac{d}{23} \tag{1}$$

with *d* being the diameter of the fastener in mm and ρ_m being the mean density of the wood in kg/m³.

The lamellas were joined together in the overlaps with a 6 mm diameter bolt and screwed together with 4 mm diameter screws continuously along the lamellas with an approximate distance of 10 cm. Consequently, two coupling types were defined (Table 2). The axial stiffness was assumed to be infinite (Lara-Bocanegra et al. [14]) and rotation around the spring axis was allowed.

Table 2: Coupling spring stiffnesses.

	d (mm)	ρ_m(kg/m^3)	k_x [kN/m]	k_y [kN/m]	k_θ [kNm/rad]
Coupling 1	6	390	1000000	2000	0
Coupling 2	4	390	1000000	1330	0

5.3. Internal stresses due to elastic bending

Elastically bending elements into their final shape, leads to residual stresses that needs to be accounted for when assessing the structural capacity of the element. However, it has been shown that these effects can be neglected when following the recommendations in Eurocode 5 [13], stating that the minimum bending radius should be limited to 200 times the element thickness (Gliniorz et al. [11]). These recommendations were considered in the design of the gridshell prototype, where the minimum bending radius was approximately 300 times the lamella thickness.

Furthermore, other studies evaluated the influence of residual stresses due to form-giving deformations in elastic gridshells and these studies have shown that such stresses have limited influence on the buckling capacity of elastic gridshells (Mesnil et al. [8], La Magna and Knippers. [15]).

Therefore, an initial form-finding step was not included in the structural analysis.

6. Results

Both numerical models exhibit approximately 25% less buckling capacity than the actual capacity registered in the test of the built prototype. These discrepancies can be mainly traced back to the definition of the coupling conditions. To investigate how the coupling stiffness influences the structural behaviour, two more analyses per model were carried out: one where the shear stiffness of the coupling springs was reduced to 50% and one where it was increased to 200%.

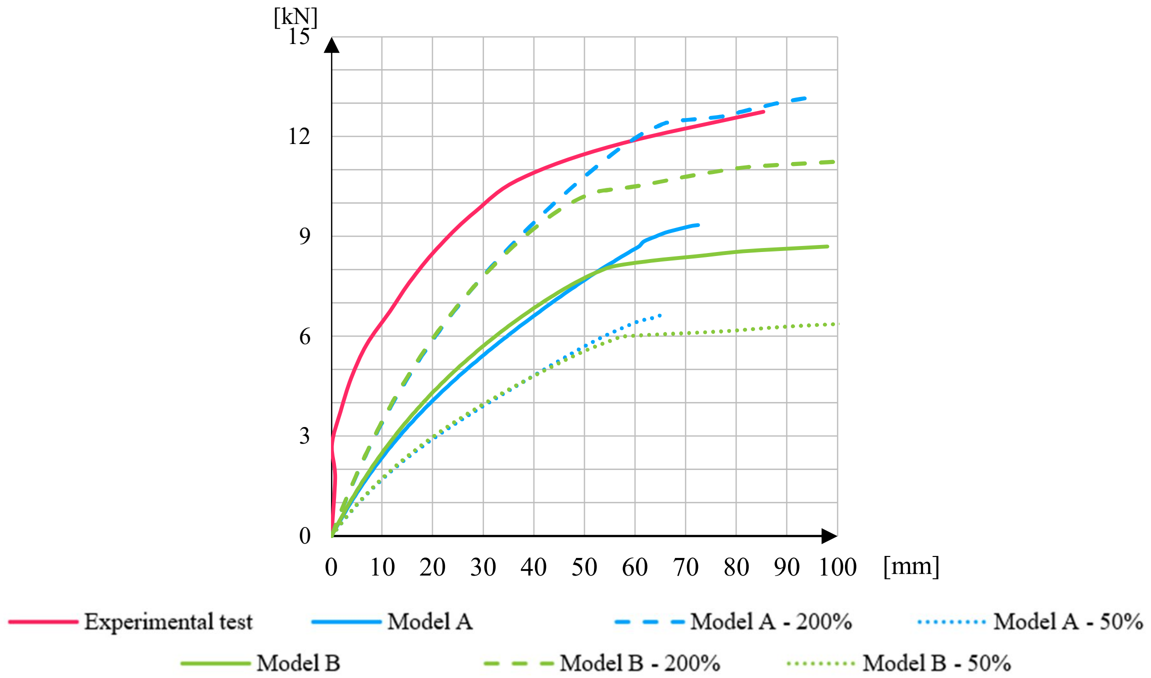


Figure 8: Load-deflection graph of the numerical models compared with the test result.

The results clearly show that both an increase and a decrease in spring stiffness has a direct influence on the buckling capacity. This indicates that simple spring coupling elements with analytically estimated spring stiffnesses is not sufficient. It is therefore necessary to model the couplings differently to assign the coupling conditions precisely and to conduct experimental tests to correctly assess the actual stiffness values of the shear planes.

It has been shown that non-continuous lamellas only partly transfer bending moments (Gliniorz et al. [16]) and thereby only implicitly contribute to the global structural stability by locally stiffening the continuous lamellas. This corresponds well with the numerical results, explaining why the inclusion of the non-continuous layers only has limited influence on the buckling capacity. A potential implementation of this knowledge in the design of elastic gridshells with compound beam sections could be to use an agile numerical model with only continuous layers in early design phases for an approximate assessment of the buckling capacity of the structure. For later design phases a more comprehensive model including all layers should be used for thorough analysis of the section forces and stresses in the individual lamellas.

4. Conclusion

The realization of the prototype presented the introduction of the half-continuous layer, as a result of the Kagome curve network. This made the construction process more challenging, however, it improves the load transfer in the structure, since two out of three lamellas are continuous over each node compared to only one out of three lamellas for a triangulated curve network. However, exactly how the half-continuous layer influences the structural behaviour and capacity is part of ongoing research work.

The presented system developments show advancements in the understanding of the production and construction of elastic Kagome gridshells with beam compound sections. Although, the simplified numerical modelling approach show consistent discrepancies when compared to the result from the structural test. Future research will focus further on investigating numerical modelling strategies to explore the full potential of these structural systems for large-scale architectural applications.

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