Geometrical appropriateness of Dieste's gaussian vault

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Abstract

Instability of structures in compression is an important design criterium. Especially for long-span structures that require lightest self-weight as possible, the resistance against instability is often sought from geometrical stiffness. The importance of geometry for spatial structures has been frequently demonstrated through past examples, and the current work had its initial interest in understanding structural behaviour of brick arch, specifically in relation to the work of Eladio Dieste. Gaussian vaults of Dieste have attracted many due to the geometries' intrinsic beauty, while the geometrical characteristics of the gaussian vaults have well-served its purpose for preventing possible buckling failure. Construction of more elaborate geometries can come with additional materials, time and labour for the constructions, and therefore, more systematic justification of such design must be in place.

Thus, the current paper present results of investigations in silico with the aim to discuss the geometrical appropriateness of gaussian vaults in comparison to comparable monotonous vaults, and the roof of J. Herrera Y Obes (JHO) warehouse building has been adopted for the current study. Firstly, change of buckling resistance in respect of varying sectional geometrical properties (transverse curvatures) is discussed. Secondly, the forms' sensitivity to geometrical imperfections are discussed based on the results of non-linear geometrical analysis using static Riks method. At last, the paper ends by discussing on the necessity of keeping the doubly curved form for the vault over more monotonous alternatives considering construction practicality.

Keywords: Long-span structures, Brick vault, Instability, Buckling, Spatial structures

Introduction

The roof of J. Herrera Y Obes (JHO) warehouse (Figure 1) building by Dieste is often appreciated by many for their geometrical expressiveness and aesthetics. The free-standing 'gaussian vault' of JHO has rise-to-span ratios below 0.15, and considered as a shallow structure, yet it is remarkable that the structure is only one brick-layer thick (130mm) and running 44.7m (Pedreschi & Theodossopoulos, 2007) (Pedreschi, 2014). Indeed, it is cheaper to increase the stiffness through geometry rather than materials, and it seemed the S-shaped sections were Dieste's intention to provide the stiffness against buckling; while essentially serving the architectural merit of bringing in the natural sunlight to the indoor space. The cross-sections create both negative and positive curvature (transverse curvature) along the span of an arch (longitudinal curvature), hence forming the Gaussian curvatures. In any cases, it must be emphasized that for long, shallow and slender sectioned compression structures, the aspect of geometrical correctness in both design and construction stages is of critical importance. During the design stage, the necessary calculations must correctly define the vertical shape and cross-sectional geometries to avoid collapses from instability (Virgin, et al., 2014). In the construction stage, realization of the designed structure within the acceptable tolerance is critical while understanding that such structures are geometrically sensitive (Cavalagli, et al., 2017) (Kassotakis, et al., 2021).

Gaussian curvatures of the vault may had been adopted for structural stability, despite the increased complexity to realize the required construction precision. However, it is not intuitive to understand how the curved sections are adequately contributing to the buckling resistance of the structure, while observing that the tilted-up S-shaped sections at the crown are eventually diminishes to simple rectangular sections near the supports, where buckling failures are more likely to occur. How readily can the actual contribution of curvature to the overall stability of structure be explained beyond the case of folding up a piece of pizza (Teffetani, et al., 2019)? Therefore, the presented investigation and discussions are in relation to the geometrical parameters of cross-sections (also as result of transverse curvatures), and their variations along the span. The impact of different rise-to-span ratio on stability of arch structures were shown significant in the past (Hu, et al., 2018), but the present work is in relation to the specific rise-to-span ratio of JHO roof structure. The presented work is carried out in silico, and the analysis was implemented on Abaqus (Dassault Systemes, 2019).

Figure 1. Julio Herrera Y Obes (JHO) warehouse, Montevideo (Images: © leonardo finotti)

Buckling Resistance and Geometrical Parameters of Cross-Sections

It is often mentioned in different design discourses that curvature of a body provides stiffness against deformation. Yet, it would be more relevant to state that, stiffness is measured in relation to the distribution of mass about the specific rotational axis in context. Therefore, curvature(s) over an area of a body do not provide understanding of, e.g. flexural stiffness about different axes.

One of the most common examples include curving of a piece of paper on one end preventing the other end from dropping down. There is a limit on the length that the curvature effect last, yet it demonstrates that, giving the transversal curvature on one end of the paper provides the longitudinal flexural stiffness for the paper to sustain the vertical height. In this case, the transversal curvature increases the moment of inertia, *I*, which is a geometrical property measuring the sections' resistance against deformation about the horizontal axis. So, the effect of curvature can be understood in the resultant *I* (Teffetani, et al., 2019). However, further question arises when a preliminary study showed that arches of curved sections (section with single curvature – yellow marks, with sine curve – blue marks) can have greater eigenvalues when compared with arches of rectangular sections with the equivalent *I* (Figure 2).

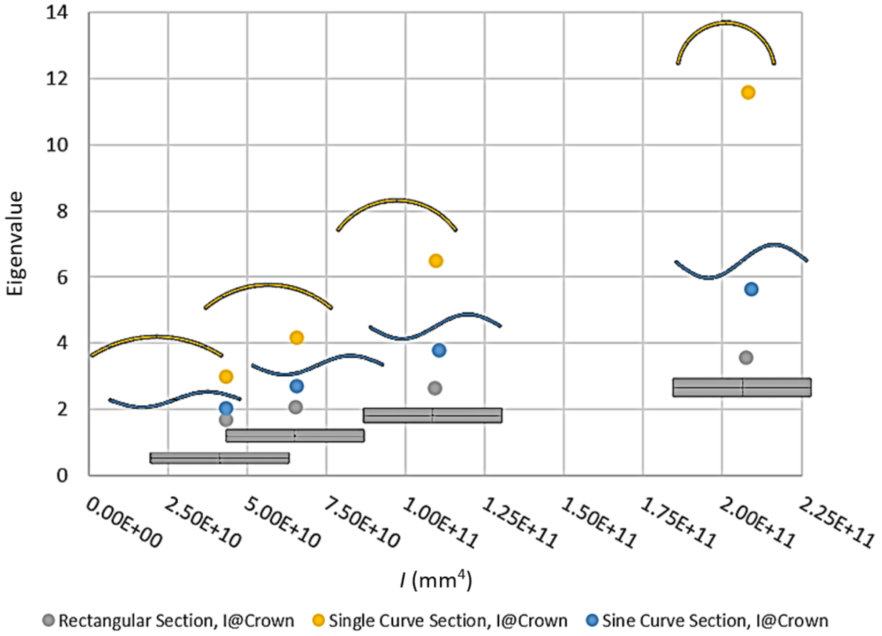

Figure 2. Comparison of arches with different section geometry

In consideration of curvature, the sine-curve sections have higher both average and maximum curvature than the one with single curvature, however, the buckling load is notably lower. The study showed that the buckling strength increases linearly with *I* values, at the same time, suggests that there are other parameters that influence the buckling strength of the structure. Therefore, an initial linear buckling analysis with 24 different arch designs is carried out in respect of the following parameters: *I* at the crown, Radii of Gyration (*G*), Length of Section (*W*), Thickness (*t*), Amplitude (*a*) and Eccentricity (*e*). The transverse curvatures were varied in relation to the eccentricity, *e* (Figure 3). The S-sections of JHO roof have varying curvatures, and both average curvature and maximum curvature were investigated for their significances. Arches with simple rectangular sections were also included in the analysis with *a* and *e* set to 0 and 0.5 respectively.

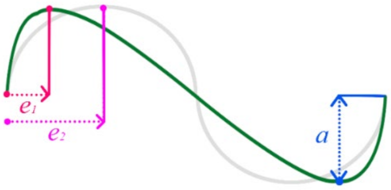

Figure 3. Description of a and e.

The investigation reviewed impacts of different parameters, and in general, the eigenvalues of the arches found to be inversely proportional to the sectional properties defined by G^a/I (Figure 4). The investigation initially used geometrical properties of the crown sections, however, later analysis showed better prediction using the geometrical properties of the critical sections near the supports, where buckling actually occurs. Based on Figure 4, one can approximate the possible buckling load for the structures, or perhaps define sectional geometry based on the design buckling load (e.g. in relation to self-weight for example). For $G^a/I < 10$ (or for the eigenvalues > 3), the discrepancy in the predictions increases with the eccentricity in section's geometry as defined in Figure 3. The general relation is that, buckling resistance of the arch drops when the sectional geometry becomes more eccentric (when e=0.5, the curve is symmetrical, and it becomes more eccentric as e moves away from 0.5, while $0 < e < 1$), and one must account for this in the approximation.

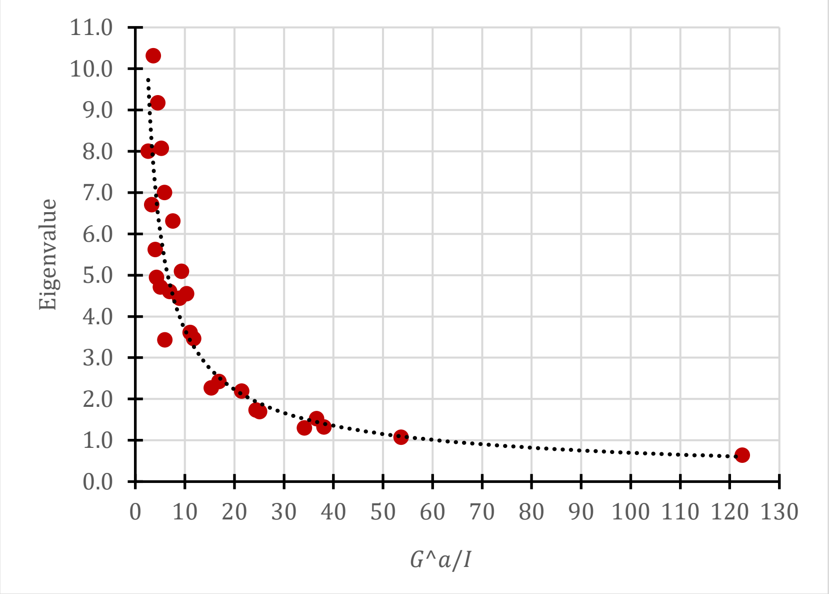

Figure 4. General trend of eigenvalues of the arches in relation to the calculated sectional properties of G^a/I.

Buckling behaviour of Arch in Monotonous Forms

The above implies that, the structures designed with sections without any transverse curvatures, but which hold the same calculated eigenvalues based on *G, a, I* and *e*, may have the comparable buckling strengths. Therefore, when construction practicality is prioritised, the sections can be designed in simpler monotonous forms while providing the same buckling strength as the roof of JHO. Figure 5 shows arch with three different sections were designed with simple three lines without any curvature in sections. Four arches (the first arch shown in Figure 5 is the roof of JHO reconstructed) have eigenvalues of 4.60, 4.44, 4.94 and 4.55 (in the order of left to right as shown in Figure 5) which are comparable to past simulation result (Pedreschi & Theodossopoulos, 2007). For the comparison, it was intended for the first (JHO roof) and second (monotonous form closely matching JHO roof) to have the same sectional properties. However, it was difficult to create the exactly matching properties, and the second structure had little higher value for G^a/I, hence the lower eigenvalue. Third arch has sections with greatest amplitudes, with the edges bent at sharpest angles resulting the highest *I* about the horizontal axis. However, with the smallest *e* value, the sections have the longest straight section in the middle. Thus, the sections have the lowest *I* about their minor-axis causing the out-of-plane buckling in different eigenmode (Figure 6). The second arch with low *I* about the minor-axis also showed an early sign of out-of-plane buckling. In fact, the tilt-up sections are susceptible to torsional deformations under vertical loads, and thus, may readily experience out-of-plane buckling in all four cases. However, the earlier occurrence of in-plane-buckling in JHO roof and the other monotonous arches may have prevented the out-of-plane buckling to kick in. In case of JHO, the continuous curvature seems to provide further resistance to out-of-plane buckling. It may be a possible explanation for, in case of JHO, the geometrical transformation is continuous between different sections due to the transverse curvature, which allows the varying stiffness of nearby sections to work together. On the contrary, the arches in monotonous forms with sharp changes in the transverse geometry can experience local stress and deformation developments which cannot be distributed over greater area. It is not yet clear how the curvature induced load distribution is beneficial for arch's buckling strength.

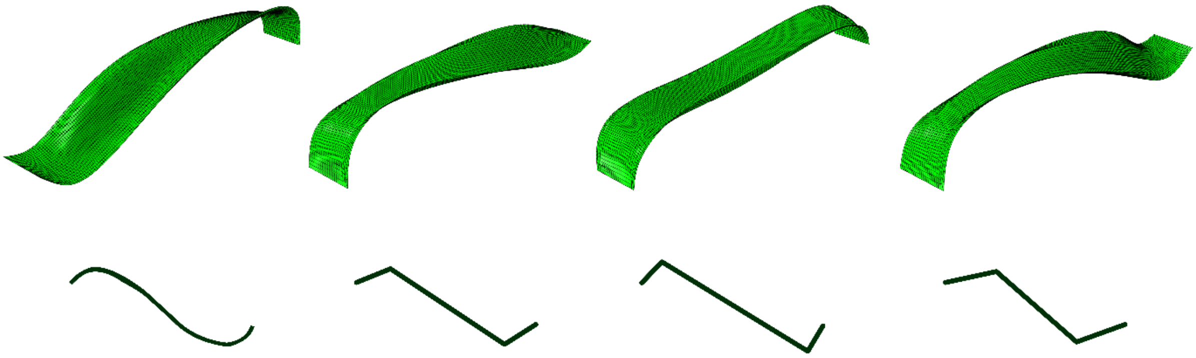

Figure 5. Arches with four different cross-sections and deformed shapes; the left-most is model of the JHO roof structure. The structures are indexed as 1st to 4th from left to right. All the four arches have similar eigenvalues.

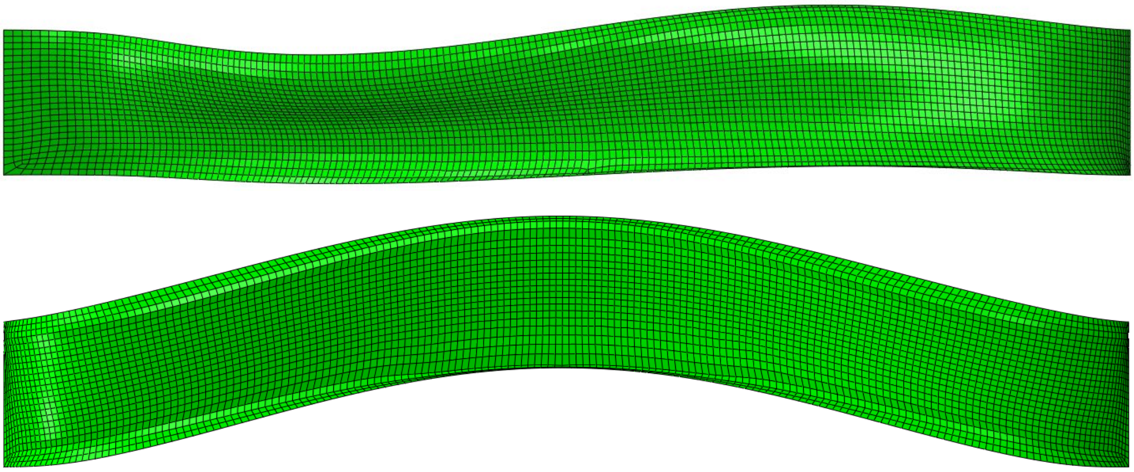

Figure 6. Plan views showing the torsional, but in-plane buckling of JHO (Top) and out-of-plane buckling of 3rd arch (Bottom)

Impact of Geometrical Imperfections

The impact of geometrical imperfection on buckling behaviour of JHO roof, and the comparable arch in the monotonous form were investigated. Imperfection amplitude applied was 65mm (t/2), which is acceptable in respect of what can be actually measured from real structures (Elferink, et al., 2016). The total load applied was 4.38 times the self-weight, and static Riks method was used for the geometrically non-linear analysis. The analysis result is shown in Figure 7.

Without imperfections introduced, the arches showed almost identical deformed shapes with buckled sections near the supports, though the monotonous arch showed much greater deflection. The maximum load of JHO roof was about 10% higher, though this is simply from lower G^a/I value. Interestingly, the monotonous form showed continual resistance to the applied load after the onset of buckling, while the load of JHO roof dropped at a faster rate.

After the geometrical imperfections were introduced, both structures showed lower strength; dropped by 18% and 8%, and increased in central deflections by 7% and 16% for JHO roof and monotonous form respectively. In summary, JHO roof was more sensitive to geometrical imperfections than the monotonous arch, while its central deflection was significantly less.

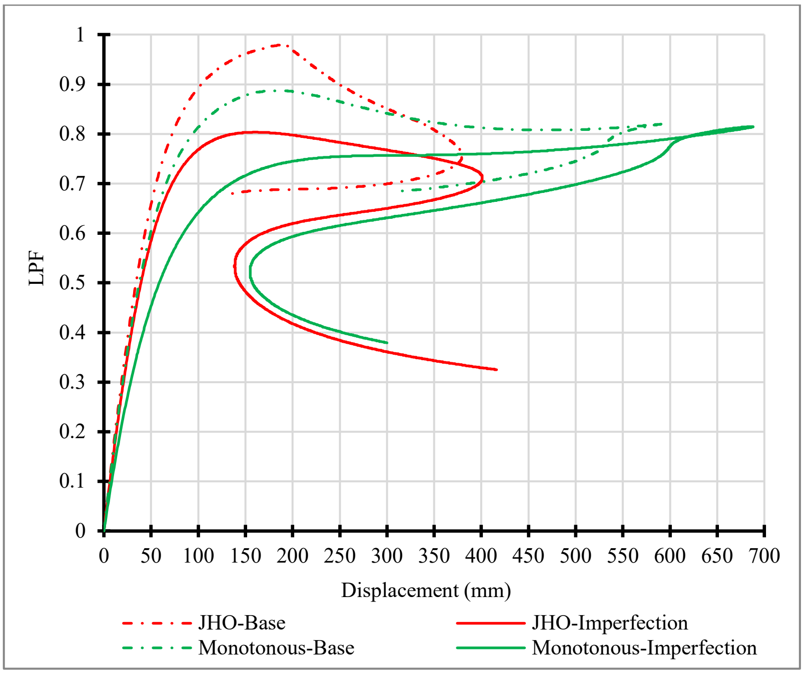

Figure 7. Central displacements of JHO roof (Red) and Monotonous form structure (Green)

Summary and Discussions

The initial buckling analysis for the listed geometrical parameters suggested that the transverse curvature was not a significant contributor to the overall strength of the arch. The equivalent monotonous arch without the transverse curvature could have similar buckling strength. However, the continuously curved form of JHO may have the advantage of distributing stresses over greater sections, preventing local stress concentrations and the resultant deformations. This can be the possible explanation behind its significantly lower deflections than the monotonous form.

The geometrical imperfection study showed that, JHO roof was more sensitive to the imperfection, which may occur in greater amplitude with increased difficulty in realising the construction precision. This may lead to adopting larger factor of safety. Furthermore, the maximum load is dropped to a lower level than the monotonous arch after the imperfection was introduced. However, the design choice of monotonous arch, for somewhat enhanced construction practicality and realising in better geometrical precision, is not immediately justified due to the significantly larger deflection, which is critical. Therefore, at this point of the research, the justification for gaussian curvature of JHO roof can be argued for its appropriate control over both the strength and deflection of the structure.

Limitation of the current investigation

The presented work is part of an on-going research that investigates into the effect of sectional geometry on buckling behaviour of shallow arch. The work is discussed in the specific context of JHO's roof structure, with the aim to provide discussions on impacts of different geometrical parameters through the relevant comparisons. On this basis, the presented work has been written in aware of the following limitations:

- The investigation is strictly restricted to width and rise-to-span ratio of JHO roof arches.
- Only the symmetric load condition is included in the investigation as part of the comparison study.
- Only the lowest eigen mode is considered in the investigation as part of the comparison study.

References

Cavalagli, N., Gusella, V. & Severini, L., 2017. The safety of masonry arches with uncertain geometry. *Computers and Structures,* Volume 188, pp. 17-31.

Elferink, B., Eigenraam, P., Hoogenboom, P. & Rots, J., 2016. Shape imperfections of reinforced concrete shell roofs. *Heron,* 61(3), pp. 177-192.

Hu, C.-F., Pi, Y.-L., Gao, W. & Li, L., 2018. In-plane non-linear elastic stability of parabolic arches with different rise-to-span ratios. *Thin-Walled Structures,* Volume 129, pp. 74-84.

Kassotakis, N., Sarhosis, V., Peppa, M. & Mills, J., 2021. Quantifying the effect of geometric uncertainty on the structural behaviour of arches developed from direct measurement and Structure-from-Motion (SfM) photogrammetry. *Engineering Structures,* Volume 230.

Pedreschi, R., 2014. Eminent Structural Engineer: Eladio Dieste - Engineer, Master Builder and Architect (1917-2000). *Structural Engineering International,* 24(2), pp. 301-304.

Pedreschi, R. & Theodossopoulos, D., 2007. The double-curvature masonry vaults of Eladio Dieste. *Structures and Buildings,* 160(1), pp. 3-11.

Teffetani, M., F, B. & Vella, A. N. a. D., 2019. Limitations of curvature-induced gravity: How a curved strip buckles under gravity. *A letters jounal exploring the frontiers of physics,* 127(14001).

Virgin, L., Wiebe, R., Spottswood, S. & T.G., 2014. Sensitivity in the structural behavior of shallow arches. *International Journal of Non-Linear Mechanics,* Volume 58, pp. 212-221.